ELSEVIER
BUTTERWORTH
HEINEMANN

First published 1987
Second edition 1995
Third edition 2006

ISBN-13: 978 07506 88796
ISBN-10: 0 7506 8879 3

British Library Cataloguing in Publication Data
A catalogue record for this book is available from the British Library.

Library of Congress Cataloging in Publication Data
A catalog record for this book is available from the Library of Congress.

The Publisher

The
Publisher's
policy is to use
paper manufactured
from sustainable forests

Printed in China

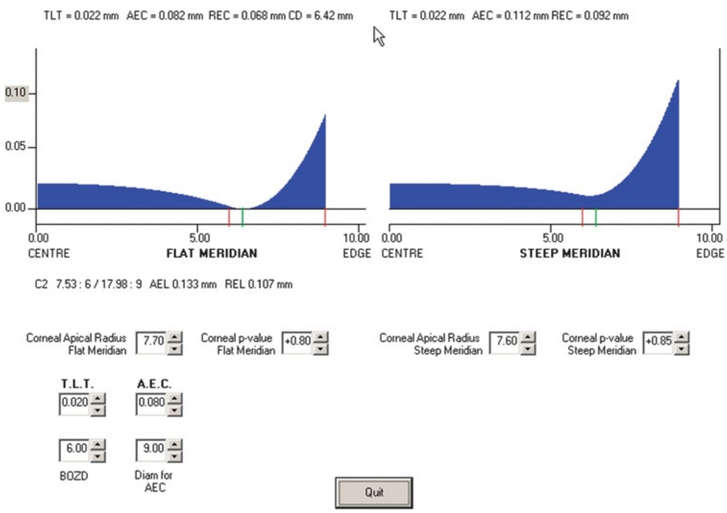
Preface

Clinical optometrists are quite rightly concerned with and interested in the pathophysiology of the visual system. In the area of contact lenses, this has generated an expertise in ocular complications of contact lens wear. Unfortunately, this admirable interest appears to have been accompanied by less enthusiastic application to the problems encountered in visual optics. This is a pity because optometrists are in a very strong position to claim the high ground in the visual optics arena. Much of what can be calculated is of direct use and relevance to the clinical optometrist. Lens thickness calculations, for example, will allow optometrists to devise and design lenses that maximize patient comfort and corneal oxygenation.

The mathematics required for most of the problems encountered is no more than basic trigonometry and algebra. The first chapter of the book will revive memories of basic visual optics and deals with topics like the effective power of a lens, ocular accommodation and retinal image size. The remaining chapters deal specifically with contact lens topics and the final chapter describes the contact lens programs that provide the clinical optometrist with the means to rapidly calculate solutions to clinical problems.

I am indebted to two colleagues who have given me significant help. The first is Henry Burek for his assistance in helping to resolve some of the more difficult topics. The second is Tony Hough for his help and advice, and for writing Chapter 8, which deals with the problem of presbyopia.

Bradford 2005 *Bill Douthwaite*



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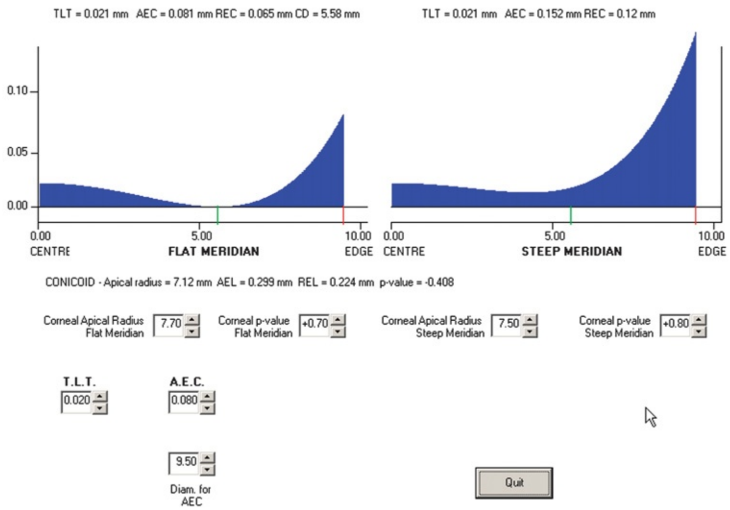

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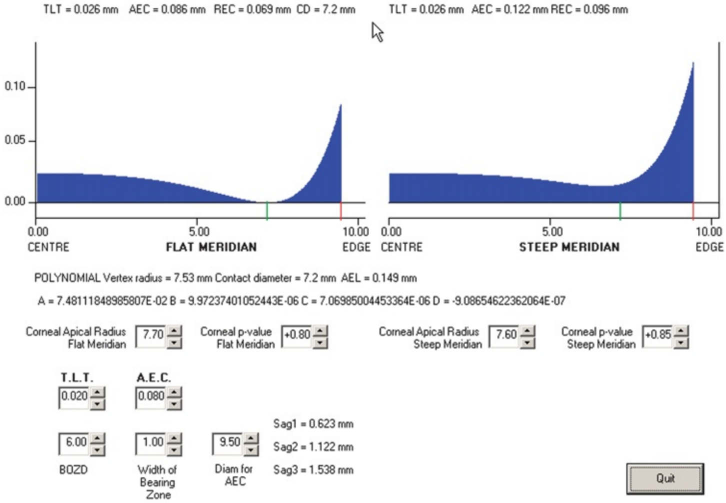

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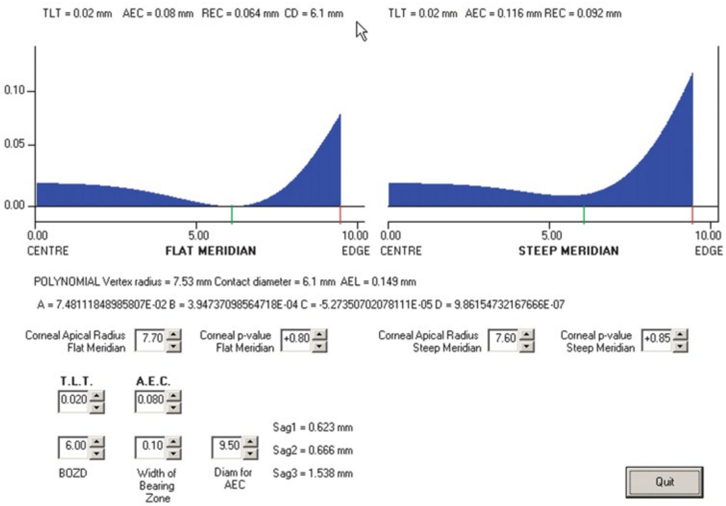

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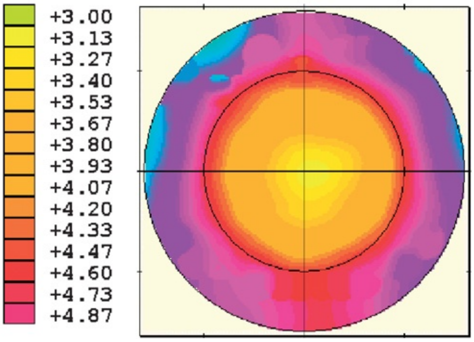

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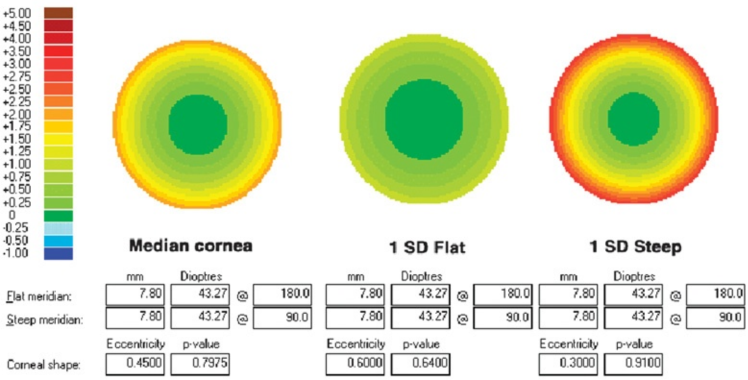

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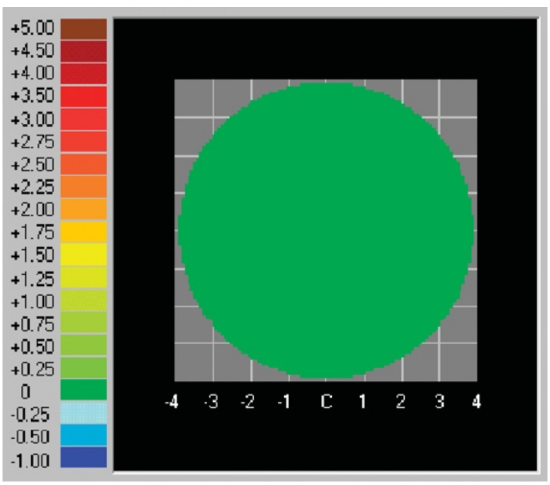

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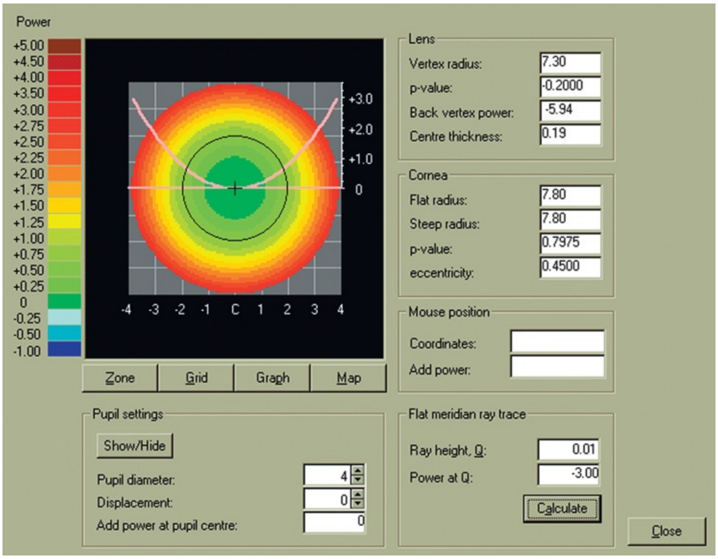

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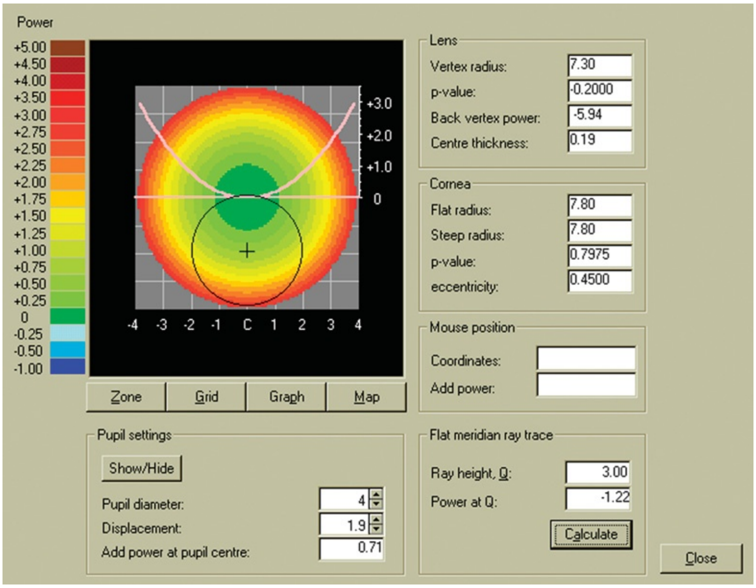

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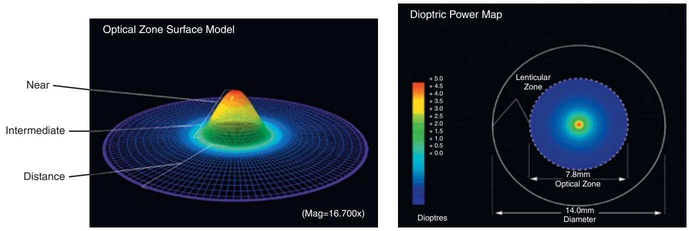

Focus Progressives power characteristics (promotional literature)

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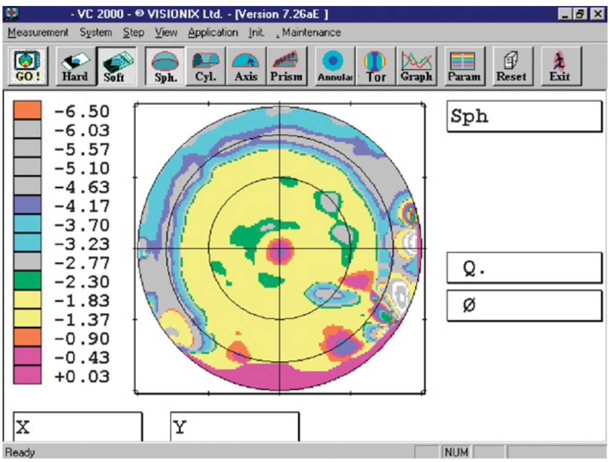

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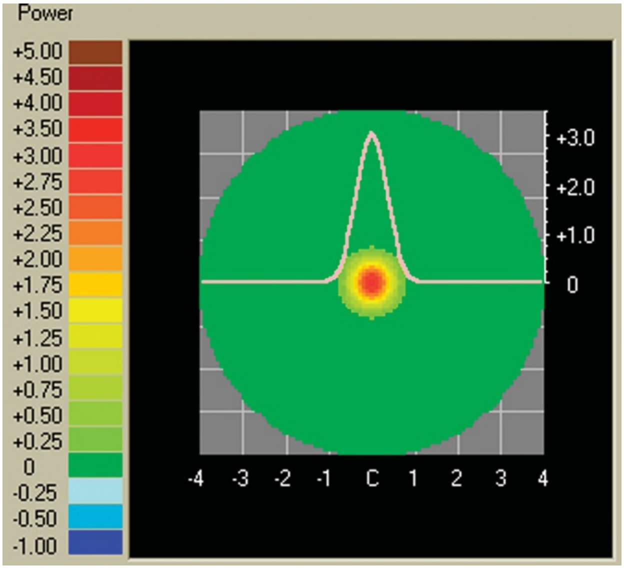

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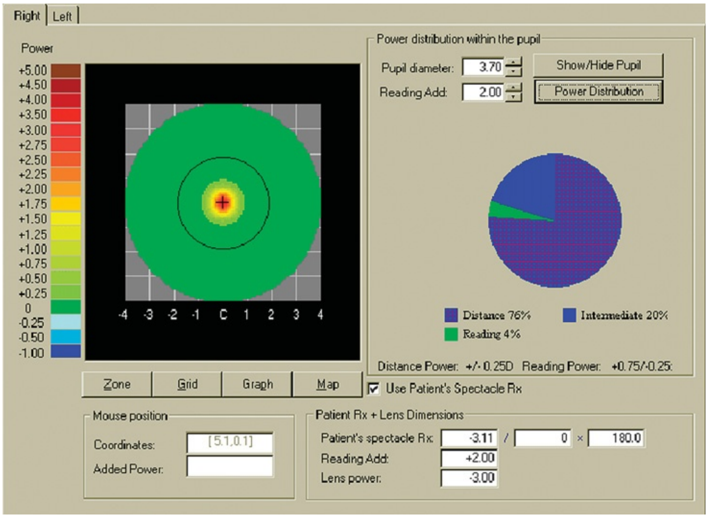

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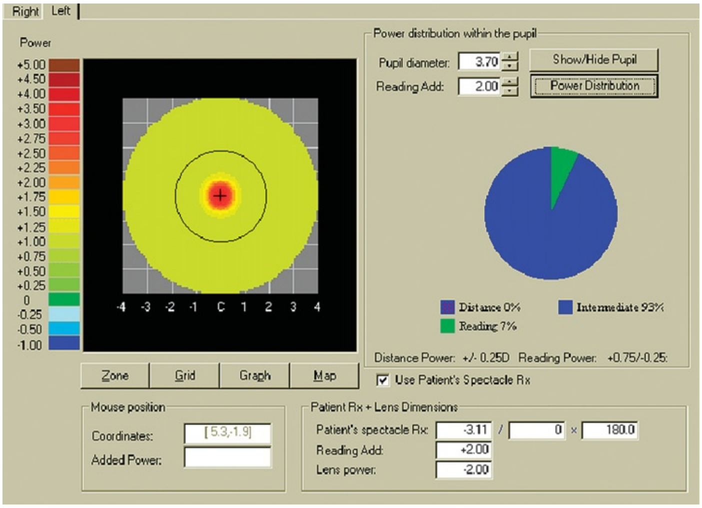

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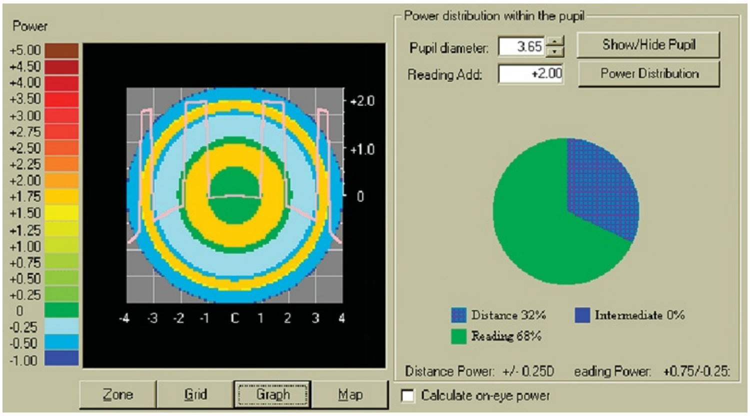

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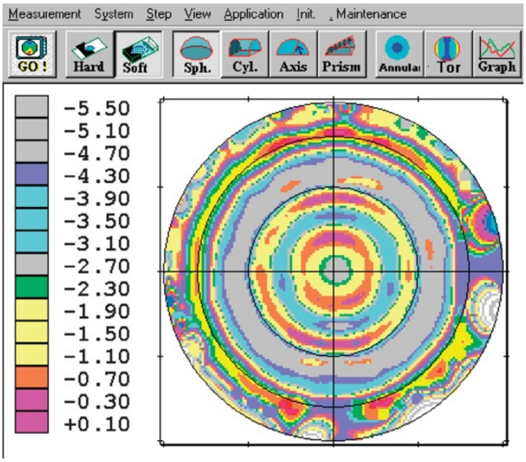

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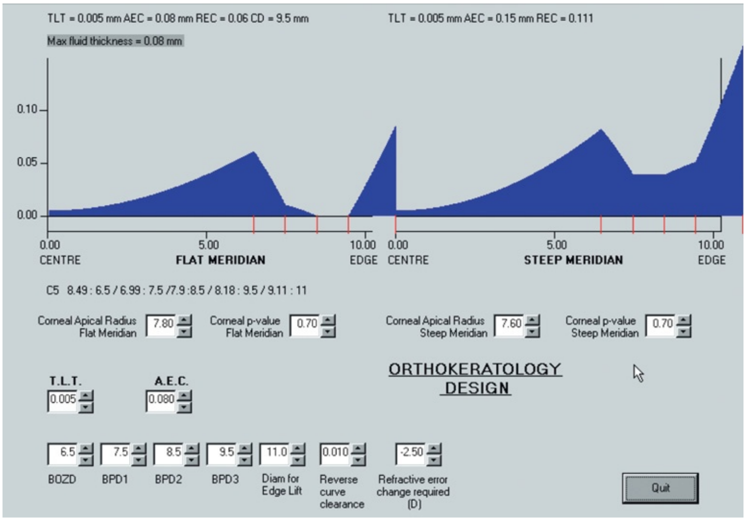

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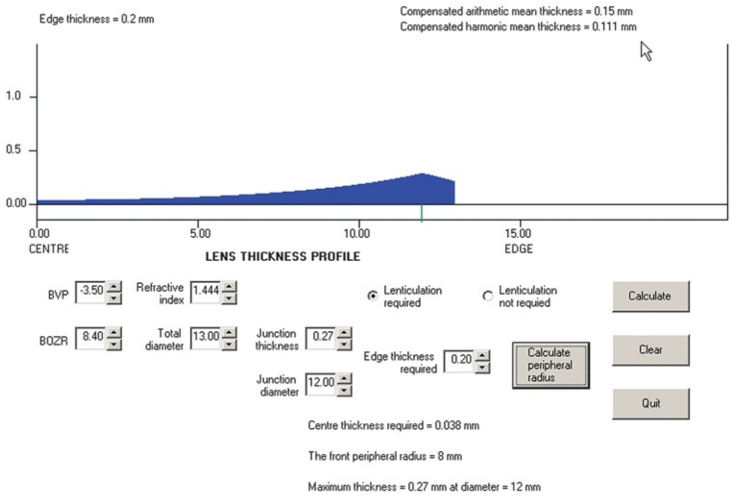

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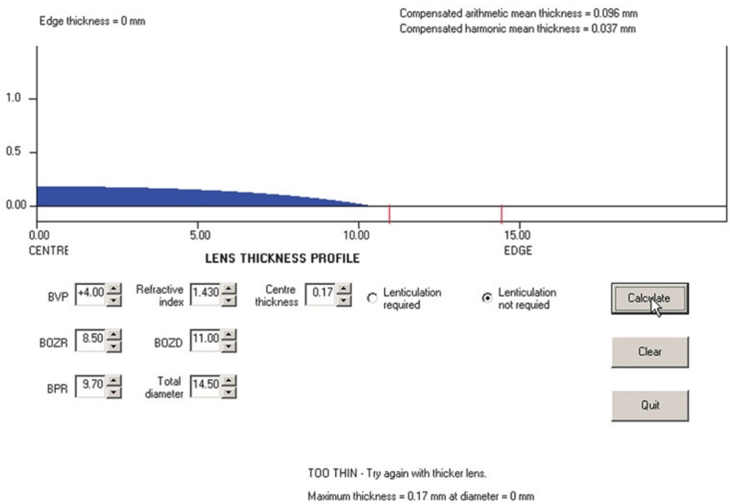

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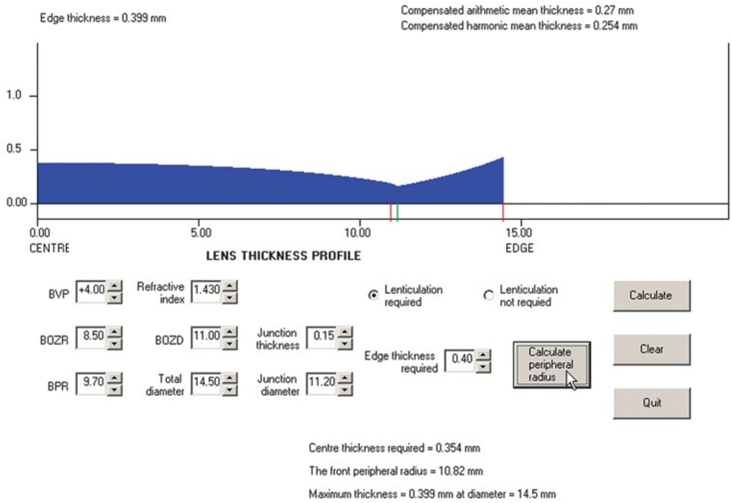

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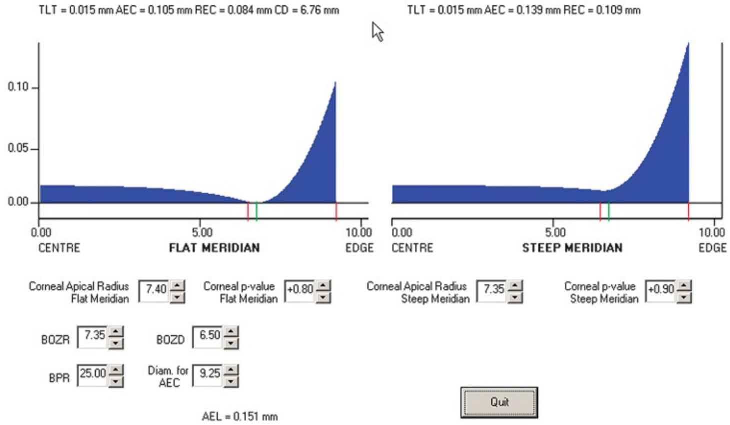

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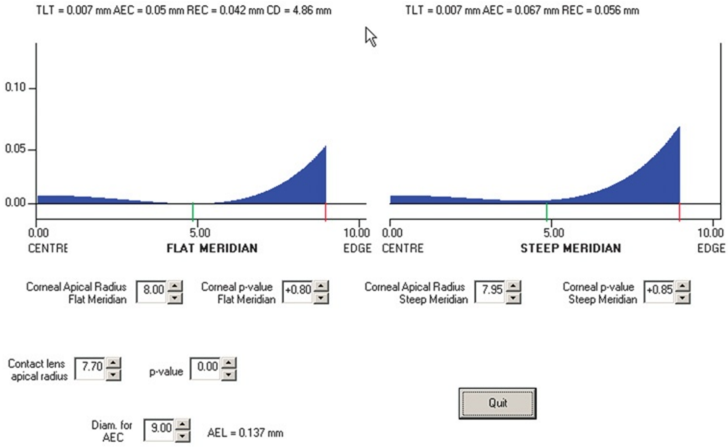

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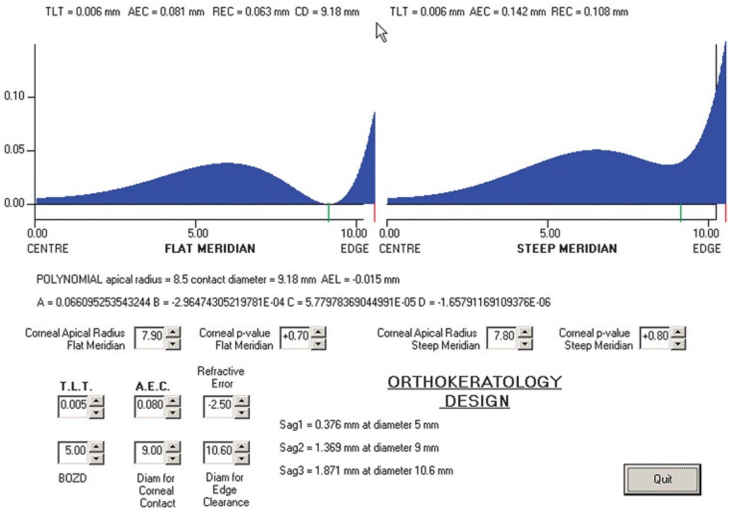

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CHAPTER 1

Basic visual optics

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1.1 Lens power and vergence

In Figure 1.1 the positive lens of power +1.00 D imparts convergence on the pencil of light rays from a point source at infinity. The vergence of light leaving the lens, vergence L_1, is +1.00 D. Since vergence is defined as the reciprocal of the distance (in metres) from a plane to the focal point, it follows that a point image is formed at a distance of 1 m from the lens. The vergence

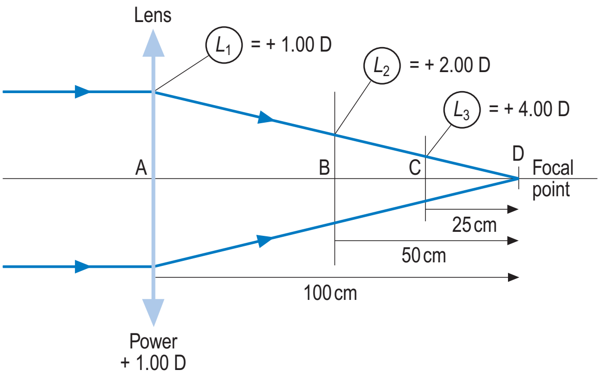

Figure 1.1 The relationship between distance and vergence. The vergences are labelled in numerical order from left to right as L_1, L_2, L_3 which represent the vergences at planes A, B and C respectively

(L_2) of light at plane B, which is 50 cm from the focal point, will be +2.00 D. The vergence (L_3) of light at plane C, which is 25 cm from the focal point, will be +4.00 D. Thus it is obvious that any vergence relates to only one plane.

It was decided to abandon the accepted convention using L_1, L_1', L_2, L_2' etc. because it is the author's experience of teaching visual optics over 25 years that this convention often leads to errors arising from neglecting to include a dash where one is required or inserting a dash in an inappropriate position, due to the similarity of labels like L_1 and L_1'. Thus throughout the book the vergences will be labelled L_1, L_2, L_3 etc. in numerical sequence, as in **Figure 1.1**.

In **Figure 1.1** the +1.00 D lens could be replaced by a +2.00 D lens at plane B or a +4.00 D lens at plane C. We can therefore state that the effective power of this lens at plane B is +2.00 D and the effective power of the lens at plane C is +4.00 D.

The distances measured in **Figure 1.1** are *from* the lens or the planes *to* the focusing point (D). In all three cases these distances are measured in the same direction as the direction of movement of the incident light and are in consequence considered as positive distances. They inevitably ensure that the pencil of light rays converge on one another. Convergence thus adopts the positive sign.

In **Figure 1.2** an eye is corrected by a +10.00 D spectacle lens placed 15 mm in front of the cornea. Light rays leave this lens with a convergence of +10.00 D. If this lens corrects the eye for distance, the light rays must be converging on the far point of the eye since the far point can be defined as the point conjugate with the retina. Thus the principal focus of the lens F′ must coincide with the far point of the eye M_R and the distance SM_R is 0.1 m. Here we are interested in the ocular refraction, that is we want to know the refractive error of the eye. As far as the *eye* is concerned, the vertex distance is measured *from the cornea* against the direction of the incident light

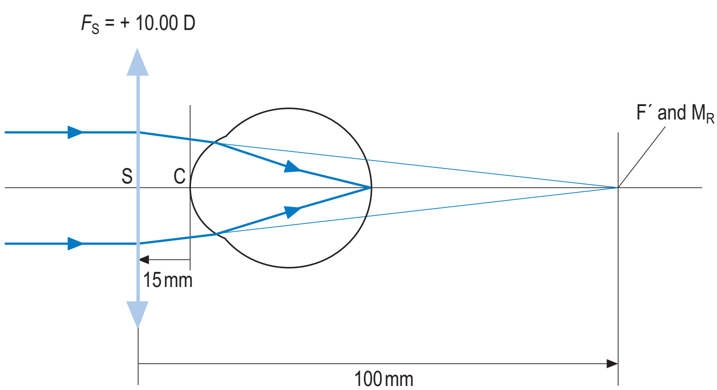

Figure 1.2 Spectacle correction of a 10.00 D hyperope. F_s is the power of the lens in the spectacle plane S. C represents the apex of the anterior cornea. M_R is the far point

rays and is therefore a negative distance. The effective power of the lens at C can be deduced as the reciprocal of the distance CM_R

$$\begin{aligned} CM_R &= 100 - 15 \\ &= 85 \text{ mm} \end{aligned}$$

Thus the effective power of the spectacle lens at C is given by

$$\begin{aligned} \frac{1}{0.085} &= \frac{1000}{85} \\ &= +11.76 \text{ D} \end{aligned}$$

Thus a hyperope will need a contact lens which is more powerful than her or his spectacle lenses.

Readers who are confused by the sign convention need only look at the diagram to see that CM_R is less than SM_R and can therefore deduce that the vertex distance must be subtracted.

In **Figure 1.3** an eye is corrected by a –10.00 D spectacle lens at S.

$$\text{Far point distance } SM_R = \frac{1}{-10} \; -100 \text{ mm}$$

Here the distance from the lens to the focal point is measured against the direction of incident light making it a negative distance. This results in a divergent pencil of light rays.

The vertex distance measured from C is also a negative distance. The algebraic addition of these distances results in

$$CM_R = -115 \text{ mm}$$

Again the reader need only look at the diagram to deduce that CM_R is negative with a value of 115 mm.

Thus the effective power of the spectacle lens at C is

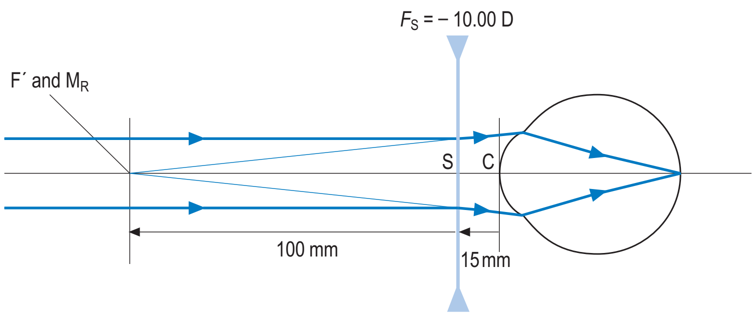

Figure 1.3 Spectacle correction of a 10.00 D myope. F_s is the power of the lens in the spectacle plane S. C represents the apex of the anterior cornea. M_R is the far point

$$\frac{1}{-0.115} = \frac{1000}{-115} - 8.70\text{ D}$$

A myope therefore will need a contact lens power which is less than her or his spectacle lenses.

From a clinical point of view this effect can be disregarded for spectacle lenses of power below 4.00 D, since a 4.00 D spectacle lens at a typical vertex distance has an effective power at the cornea of around – 3.75 D (myope) or +4.25 D (hyperope).

Chapter 11 includes a computer program which calculates the effective power at the eye of any spectacle lens at any vertex distance.

1.2 Accommodation

Clinically, accommodation is measured with a near-point rule and this is, strictly speaking, the spectacle accommodation. When dealing with higher degrees of refractive error, the spectacle accommodation may be an inadequate measurement. The problem is best illustrated by the following examples.

1.2.1 Example 1

A myope is corrected by a –6.00 D spectacle lens placed 14 mm from the cornea. What accommodation is exerted by the eye when looking at an object 400 mm from the spectacle plane

(a) when using the spectacle correction?
(b) when corrected by a lens placed in the plane of the cornea?

This example asks for the accommodation exerted by the eye. Strictly speaking, therefore, we must deduce the ocular accommodation and this is referred to the principal refracting plane of a schematic eye. The cornea is less than 2 mm in front of this plane. In order to simplify the treatment of the problem we will assume that the principal refracting plane and the cornea coincide. Strictly speaking we will be comparing spectacle accommodation with corneal accommodation. One other assumption that is made in this treatment is that the lenses involved are thin lenses.

In order to clarify the difference between spectacle and ocular accommodation, a single diagram is drawn below with a light ray tracking through the distance path (accommodation relaxed) above the optical axis, and a light ray following the near path (accommodation active making the eye power more positive) below the optical axis.

1.2.1.1 The spectacle lens

We measure spectacle accommodation with a near point rule, i.e. we note (in dioptres) the reciprocal of the shortest distance at which a target can be

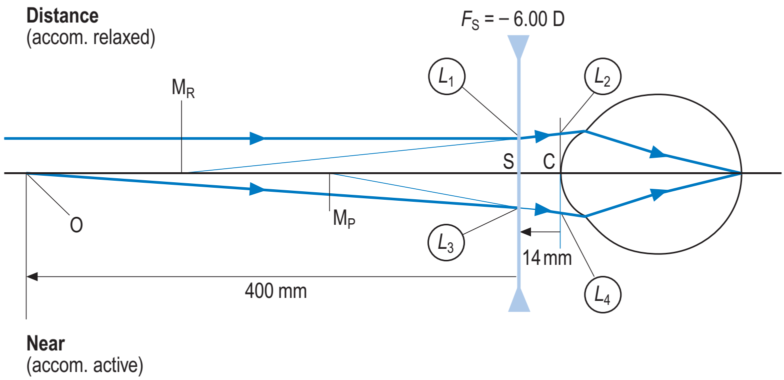

Figure 1.4 The spectacle correction in distance and near vision for a myope. The distant object is a point object at infinity. O is the point object for near vision. The symbols enclosed in circles are the vergences

clearly seen. The spectacle accommodation is the difference between the incident vergence on the spectacle lens subtracting the near vergence from the distance vergence.

In Figure 1.4

spectacle accommodation = vergence L_1 – vergence L_3
vergence L_1 = 0.00 D
distance OS = – 400 mm

$$\text{vergence } L_3 = \frac{1000}{-400} = -2.50 \text{ D}$$

Therefore

spectacle accommodation = 2.50 D

What we are really saying here is that, at near, the incident vergence on the spectacle lens has become more negative than the distance vergence by 2.50 D. If the light rays are still focused on the retina then the system must have become more positive (more converging) by this amount in the spectacle plane and this is the spectacle accommodation. This digression has nothing to do with the question but is included here for completeness.

The example asks for the accommodation exerted at the eye, and it does not take great powers of deduction to conclude that we need to concern ourselves with the incident vergences for distance and near at the eye rather than the spectacle lens.

Thus

accommodation at the eye = vergence L_2 – vergence L_4

Vergence L_2

distance $SM_R = \frac{1000}{-6} = -166.7$ mm

distance $CM_R = -166.7 - 14 = -180.7$ mm

Therefore vergence $L_2 = \frac{1000}{-180.7} = -5.53$ D

Vergence L_4

Incident light on the spectacle lens from the near object

(vergence L_3) $= \frac{1000}{-400} = -2.50$ D

Lens power $= -6.00$ D

Emergent vergence from the spectacle lens $= -8.50$ D

Therefore distance $SM_P = \frac{1000}{-8.5} = -117.6$ mm

distance $CM_P = -117.6 - 14 = -131.6$ mm

Therefore vergence $L_4 = \frac{1000}{131.6} = -7.60$ D

accommodation at the eye = vergence L_2 – vergence L_4
$= -5.53 - (-7.60)$
$= 2.07$ D

i.e. since the incident vergence at the eye becomes more negative by 2.07 D when changing from distance to near, the converging power of the eye must become more positive by the same amount, in the plane of the refracting surface, if the light rays are to maintain a focus on the retina.

1.2.1.2 The contact lens

Figure 1.5 illustrates the arrangement for a correcting lens in the plane of the cornea. Once again

accommodation at the eye = vergence L_2 – vergence L_4

Vergence L_2

contact lens refraction = vergence $L_2 = -5.53$ D

This was deduced in the previous section of this example and must be the back vertex power (BVP) of the correcting lens at the cornea.

Vergence L_4

distance OS $= -400$ mm
distance OC $= -414$ mm

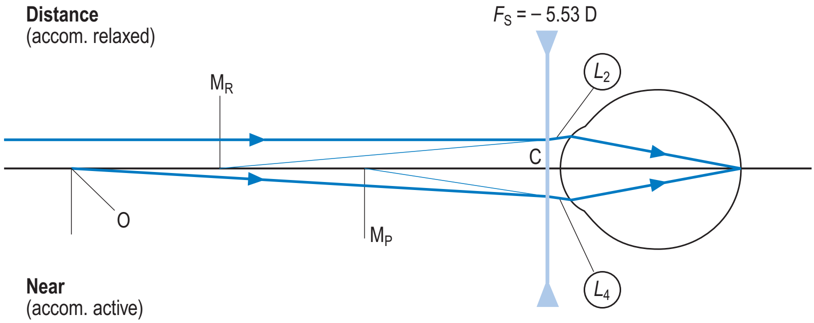

Figure 1.5 The contact lens correction in distance and near vision for a myope. The gap between the contact lens and the cornea is assumed to be an infinitely thin air gap and the contact lens is assumed to be a thin lens

Therefore

$$\begin{aligned}
&\text{incident vergence on the correcting lens} = \frac{1000}{-414} && = -2.42\ \text{D}\\
&\text{correcting lens BVP} && = \underline{-5.53\ \text{D}}\\
&\text{Therefore emergent vergence } L_4 && = -7.95\ \text{D}\\
&\text{vergence } L_4 && = \text{incident vergence on the cornea}\\
&\text{accommodation at the eye} && = -5.53 - (-7.95) = 2.42\ \text{D}
\end{aligned}$$

It is apparent from this example that myopes must accommodate more with contact lenses than when wearing a spectacle correction. It will be useful to repeat this exercise for the hyperope.

1.2.2 Example 2

A hyperope is corrected by a +6.00 D spectacle lens at a vertex distance of 14 mm. What accommodation is exerted by the eye when looking at an object 400 mm from the spectacle plane

a) when using the spectacle correction?
b) when corrected by a lens in the plane of the cornea?

1.2.2.1 The spectacle lens

Figure 1.6 illustrates that once again the spectacle accommodation is 2.50 D, i.e. the same as the value for the myopic eye

$$\text{accommodation at the eye} = \text{vergence } L_2 - \text{vergence } L_4$$

Vergence L_2

Since the lens power is +6.00 D, this must also be the emergent vergence from the lens at S. Therefore

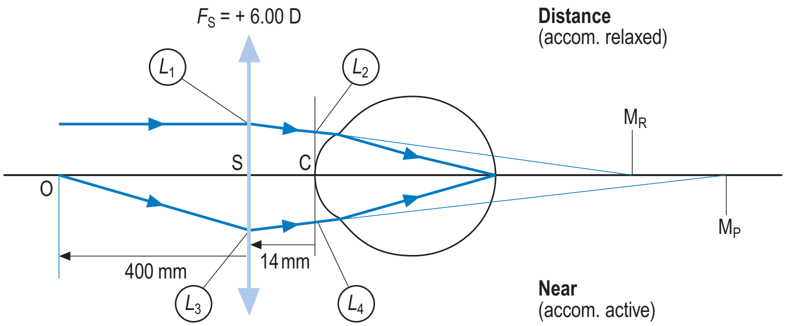

Figure 1.6 The spectacle correction in distance and near vision for a hyperope. The distant object is a point object at infinity. O is the point object for near vision. The symbols enclosed in circles are the vergences

$$\text{distance } SM_R = \frac{1000}{6} = 166.7 \text{ mm}$$

$$\text{distance } CM_R = 166.7 - 14 = 152.7 \text{ mm}$$

Therefore

$$\text{vergence } L_2 = \frac{1000}{152.7} = +6.55 \text{ D}$$

Vergence L_4

$$\text{vergence } L_3 = \frac{1000}{-400} = -2.50 \text{ D}$$

$$\text{lens power} = \underline{+6.00 \text{ D}}$$

$$\text{emergent vergence from the lens at S} = +3.50 \text{ D}$$

$$\text{distance } SM_P = \frac{1000}{3.5} = 285.7 \text{ mm}$$

$$\text{distance } CM_P = 285.7 - 14 = 271.7 \text{ mm}$$

Therefore

$$\text{vergence } L_4 = \frac{1000}{271.7} = +3.68 \text{ D}$$

$$\text{accommodation at the eye} = 6.55 - 3.68 = 2.87 \text{ D}$$

1.2.2.2 The contact lens

Vergence L_2

$$\text{vergence } L_2 = +6.55 \text{ D}$$

This was deduced in the previous section of this example and it gives the BVP of the correcting lens at the cornea (**Figure 1.7**).

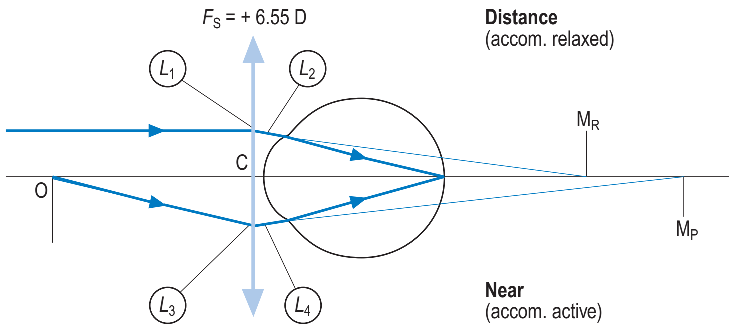

Figure 1.7 The contact lens correction in distance and near vision for a hyperope. The gap between the contact lens and the cornea is assumed to be an infinitely thin air gap and the contact lens is assumed to be a thin lens

Vergence L_4

$$\text{vergence } L_3 = \frac{1000}{-414} = -2.42 \text{ D}$$

correcting lens BVP = $\underline{+6.55 \text{ D}}$
vergence L_4 = +4.13 D
accommodation at the cornea = 6.55 – 4.13 = 2.42 D

N.B. This is the same value as that in the myopic eye when corrected by a contact lens.

1.2.3 Summary

A summary of the results is given in **Table 1.1**, which may help to clarify the picture.

The summary illustrates that, with contact lenses, both the myope and hyperope require the same accommodative effort at the eye. However, when wearing spectacles the effort is reduced for the myope but increased for the hyperope. Thus myopes need more accommodation for contact lens wear than for spectacle wear, hyperopes need less.

If the form and thickness of the lenses is taken into account by precise calculation, the results are found to be very similar for negative powers, with the discrepancy between ocular and spectacle accommodation increasing in the case of positive powers. The results above have assumed that the lenses are thin and this may be considered to be a close approximation.

1.3 Reduced thickness

In **Figure 1.8** a point object O is emitting light rays from the bottom of a glass block. The rays track towards the eye of an observer. Refraction occurs at the

Table 1.1 Summary of results

Correction	Accommodation at the cornea
Myope (spectacle correction)	2.07 D
Hyperope (spectacle correction)	2.87 D
Myope (contact lens correction)	2.42 D
Hyperope (contact lens correction)	2.42 D

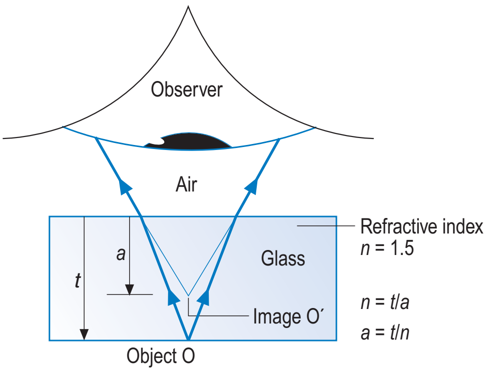

Figure 1.8 Refraction at a plane surface producing the apparent depth effect

upper surface of the block so the light appears to be coming from O′. Thus the real thickness t of the block has an apparent value of a as far as the observer is concerned.

The refractive index n of the glass can be deduced from the equation

$$n = \frac{\textit{real depth}}{\textit{apparent depth}} = \frac{t}{a}$$

Thus

$$\text{apparent thickness} = \frac{\textit{real thickness}}{\textit{refractive index}} = \frac{t}{n}$$

The value t/n is called the reduced thickness.

Considering the vergences in **Figure 1.8**. If the vergence of light from O at the upper surface of the block is $1/t$ and the surface has no power, then the emergent vergence is $1/t$. This is obviously incorrect.

The correct approach in these circumstances is to assume that the block has a reduced thickness t/n. We can then say that the vergence of light from O is n/t. The surface power is zero. Therefore the emergent vergence is n/t. This is obviously correct since we have just deduced that the apparent

thickness a is t/n and the emergent vergence dictates that the light appears to be coming from O′.

Alternatively we could state that the light within the glass block has an incident reduced vergence of n/t. This once again results in an emergent vergence of n/t.

Therefore either we consider

a) A reduced thickness – this is the most useful approach when dealing with contact lenses.

or

b) We use the concept of reduced vergences – this is convenient for light rays travelling within the eye.

1.4 Blur circles

Figure 1.9 can be used to illustrate how to calculate blur circle diameters. The reduced vergence of the light rays emerging from P is n/f'. Thus

$$F = \frac{n}{f'}$$

Therefore

$$f' = \frac{n}{F} = \frac{4}{3} \cdot \frac{1}{60} = 22.22 \text{ mm}$$

By similar triangles

$$\frac{\text{pupil diameter}}{f'} = \frac{\text{blur circle diameter}}{e'}$$

Therefore

$$\text{blur circle diameter} = \frac{\text{pupil diameter}}{f'} \cdot e'$$

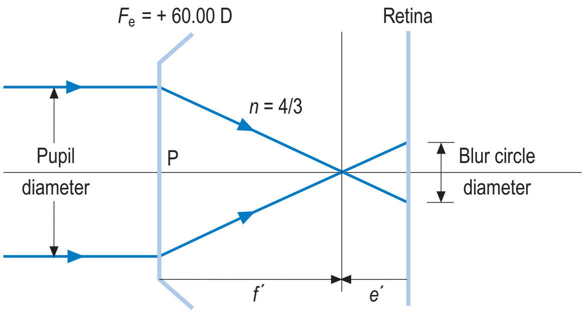

Figure 1.9 Blur circle formation on the retina of an axially myopic reduced eye of refractive power F_e and refractive index n

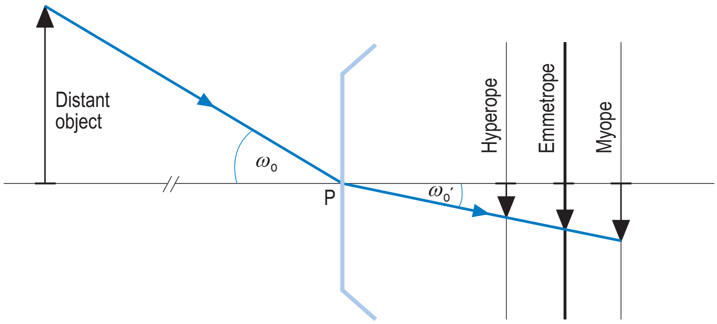

Figure 1.10 The basic retinal image size in uncorrected reduced eyes with axial ametropia. The light ray illustrated is the principal ray. The effect of the blur circles is not considered. ω_o is the visual angle of the distant object. This decreases to ω_o' after refraction at the reduced surface

1.5 Spectacle magnification

In **Figure 1.10**, a distant object is standing on the optical axis of the eye. Using the reduced eye, this object subtends a visual angle ω_o at P, the reduced surface. The light ray in **Figure 1.10** is the principal light ray which, after refraction at P subtends an angle ω_o'. This ray is used to determine the basic retinal image size. The assumption is made that the entrance and exit pupils coincide with the reduced surface P. The diagram illustrates that, for *axial* defects, the uncorrected basic retinal image size is largest for the myope and smallest for the hyperope. Since the image size variation is due to changes in axial length, it follows that in *refractive* defects the basic retinal image size does not alter with refractive error variations.

In order to calculate the actual retinal image size it is necessary to add one-half of a blur circle diameter to each end of the basic image, i.e. take the basic retinal image size and add one blur circle diameter.

1.5.1 The corrected eye

As shown in **Figure 1.11**, the spectacle lens which corrects the eye will produce an image (size h_1) of the distant object at the far point plane M_R. This image becomes the object for the corrected eye which will focus an image of height h_2 on the retina. In **Figure 1.11(a)** the positive correcting lens increases the unaided visual angle ($\omega > \omega_o$) and this results in an increase in image size with correction. In **Figure 1.11(b)** the negative correcting lens decreases the retinal image size ($\omega < \omega_o$). These diagrams illustrate that positive spectacle lenses make things appear larger, and negative lenses have the opposite effect. Also that the effect is increased by increasing the vertex distance.

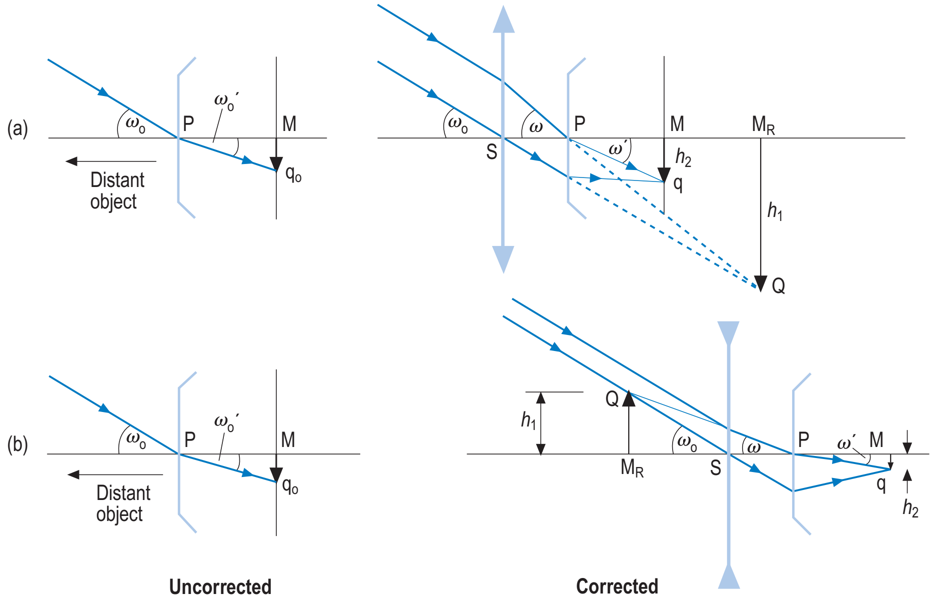

Figure 1.11 Change of retinal image size with the spectacle correction for (a) a hyperope and (b) a myope. The correcting spectacle lens produces an image (size h_1) of the distant object in the far point plane of the eye. This image becomes the object for the eye

Spectacle magnification is defined as

$$\frac{\text{retinal image size corrected}}{\text{retinal image size uncorrected}}$$

From Figure 1.11(a) and (b)

$$\text{spectacle magnification} = \frac{Mq}{Mq_o}$$

but

$$\tan \omega' = \frac{Mq}{PM}$$

and

$$\tan \omega_o' = \frac{Mq_o}{PM}$$

Therefore

$$\text{spectacle magnification} = \frac{PM \tan \omega'}{PM \tan \omega_o'} = \frac{\tan \omega'}{\tan \omega_o'}$$

For small angles

$$\tan \omega = \sin \omega$$

and

$$\text{refractive index } n = \frac{\sin \omega}{\sin \omega'} = \frac{\sin \omega_o}{\sin \omega_o'}$$

Therefore

$$\text{spectacle magnification} = \frac{\tan \omega / n}{\tan \omega_o / n} \cong \frac{\tan \omega}{\tan \omega_o}$$

$$\frac{\tan \omega}{\tan \omega_o} = \frac{M_RQ/PM_R}{M_RQ/SM_R} = \frac{M_RQ \cdot SM_R}{PM_R \cdot M_RQ} = \frac{SM_R}{PM_R}$$

Thus

$$\text{spectacle magnification} = SM_R/PM_R$$

If the distances PM_R and SM_R are measured in metres their reciprocals will be the ocular and spectacle refraction respectively. Therefore

$$\text{spectacle magnification} = \frac{1/PM_R}{1/SM_R}$$

$$\text{spectacle magnification} = \frac{\text{ocular refraction}}{\text{spectacle refraction}}$$

This equation applies to hyperopia and myopia, both axial and refractive. A contact lens correction results in the equation

$$\text{contact lens magnification} = \frac{\text{ocular refraction}}{\text{contact lens refraction}}$$

With the positions of S and P almost coinciding in contact lens wear, we can state that the spectacle magnification approximates to unity. Thus contact lenses do not alter the retinal image size significantly from that of the uncorrected eye. The foregoing is an approximation which arises from the use of the reduced eye. Strictly speaking, unity is achieved when the correcting lens is in the plane of the entrance pupil of the eye. The contact lens will be around 3 mm in front of the entrance pupil and it must be considered as a thick lens which is most certainly not flat. The fact that the lens is curved leads to displacement of the lens principal planes. Let us consider the influence of these factors.

1.5.2 The entrance pupil and the power factor

In **Figure 1.12** the distant object which stands on the axis of the system subtends an angle ω_o at the contact lens at C. If the eye were removed from the diagram this lens would form an image h' in the focal plane of the lens. The light ray in **Figure 1.12** passes through the optical centre of the lens, it therefore continues undeviated to emerge at an angle ω_o to the optical axis. However, when the eye is as shown in the diagram, h' becomes the object for

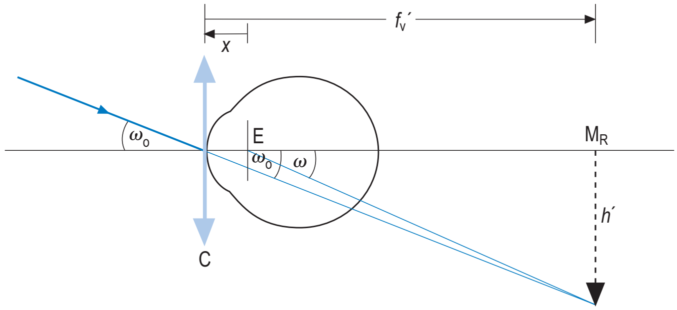

Figure 1.12 Spectacle magnification of a contact lens. The visual angle of the distant object is ω_o. The correcting contact lens produces an image at the far point plane. This becomes the object for the eye and subtends a visual angle of ω at the eye's entrance pupil (E)

the eye in the far point plane M_R. The corrected visual angle is ω measured at the eye's entrance pupil. From the previous definition of spectacle magnification we can deduce

$$\text{spectacle magnification} = \frac{\text{corrected visual angle}}{\text{uncorrected visual angle}}$$

Thus

$$\text{spectacle magnification} = \frac{\omega}{\omega_o}$$

$$\tan \omega_o = -h' / f_v' \text{ (image is inverted)}$$

and

$$\tan \omega = \frac{-h'}{fv' - x}$$

Therefore

$$\text{spectacle magnification} = \frac{-h'/(fv' - x)}{-h'/fv'}$$

$$= \frac{fv'}{fv' - x}$$

Dividing through by f_v'

$$\text{spectacle magnification} = \frac{1}{1 - xFv'} \qquad (1.1)$$

where F_v' is the back vertex power (BVP) of the contact lens and x is the distance from the lens back vertex to the entrance pupil of the eye (in metres).

The value $\dfrac{1}{1 - xFv'}$ is called the *power factor* of the lens.

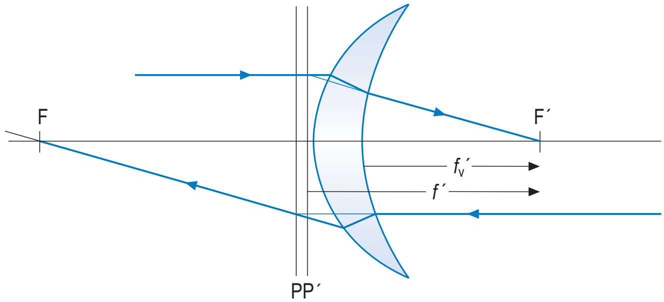

Figure 1.13 The principal planes (P and P′) of a thick curved lens. f_v' is the back vertex focal length. f' is the equivalent focal length

1.5.3 The principal planes and the shape factor

In a thick lens the planes P and P′ can be located by ray tracing as in **Figure 1.13**. The plane P′ is of interest to the lens wearer since, for a parallel pencil of light coming from a distant object, the emergent light from the back surface of the lens follows the same track as that which emerges from a thin lens of focal length f' (equivalent focal length) placed in the plane P′. Thus the contact lens could be replaced by this thin lens at P′. The fact that the thick positive lens is curved results in P′ being further from the eye and therefore increasing the spectacle magnification and hence the retinal image size.

1.5.3.1 The shape factor

If we assume in **Figure 1.14** that the contact lens is thin and of focal length f_v' then we can see that the image size at the far point of the eye is h_1. However, the thick curved contact lens will behave like an equivalent thin lens placed at P′ of focal length f' (the equivalent focal length). Thus the contact lens actually produces an image size h_2. The ratio of these two image sizes is called the shape factor. Thus

$$\text{shape factor} = \frac{h_2}{h_1}$$

$$\tan \omega_o = \frac{-h_1}{fv'}$$

and

$$\tan \omega_o = \frac{-h_2}{P'}$$

Therefore

$$\text{shape factor} = \frac{-f' \tan \omega_o}{-fv' \tan \omega_o} = \frac{f'}{fv'}$$

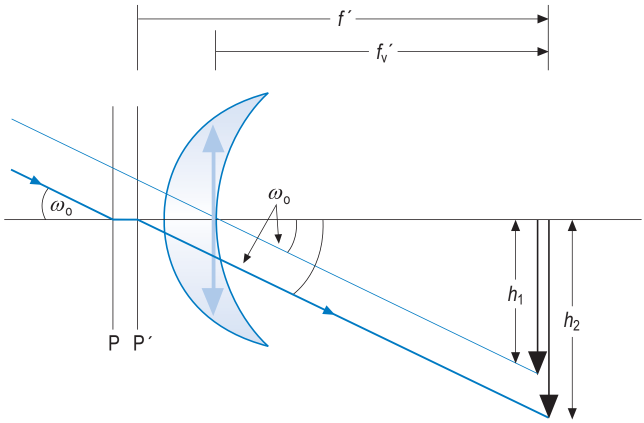

Figure 1.14 The shape factor. This diagram compares the image sizes produced by a thin flat lens and a thick curved lens with the same back vertex focal length. The object is standing on the axis of the system at infinity producing a visual angle ω_o

If these distances are expressed in metres, then their reciprocals give the lens powers in dioptres. Thus

$$\text{shape factor} = \frac{Fv'}{F'} \tag{1.2}$$

where F_v' is the back vertex power and F' is the power of the equivalent thin lens at P′.

For a single thick lens we can use the standard thick lens expressions

$$\text{back vertex power } F_v' = \frac{F_1 + F_2 - (t/n)\,F_1F_2}{1-(t/n)\,\mathrm{F}_1} \tag{1.3}$$

and

$$\text{equivalent power} \quad F' = F_1 + F_2 - (t/n)\,F_1F_2 \tag{1.4}$$

where F_1 is the front surface power, F_2 is the back surface power and t/n is the reduced central thickness of the lens. Substituting equations (1.3) and (1.4) into (1.2) we arrive at

$$\textit{shape factor} = \frac{1}{1-(t/n)\,F_1} \tag{1.5}$$

1.5.4 The fluid lens

We may also need to consider the influence of the fluid lens. This can be done by calculating the equivalent power of the contact lens and fluid lens in combination. Suppose in **Figure 1.15** that

$F_1 = +54.69$ D, $F_2 = -19.25$ D, $F_3 = -43.08$ D,
$t_p = 0.6$ mm, $t_f = 0.4$ mm, $n_p = 1.490$, $n_f = 1.336$

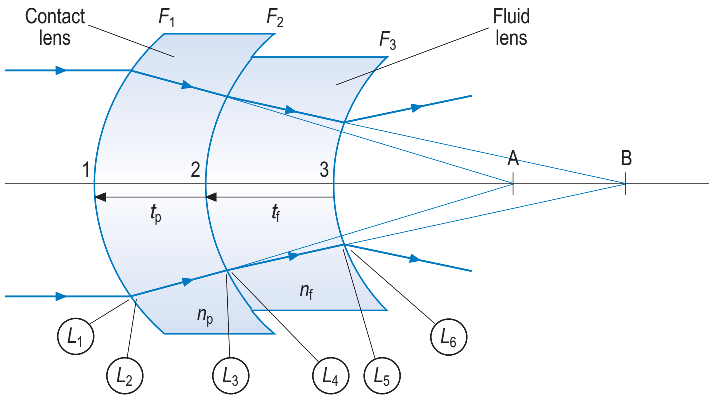

Figure 1.15 The shape factor for a contact lens in combination with a thick fluid lens. F_1, F_2 and F_3 are the surface powers. t_p and t_f are the centre thicknesses of the contact and the fluid lenses. n_p and n_f are the two refractive indices

Using the 'step along' method which converts vergences to distances, alters the distances and converts them back to vergences we have

Vergence (D)				**Distance (mm)**
$L_1 = 0.00$				
$F_1 = \underline{+54.69}$				
$L_2 = +54.69$	$\longrightarrow$	$\frac{1000}{54.69}$	$\longrightarrow$	$18.28 = 1A$
				$\underline{-0.4} = t_p/n_p$
$L_3 = +55.93$	$\longleftarrow$	$\frac{1000}{17.88}$	$\longleftarrow$	$17.88 = 2A$
$F_2 = \underline{-19.25}$				
$L_4 = +36.58$	$\longrightarrow$	$\frac{1000}{36.58}$	$\longrightarrow$	$27.26 = 2B$
				$\underline{-0.3} = t_f/n_f$
$L_5 = +37.09$	$\longleftarrow$	$\frac{1000}{26.96}$	$\longleftarrow$	$26.96 = 3B$
$F_3 = \underline{-43.08}$				
$L_6 = -5.99$				

Again the shape factor is $\frac{Fv'}{F'}$ and $L_6 = F_v'$

F' can be deduced from the general thick lens equation

$$F' = F_1 \frac{L_4 L_6}{L_3 L_5} \tag{1.6}$$

$$F' = +54.69 \; \frac{(+36.68)\;(-5.99)}{(+55.93)\;(+37.09)}$$

$$= -5.79 \text{ D}$$

Therefore, for a thick contact lens in combination with a thick fluid lens

$$\text{the shape factor} = \frac{-5.99}{-5.79} = 1.035$$

1.5.5 Total spectacle magnification

The total spectacle magnification must be the product of spectacle magnification (assuming thin lenses) and the shape factor of the system.

Total spectacle magnification = power factor × shape factor

1.6 Relative spectacle magnification

Relative spectacle magnification is defined as

$$\frac{\text{size of retinal image in corrected ametropic eye}}{\text{size of retinal image in standard emmetropic eye}}$$

1.6.1 Size of image in corrected ametropic eye

In **Figure 1.16**, consider light incident on the correcting lens (power F_1) from a distant object, then

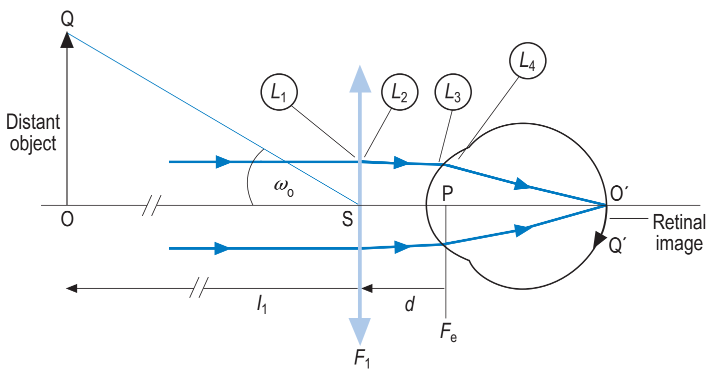

Figure 1.16 The retinal image size in the corrected eye. The distant object OQ subtends a visual angle ω_o and produces a retinal image O′Q′. F_1 is the power of the correcting lens, distance d from the refracting surface of the reduced eye which has a power F_e

$$L_2 = L_1 + F_1$$

$$L_3 = \frac{1}{1/(L_2 - d)} = \frac{L_2}{1 - dL_2} = \frac{L_1 + F_1}{1 - d(L_1 + F_1)}$$

and

$$L_4 = \frac{L_1 + F_1}{1 - d(L_1 + F_1)} + F_\mathrm{e}$$

If we take the general equation for magnification as

$$\text{magnification} = \frac{L_1}{L_2} \cdot \frac{L_3}{L_4} \tag{1.7}$$

and substitute the values above in this equation

$$\text{magnification} = \frac{L_1}{L_1 + F_1} \cdot \frac{(L_1 + F_1)/(1 - d(L_1 + F_1))}{(L_1 + F_1)/(1 - d(L_1 + F_1)) + F_\mathrm{e}}$$

$$= \frac{L_1/(1 - d(L_1 + F_1))}{L_1 + F_1\ /(1 - d(L_1 + F_1)) + Fe}$$

$$= \frac{L_1/(1 - d(L_1 + F_1))}{(L_1 + F_1 + F_\mathrm{e} - dF_\mathrm{e}\,(L_1 + F_1))/(1 - d(L_1 + F_1))}$$

$$= \frac{L_1}{1 - d(L_1 + F_1)} \cdot \frac{1 - d(L_1 + F_1)}{L_1 + F_1 + F_\mathrm{e} - dF_\mathrm{e}\,(L_1 + F_1)}$$

$$\text{magnification} = \frac{L_1}{L_1 + F_1 + F_\mathrm{e} - dL_1F_\mathrm{e} - dF_1F_\mathrm{e}}$$

But

image size = object size × magnification

If ω_o is expressed as OQ/l_1 then

$$\text{object size} = \omega_\mathrm{o} \,.\, l_1 = \frac{\omega_\mathrm{o}}{L_1}$$

Therefore

$$\text{image size} = \frac{\omega_\mathrm{o}}{L_1} \cdot \frac{L_1}{L_1 + F_1 + F_\mathrm{e} - dL_1F_\mathrm{e} - dF_1F_\mathrm{e}}$$

$$= \frac{\omega_\mathrm{o}}{L_1 + F_1 + F_\mathrm{e} - dL_1F_\mathrm{e} - dF_1F_\mathrm{e}}$$

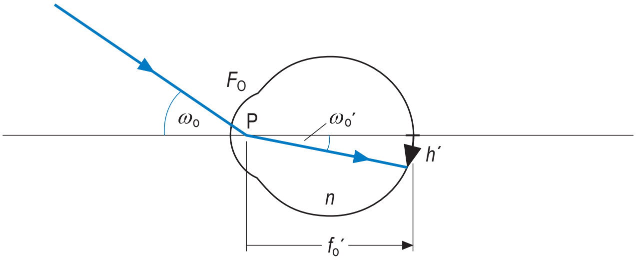

Figure 1.17 The retinal image in the standard emmetropic eye. The distant object subtends a visual angle ω_o at the reduced eye refracting surface which has a power F_o. The refractive index of the eye is n and the retinal image size is h'

If the object is considered to be at infinity then $L_1 = 0$. Therefore

$$\text{image size in the corrected ametropic eye} = \frac{\omega_o}{F_1 + F_e - dF_1 F_e} \tag{1.8}$$

1.6.2 Size of image in the standard emmetropic eye

In Figure 1.17

$$\omega_o' = \frac{h'}{f_o'} \quad \text{therefore} \quad h' = f_o' . \omega_o'$$

but

$$f_o' = \frac{n}{F_o} \quad \text{therefore} \quad h' = \frac{n}{F_o} . \omega_o'$$

but

$$\omega_o' = \frac{\omega_o}{n} \quad \text{therefore} \quad h' = \frac{\omega_o}{F_o} \tag{1.9}$$

1.6.3 Relative spectacle magnification

If we now substitute the expressions for the two retinal image sizes (equations (1.8) and (1.9)) into the relative spectacle magnification definition we deduce that

$$\text{relative spectacle magnification} = \omega_o \frac{(F_1 + F_e - dF_1F_e)}{\omega_o/\mathrm{F}_o}$$

$$= \frac{F_o}{F_1 + F_e - dF_1F_e} \tag{1.10}$$

where

F_o = power of emmetropic eye F_1 = spectacle lens power
F_e = power of ametropic eye d = vertex distance

1.6.4 Axial ametropia

In axial ametropia

$$F_e = F_o$$

Therefore

$$\text{relative spectacle magnification} = \frac{F_o}{F_1 + F_o - dF_1F_o}$$

$$= \frac{1}{1 + F_1\,(f_o - d)} \tag{1.11}$$

If d equals f_o then the relative spectacle magnification is unity, i.e. the corrected image size is the same as that of the standard emmetropic eye. d is usually less than f_o. Therefore $f_o - d$ results in a positive value.

If the eye is hyperopic, F_1 will be positive and the relative spectacle magnification will be less than unity. Thus the corrected image in an axially hyperopic eye is increased in size by the spectacle correction but it is still smaller than the image in the standard emmetropic eye. In axially myopic eyes the minifying correction still leaves an image which is larger than that of the standard emmetropic eye.

1.6.5 Refractive ametropia

Since the image size in the uncorrected eye is equal to the image size in the standard emmetropic eye (both have the same axial lengths) then the spectacle magnification must be equal to the relative spectacle magnification. Therefore the relative spectacle magnification is greater than unity in hyperopia and less than unity in myopia.

1.7 Anisometropia

Correction of an anisometrope with contact lenses results in the obvious advantage of a correcting lens that moves with the eye. Thus the prismatic effects that result from looking through an area of a correcting lens peripheral to its optical centre are minimized. Prism induced by lens decentration produces relative vertical and horizontal prism in the eyes of anisometropes with the relative vertical prism being particularly troublesome. However, we must also consider the effects of changes in retinal image size and this will be influenced by the type of ametropia (axial or refractive).

1.7.1 Refractive anisometropia

In refractive anisometropia the uncorrected retinal image size will be the same in the two eyes. Positive correcting lenses will magnify, and negative

correcting lenses will minify, the retinal image. Thus correcting lenses will induce an image size disparity which may result in binocular vision problems. A pair of contact lenses will induce a much smaller change than a pair of spectacle lenses. Therefore a contact lens correction appears to be the more satisfactory of the two alternatives.

1.7.2 *Axial anisometropia*

If we consider the antimetrope for illustration purposes, then in axial anisometropia the uncorrected myopic eye will possess a larger than normal retinal image size which will be reduced by a correcting spectacle lens. However, the relative spectacle magnification will still be greater than unity, i.e. the correcting lens is only partially successful in returning the image to a normal size. A contact lens will hardly alter the image size at all and so we can see that the large uncorrected image will be returned to a more normal size by a spectacle lens, with the contact lens having a much less pronounced influence. The axially hyperopic eye will also be better corrected by a spectacle lens since the small uncorrected retinal image will be magnified. It therefore appears on strictly optical grounds that the spectacle correction reduces the disparity between the image sizes in the two eyes more effectively than a contact lens correction.

However, suggestions have been made in the past that an axially myopic eye will have its retina stretched over a large area (owing to the larger than normal size of the eye) and this may increase the receptor spacing. If this notion is accepted then a particular object at a particular distance may, for example, produce an image on the uncorrected myopic eye retina which subtends ten receptors. In the axially hyperopic eye where the receptors are more densely packed, the same object will produce a smaller uncorrected image which could well still stimulate ten receptors. The neural image size in these two eyes is thus identical and an ideal correction would be one that did not alter the uncorrected retinal image sizes. Thus a contact lens correction would be preferable to a spectacle lens correction.

Anisometropia is more likely to be axial than refractive and the foregoing indicates unfortunately that the fundamentals have not yet been unequivocally resolved. The only thing that can be stated with some certainty is that, if an anisometrope has comfortable binocular vision with a spectacle correction, she or he can expect to encounter adaptation problems when transferring to contact lenses.

A study by Winn *et al.* (1986) has concluded that all anisometropes are better corrected by contact lenses if the aim is to minimize aniseikonic effects. They propose that in the case of the axial anisometrope the receptive field sizes compensate for the difference in retinal image size and this results in congruous cortical images.

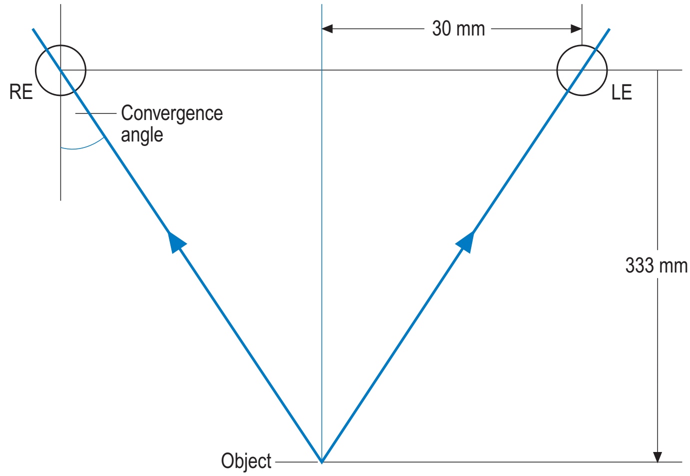

Figure 1.18 Convergence onto a point object 333 mm from a pair of uncorrected eyes

1.8 Convergence

Let us take as an example an uncorrected myope observing a point object at a distance of 333 mm from the plane running through the eyes' centres of rotation. The eyes are converging as shown in **Figure 1.18**. It can be seen that the convergence required is greater for larger pupillary distances (PDs). Let us assume that the distance PD is 60 mm. The convergence unit most commonly encountered is the prism dioptre (Δ) which is defined as the displacement of a light ray by 10 mm when travelling a distance of 1 metre. Thus the angle of deviation in prism dioptres is the tangent of the convergence angle multiplied by 100. Therefore in **Figure 1.18**

$$\text{convergence angle in prism dioptres} = \frac{3000}{333} = 9\Delta$$

If the myope is corrected by contact lenses and we assume that the lenses remain in a central position on converging, then the myope will require 9 Δ of convergence in each eye in order to view the object of interest at 333 mm.

Let us suppose that the myope in question requires a spectacle correction of –6.00 DS at a fitting distance (distance from the lens back vertex to the eye's centre of rotation) of 25 mm. In **Figure 1.19** the myope is corrected by lenses centred for the distance PD of 60 mm. When the eyes converge to look at the object 333 mm away the visual axes pass through a point on the nasal side of the optical centre of both lenses. This introduces a base in prism which reduces the convergence required.

The –6.00 DS lens will produce an image B′ of the object B and this image must lie on the line which joins the object to the optical centre of the lens O.

$$\text{Object distance } l = -333 + 25 \quad = -308 \text{ mm}$$

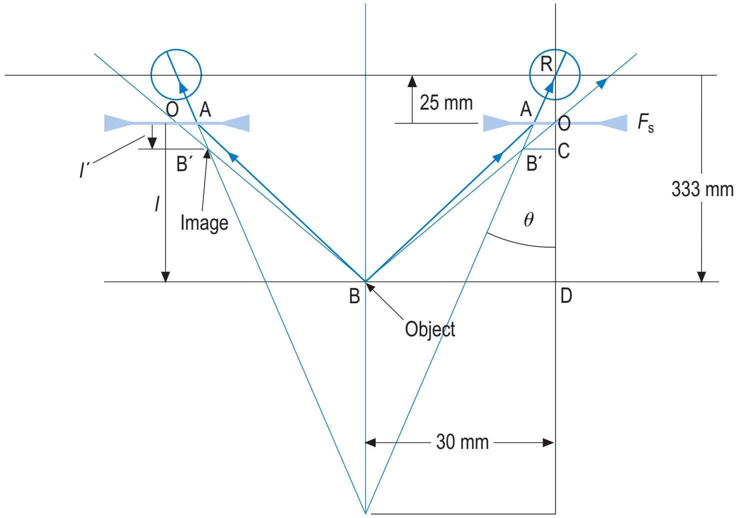

Figure 1.19 Convergence onto a point object 333 mm from a pair of corrected myopic eyes. The spectacle lens power is – 6.00 D and the lens is situated 25 mm in front of the centres of rotation of the eyes. The spectacle lenses have their optical centres (point O) set at the distance PD of 60 mm

Giving an object vergence of $\dfrac{1000}{-308} = -3.25$ D

lens power $= -6.00$ D

Therefore

image vergence $= -9.25$ D

giving an image distance $l' = \dfrac{1000}{-9.25}$

$l' = -108.11$ mm

From **Figure 1.19**, in the similar triangles BOD and B′OC

$$\frac{B'C}{BD} = \frac{l'}{l}$$

Therefore

$$B'C = BD \cdot \frac{108.11}{308} = 10.53 \text{ mm}$$

$$CR = 108.11 + 25 = 133.11 \text{ mm}$$

$$\tan\theta = \frac{B'C}{CR} = \frac{10.53}{133.11} = 0.0791$$

Convergence of each eye in prism dioptres = tan $\theta \times 100$

Therefore

each eye converges 7.91Δ

Thus the 9 D of convergence required in each eye when uncorrected or corrected by a well-centred contact lens is reduced by just over 1Δ when the eye is corrected by the spectacle lens.

We can conclude that negative spectacle lenses reduce the convergence required. Positive spectacle lenses centred for distance vision will require an increase of convergence over that required when uncorrected. Therefore a myope wearing contact lenses requires more convergence than when wearing spectacles, and vice versa for the hyperope.

It will be recalled that myopes accommodate more, hyperopes accommodate less, when wearing contact lenses instead of a spectacle correction. This means that the accommodation–convergence ratio is only minimally disturbed.

1.9 Summary

1. Vergence in dioptres is the reciprocal of distance in metres and vice versa.
2. Convergence is positive, divergence is negative. A positive vergence will be accompanied by a positive distance. Divergence will be accompanied by negative distances.
3. Hyperopes need more powerful contact lenses. Myopes need more powerful spectacle lenses.
4. Clinically, the effective power of a spectacle lens at the eye is of no consequence under ± 4.00 D.
5. The uncorrected basic retinal image size is determined by the axial length of the eye. The longer eye produces a larger retinal image.
6. Positive spectacle lenses increase the retinal image size. Negative lenses decrease the retinal image size. The effect is more pronounced with large vertex distances.
7. In axial ametropia, the increase in retinal image size due to a positive spectacle lens produces an image which is still smaller than that of an emmetropic eye. Corrected axial myopes possess retinal image sizes which are larger than those of the emmetrope.
8. Hyperopes require less accommodation and less convergence, having transferred to contact lenses from spectacle lenses. Myopes accommodate more and converge more with contact lenses than with spectacles.

Reference

Winn, B., Ackerly, R.G., Brown, C.A., Murray, F.K., Prais J. and St John, M.F. (1986) The superiority of contact lenses in the correction of all anisometropia. *Trans. Br. Cont. Lens Ass. Annu. Clin. Conf.*, 95–100

CHAPTER 2

The contact lens

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2.1 Terms, symbols and abbreviations for contact lens dimensions

Figure 2.1 illustrates a tricurve rigid gas permeable (RGP) contact lens seen in cross-section. The terms symbols and abbreviations are as specified in ISO 8320 – 1986 *Optics and Optical Instruments – Contact Lenses – Vocabulary and Symbols*.

Table 2.1 lists the dimensions, symbols and abbreviations.

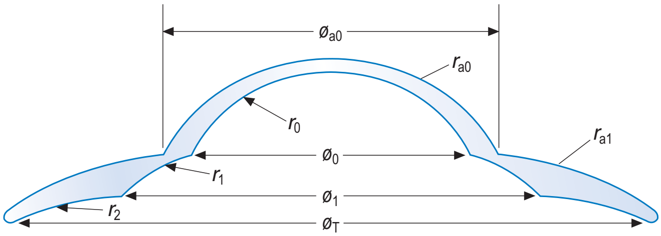

Figure 2.1 The symbols for radii and diameters applied to a tricurve lenticulated RGP lens. This figure should be inspected with reference to Table 2.1

2.2 The contact/fluid lens system

When a contact lens is placed on the eye the tear fluid trapped between the contact lens and the cornea forms a fluid lens as shown in **Figure 2.2 (a)**. However, it is often convenient to 'pretend' that the system is as shown in **Figure 2.2(b)** where the contact lens, the thin fluid lens and cornea are separated by two infinitely thin air films. In these circumstances we can readily see that a contact lens with a back vertex power (BVP) of –2.00 D in air will correct a 2.00 D myope when placed in contact with the eye (this assumes that the fluid lens is afocal).

Let us assume that the contact lens back optic zone radius (BOZR) r_0 is 8.70 mm, with a refractive index n_p of 1.450. The tear fluid refractive index n_f, is assumed to be 1.336.

In **Figure 2.2(a)** the power of the contact lens/fluid lens interface is

$$\frac{n_f - n_p}{r_0\,(\text{metres})} = \frac{-0.114 \times 1000}{8.70} = \frac{-114}{8.70}$$

$$= -13.10\ \text{D}$$

Thus light rays undergo a vergence change of –13.10 D as they cross the contact lens/fluid lens interface.

There is a computer program in Chapter 11 which performs this calculation for media of any refractive index.

In **Figure 2.2(b)** the power of the back surface of the contact lens in air is

$$\frac{1 - n_p}{r_0\,(\text{metres})} = \frac{-0.45 \times 1000}{8.70} = \frac{-450}{8.70}$$

$$F_0 = -51.72\ \text{D}$$

The power of the front surface of the fluid lens in air is

$$\frac{n_f - 1}{r_0\,(\text{metres})} = \frac{0.336 \times 1000}{8.70} = \frac{336}{8.70}$$

$$F_{af} = +38.62\ \text{D}$$

Table 2.1 Contact lens dimensions, symbols and suggested abbreviations

Dimension	Symbol	Suggested abbreviation
Back optic zone radius	r_0	BOZR
Back peripheral radius	r_1, r_2, r_3, etc.	BPR1, BPR2, BPR3
Front optic zone radius	r_{a0}	FOZR
Front peripheral radius	r_{a1}, r_{a2}, etc.	FPR1, FPR2, etc.
Back optic zone diameter	ϕ_0	BOZD
Back peripheral zone diameters	ϕ_1, ϕ_2, ϕ_3, etc.	BPD1, BPD2, BPD3
Total diameter	ϕ_T	TD
Front optic zone diameter	ϕ_{a0}	FOZD
Front peripheral diameters	ϕ_{a1}, ϕ_{a2}, etc.	FPD1, FPD2, etc.
Geometric centre thickness	t_c	TC
Carrier junction thickness	t_{a0}, t_{a1}, etc.	TA0, TA1, etc. I would use TJ to mean t_{a0} due to its wide usage
Radial edge thickness	t_e	RET
Axial edge thickness	t_{ak}	AET
Radial edge lift	l_r	REL
Axial edge lift	l_a	AEL
Front vertex power	F_v	FVP
Back vertex power	F'_v	BVP
Oxygen permeability	Dk	Does not need an abbreviation
Oxygen transmissibility	Dk/t	Does not need an abbreviation

In **Figure 2.2(b)** the light rays undergo a total vergence change of:

$$-51.72 + 38.62 = -13.10\ \text{D}$$

as they leave the contact lens and enter the fluid lens. Clearly the presence of an infinitely thin film of air has no effect on the path of the light rays through the system, and provided the fluid lens has no power (as is the case in an

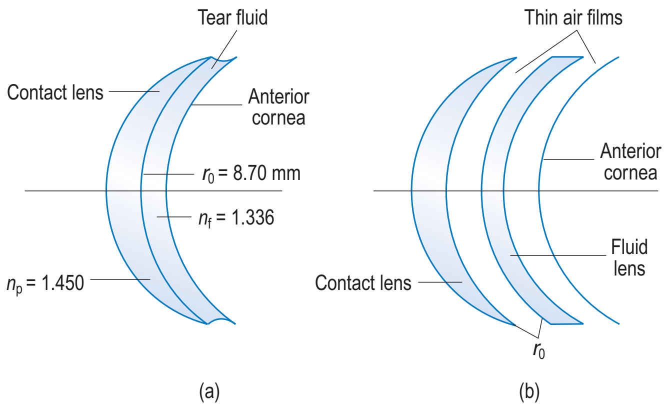

Figure 2.2 The contact lens on the eye. n_p is the refractive index of the plastic (1.450). n_f is the refractive index of the fluid (1.336) and r_0 is the radius of curvature of the interface

alignment fitting) then the power of the contact lens in air should equal the ocular refraction.

2.3 Calculation of surface radii

Contact lens surfaces are of very short radius and therefore very high power. In consequence, we usually consider contact lenses as thick lenses. In practice an optometrist decides on a suitable back optic zone radius (BOZR) in order to ensure an acceptable fit. S/he goes on to perform a contact lens over-refraction in order to deduce the contact lens BVP to be ordered. The contact lens laboratory must, in manufacturing the lens, provide it with the correct BOZR and deduce the front surface radius required to provide the BVP requested. The calculation of front surface radius in these circumstances may best be illustrated by the following example.

An optometrist decides that a fitting set soft lens with a BOZR of 8.70 mm provides a good fit. The trial set lens has a BVP of +1.00 D. The over-refraction results in a +2.00 DS in the refractor head. What front surface radius is required for this soft lens if the refractive index of the material is 1.444 and the final centre thickness is 0.6 mm?

Since the lens in the refractor head is less than 4.00 D we can ignore the effect of vertex distance. Therefore the contact lens BVP to be ordered is

$$+ 1.00 + 2.00 = +3.00 \text{ D}$$

If the central thickness t is 0.6 mm, and the refractive index n_p for the contact lens material is 1.444, then the reduced thickness $^t/n_p$ is 0.42 mm (to two decimal places). In **Figure 2.3** the vergence L_4 is the BVP of the contact lens (+3.00 D).

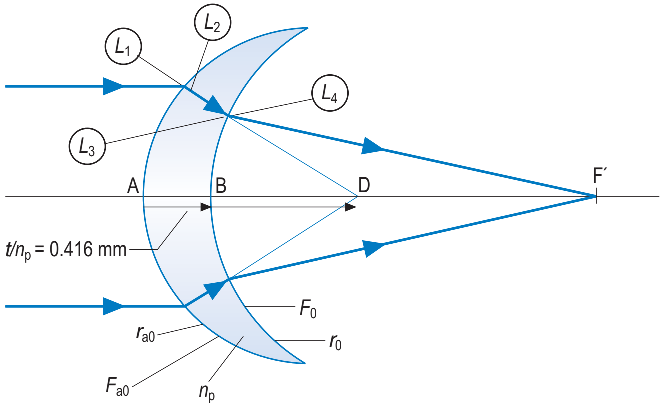

Figure 2.3 Deduction of the front surface radius. Point D is the place where the point image is formed by refraction at the first surface only. The vergences are enclosed in circles

We can use the step along method to work through the system to deduce the front surface radius as follows.

The contact lens back surface power is F_0

$$F_0 = \frac{1 - n_p}{r_0 \text{ (metres)}} = \frac{-444}{8.7}$$
$$= -51.03 \text{ D}$$

Vergence (D) | **Distance (mm)**

$L_4 = +3.00$

$F_0 = \underline{-51.03}$ subtract

$L_3 = +54.03 \longrightarrow \dfrac{1000}{54.03} \longrightarrow$ 18.51 = BD

$\underline{0.42} = {}^t/n_p$

$L_2 = +52.83 \longleftarrow \dfrac{1000}{18.93} \longleftarrow$ 18.93 = AD

$L_1 = \underline{00.00}$ subtract

$F_{a0} = +52.83$

$$r_{a0} = \frac{n_p - 1}{F_{a0}} = \frac{444}{52.83} = 8.40 \text{ mm}$$

The deduction of L_3 may not be immediately obvious until we look at the lens. An incident vergence of +54.03 D on a surface of power –51.03 D will result in an emergent vergence of +3.00 D (by algebraic addition). Indeed it is good practice to cross check a result in this way before proceeding further. The point to remember is that the algebraic addition of the incident vergence and the surface power gives the emergent vergence.

That is

$$L_3 + F_0 = L_4$$

Therefore

$$L_3 = L_4 - F_0$$

Also, it is obvious in **Figure 2.3** that the distance AD is acquired by adding the reduced thickness to the distance BD. The sign convention assumes that distances are measured *from* the optical surfaces. As far as surface A is concerned the reduced thickness is a positive distance, as is distance BD. Therefore BD and ${}^{t}/n_{\mathrm{p}}$ are added together.

2.4 The relationship between front and back surfaces for a thick lens

In the last example the step along method was used to determine the front surface radius of the contact lens, which was assumed to be thick. An alternative approach to this problem is to assume, initially, that the lens is thin and then proceed with calculating the front surface radius on that basis. The radius is then finally adjusted by adding a correction factor which takes the lens thickness into account. This approach is quicker than the step along method. The correction factor is most conveniently derived by considering an afocal lens as described below.

In **Figure 2.4**

$$L_1 = 0.00$$
$$L_2 = F_1$$

The lens thickness is a negative distance as far as the lens back surface is concerned. Therefore

$$L_3 = \frac{1}{1/F_1 - {}^{t}/_{n}} = \frac{F_1}{1 - ({}^{t}/_{n})F_1}$$

and

$$L_4 = \frac{F_1}{1 - ({}^{t}/_{n})F_1} + F_2 = 0.00 \text{ D for an afocal lens}$$

Therefore

$$-F_2 = \frac{F_1}{1 - ({}^{t}/_{n})F_1} \quad \text{where } F_2 = \frac{1-n}{r_2}$$

$$\frac{n-1}{r_2} = \frac{F_1}{1 - ({}^{t}/_{n})F_1}$$

but

$$F_1 = \frac{n-1}{r_1}$$

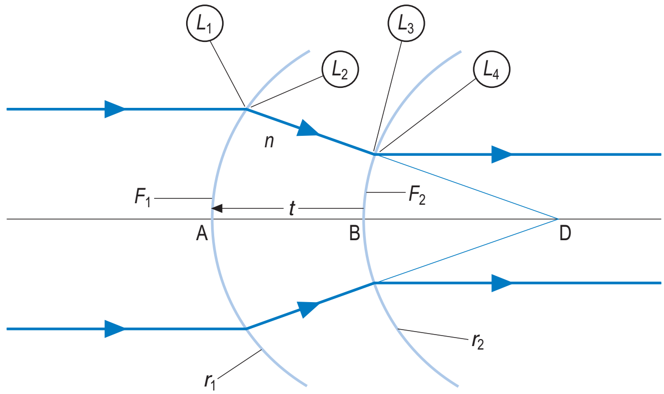

Figure 2.4 An afocal thick lens. F_1 and F_2 are the surface powers arising from surface radii of r_1 and r_2 respectively, t is the central thickness and n is the refractive index

Therefore

$$\frac{n-1}{r_2} = \frac{(n-1)/r_1}{1-(t/n)(n-1)/r_1}$$

Divide each side by $(n-1)$ to get:

$$\frac{1}{r_2} = \frac{1/r_1}{1-(t/n)(n-1)/r_1}$$

$$r_2 = \frac{1-(t/n)(n-1)/r_1}{1/r_1}$$

$$r_2 = r_1\{1-(t/n)(n-1)/r_1\}$$

$$r_2 = r_1 - \frac{t}{n}(n-1)$$

$$r_2 = r_1 - \frac{(n-1)}{n}t \qquad (2.1)$$

and

$$r_1 = r_2 + \frac{(n-1)}{n}t \qquad (2.2)$$

For a *thin afocal* lens

$$r_1 = r_2$$

For a *thick afocal* lens

$$r_1 = r_2 + \frac{(n-1)}{n}t$$

i.e. the front surface curve must be flatter than the back by an amount $\{(n-1)/n\}t$ which is equal to one-third of the lens centre thickness for a refractive index of 1.5.

The last two equations indicate the difference between a thin and a thick lens respectively. We can therefore state

$$r_{1\ \text{thick}} = r_{1\ \text{thin}} + \frac{(n-1)}{n}\,t$$

This relationship, derived by Bennett (1985), can be applied to thick powered lenses, shown by repeating the previous example which requested calculation of the front surface radius.

In **Figure 2.5** we commence the calculation with the assumption that the lens is thin.

$$F_0 = \frac{1-n_p}{r_0} = \frac{-444}{8.7} = -51.03\ \text{D}$$

$$\begin{aligned} L_4 &= +\ 3.00\ \text{D} \\ F_0 &= \underline{-\ 51.03\ \text{D}}\ \text{subtract} \\ L_3 &= +\ 54.03\ \text{D} \end{aligned}$$

Since the lens is thin, this vergence is not only the incident vergence on the back surface but also the emergent vergence L_2 from the front surface of the lens, i.e.

$$\begin{aligned} L_2 &= +\ 54.03\ \text{D} \\ L_1 &= \underline{\quad 00.00\ \text{D}}\ \text{subtract} \\ F_{a0} &= +\ 54.03\ \text{D} \\ r_{a0} &= \frac{n_p-1}{F_{a0}} = \frac{444}{54.03} = 8.22\ \text{mm} \end{aligned}$$

But the lens is thick so we must add a correction factor of $\{(n_p-1)/n_p\}t$ to this front surface radius because r_1 in a thick lens must be longer than r_1 in a thin lens by the amount $\{(n_p-1)/n_p\}t$.

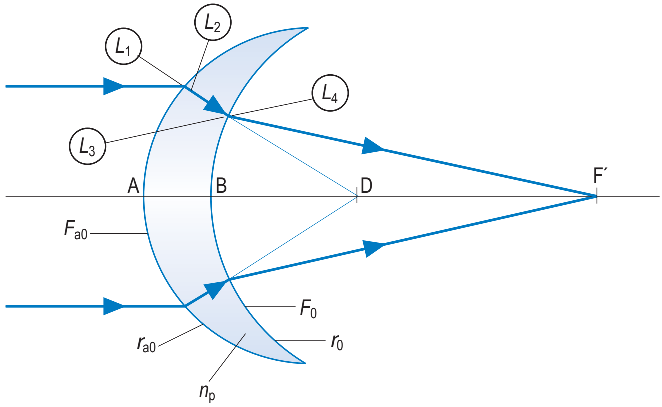

Figure 2.5 Deduction of front surface radius for a thin lens where $L_2 = L_3$

$$\frac{n_p - 1}{n_p} t = \left(\frac{0.444}{1.444}\right) 0.6 = 0.18 \text{ mm}$$

Therefore, for the thick lens

$$r_{a0} = 8.22 + 0.18 = 8.40 \text{ mm}$$

It will be noted that this is in agreement with the answer obtained by the step along method (Section 2.3).

There is a computer program in Chapter 11 which calculates the front surface radius when the BVP, BOZR, refractive index and lens centre thickness are known.

2.5 Toric lenses

A toric contact lens introduces no extra problems for deduction of the radii of one surface when the other surface radii, lens power, refractive index and centre thickness are known. It is *simply* a matter of regarding the two principal meridians as separate lenses so that the calculation is performed as above for the first meridian and repeated for the second.

2.6 Contact lens over-refraction using an RGP lens with a BOZR different from that of the lens to be ordered

Occasionally the contact lens practitioner will not have an RGP contact lens of the desired BOZR for the purpose of over-refracting the patient. In these circumstances an RGP lens with a BOZR as near as possible to that to be ordered can be used and the final BVP adjusted to compensate for the fact that the over-refraction was performed with a contact lens of BOZR different from that of the ordered lens.

This is best illustrated by the following example.

A trial RGP lens with a BOZR of 8.00 mm and BVP of –2.00 D is worn by an eye and an over-refraction is performed. When the end point is reached the lens in the refractor head is +0.50 DS. We intend to order a lens with a BOZR of 8.25 mm. What BVP should be ordered for this lens?

In **Figure 2.6** we see the contact lens that would have been ordered if the BOZR was to remain unchanged at 8.00 mm. In this case the BVP requested would be

$$-2.00 + 0.50 = -1.50 \text{ D}$$

Thus

$$L_4 = -1.50 \text{ D}$$

This is the vergence of light in the thin air film between the contact lens and the fluid lens which is equal to the lens BVP in air.

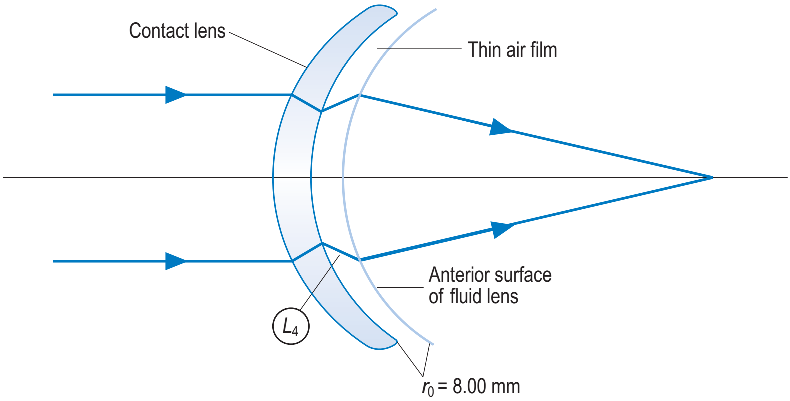

Figure 2.6 The lens to order for a BOZR of 8.00 mm. L_4 is the BVP of the contact lens in air

The power of the fluid lens anterior surface is

$$F_{af} = \frac{n_f - 1}{r_0} = \frac{336}{8} = +42.00 \text{ D}$$

2.6.1 *The lens to be ordered*

When the BOZR is changed this alters the power of the anterior surface of the fluid lens. However, all components posterior to this surface remain unchanged when the new BOZR is substituted.

The contact lens must correct the system. The system is the refractive error of the eye *and* the power of the fluid lens. Changing the BOZR changes the power of the fluid lens by changing the power of the fluid lens anterior surface. The fluid lens is assumed to be a thin lens so the change in surface power and change in lens power will be the same.

So all we need to know is what is the power change of the anterior fluid lens surface when its radius of curvature is altered.

In **Figure 2.7**, the power of the fluid lens anterior surface has changed to:

$$F_{af} = \frac{n_f - 1}{r_2} = \frac{336}{8.25} = +40.73 \text{ D}$$

The flatter BOZR has made the fluid lens less positive (more negative).

The power change in the fluid lens = 40.73 – 42.00 = –1.27 D

Thus the contact lens must be made 1.27 D more positive to neutralize the change in power of the fluid lens.

L_4 = –1.50 + 1.27 = –0.23 D

Therefore

L_4 = –0.23 D = BVP of contact lens in air

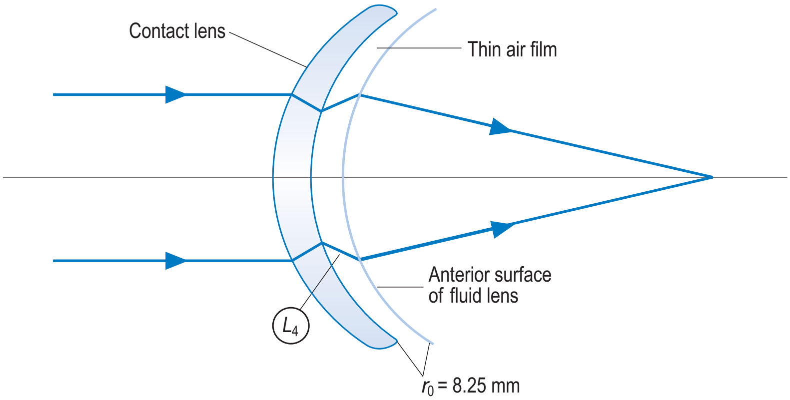

Figure 2.7 The lens to order for a BOZR of 8.25 mm. L_4 is the BVP of the contact lens in air

N.B. We have calculated that a BOZR change of 0.25 mm must be accompanied by a power change of approximately 1.25 D. **Thus a change of 0.05 mm in the BOZR must be accompanied by a fluid lens power change of 0.25 D**.

Thus:

(a) The contact lens that corrects the eye is correcting a system that consists of the refractive error and the fluid lens power. The fluid lens power arises from differences in the radius of curvature of the cornea and the contact lens back surface.
(b) When the contact lens BOZR is changed, then the only change induced in the fluid lens power is a change of front surface power. If the fluid lens is assumed to be thin, then the change in fluid lens power will be identical to the change in fluid lens front surface power.

In the above example the fluid lens front surface power changed from

$$\frac{336}{8} = +42.00\ \text{D} \quad \text{to} \quad \frac{336}{8.25} = +40.73\ \text{D}$$

The fluid lens power change is

$$40.73 - 42.00 = -1.27\ \text{D}$$

The fluid lens has become more negative by 1.27 D and so the contact lens must become more positive by 1.27 D if it is still to correct the system. So the new BVP for the contact lens must be

$$-1.50 + 1.27 = -0.23\ \text{D}$$

Thus, **the relationship established in the approximate rule arises from the change in power of the anterior surface of the fluid lens**.

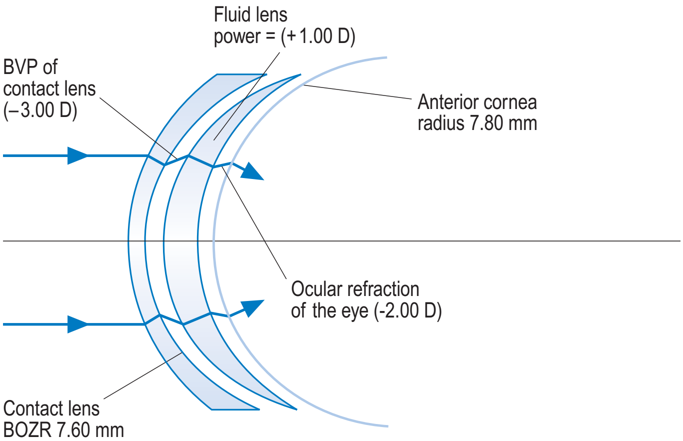

Figure 2.8 An RGP lens fitting 0.2 mm steep. Note that the vergence of light in the posterior thin air film is equal to the ocular refraction. The vergence in the anterior air film is equal to the contact lens BVP. The difference between these two is the fluid lens power

This approximate rule is well worth remembering since it not only allows deduction of the power to order but it tells the contact lens fitter whether the lens is fitting steep, aligned or flat.

If, for example, a 2.00 D myope was fitted with a contact lens and the over-refraction indicated that a BVP of – 3.00 D was required, then this suggests that the fluid lens power is +1.00 D. If we use the approximate rule above we deduce that the contact lens is fitting 0.2 mm steep. The rule applies to scleral and corneal lenses, both hard (PMMA) and RGP, since the contact lens refractive index is not considered in the calculation. **Figure 2.8** illustrates the situation for the lens that is fitting 0.2 mm steep. The light is coming from an infinitely distant point object. The fluid lens power is approximately +1.00 D. The components of the system are separated by two infinitely thin air films. The eye is 2.00 D myopic. The vergence of light in the thin air film between the fluid lens and the cornea is always equal to the ocular refraction. The vergence of light in the other film will always be the contact lens BVP. The difference between the ocular refraction and the contact lens BVP indicates the power of the fluid lens.

2.7 Soft lenses on the eye

The approximate rule cannot be applied to soft lenses because they tend to take up the curvature of the central cornea on which they rest. Thus the power of the fluid lens is unpredictable but is likely to be approximately afocal and to remain largely unaltered by changes in the contact lens specification. The flexibility of the soft contact lens inevitably means that on-eye optical calculations are open to question. The fluid lens formed behind a soft contact lens on the eye is likely to have a power around plano. However,

this cannot be reliably predicted and the change occurring in the fluid lens power when the BOZR is changed is unpredictable but is generally assumed to remain unchanged at the plano value. The assumption is that no matter what the soft lens BOZR value, the back surface of the contact lens will drape over the cornea and take up the radius of curvature of the cornea on which it rests. It must be noted that in a typical soft contact lens fitting, the BOZR will be considerably flatter than the cornea. This will, if anything, encourage the formation of a negative powered fluid lens particularly with thicker or more rigid lens materials. Thus in practice the fluid lens power is likely to be plano but it may have a small negative power, up to around –0.50 D, in some circumstances. If a positive fluid lens is found during the fitting of a soft lens then this is suggestive of a tight fit.

The usual methods of soft (hydrogel) lens manufacture are cast moulding, fully or partially hydrated wet moulding, spin casting and lathe cutting. Soft silicone hydrogel lenses are cast moulded.

In the case of cast moulding, spin casting and lathing the lens must be hydrated following manufacture so that the dimensions of the dry (xerotic) lens are based on the expansion factor of the material being used. Typical expansion factors are 1.19 for 38% water content (polyHema) and 1.40 for 58% water content hydrogel materials. Therefore, a 38% water content soft lens having a BOZR of 8.70 mm and a total diameter of 14.00 mm would have corresponding dry dimensions of BOZR = 7.31 mm and TD = 11.76 mm. Two examples of a 59% water content material soft lens are given in **Tables 2.2 and 2.3**. The tables illustrate the specifications of a negative and positive powered lens.

There are now examples of commercial soft lens products which are moulded either fully or partially hydrated. CIBA Vision's daily disposable lens which is marketed under the name Focus Dailies is cast moulded in glass reusable moulds in a fully hydrated state. After casting, the lens is polymerized using ultraviolet light. The Johnson & Johnson Vision Care range of 58% water content hydrogel lenses are cast moulded in a partially hydrated state, a process described by the company as *stabilized soft moulding*. In this case the water content of the lenses is increased to the 58% value following the cast moulding.

Lens manufacturers will normally have carefully controlled high-precision process control and measurement systems in place for all stages of lens production, but the dimensions of soft lenses will be confirmed in the hydrated state. In the case of high volume products this will be based on batch sampling while in the case of specialist or made-to-order lenses measurement will be carried out on individual lenses.

In general, the dimensional tolerances for soft lenses allow more leeway than is the case for rigid lenses; for example, the tolerance for total diameter for rigid lenses is ±0.10 mm while for soft lenses it is ±0.20 mm. This is based on considered clinical assessment of product functionality and also reflects the limitations of measurement methods. The flexibility of soft materials

Table 2.2 Dry and wet lens specification for a typical negative power soft lens made of material with a water content of 59%

Dry lens			Wet lens		
Back surface			**Back surface**		
Radius (mm)	**Diameter (mm)**		**Radius (mm)**	**Diameter (mm)**	
6.10	9.22		8.60	13.00	
6.67	10.07		9.40	14.20	
Power (D)			**Power (D)**		
–5.82			–3.25		
Centre thick (mm)			**Centre thick (mm)**		
0.060			0.085		
Front surface			**Front surface**		
Radius (mm)	**Diameter (mm)**	**Thickness (mm)**	**Radius (mm)**	**Diameter (mm)**	**Thickness (mm)**
6.58	5.67	0.12	9.28	7.99	0.17
6.13	9.36	0.11	8.64	13.20	0.16
6.45	10.07	0.09	9.09	14.20	0.13

is generally more forgiving than rigid materials when worn, so the wider dimensional tolerances do not influence clinical performance. Overall, the precision of soft lens manufacture is not as high as that of rigid lenses but that is not clinically significant due to the different fitting characteristics of the products.

2.7.1 *Power changes of soft lenses*

If a soft lens is placed on a particular eye it will warp into a steeper form so that the back surface takes up the curvature of the cornea on which it rests. This is called lens flexure. In these circumstances the front surface radius will also alter and these induced changes may well affect the power of the contact lens. In order to investigate the consequence of power changes due to the lens bending on the eye we must make some assumptions about induced changes in the lens parameters. Strachan (1973) claims that the 'wrap factor' remains

Table 2.3 Dry and wet lens specification for a typical positive power soft lens made of material with a water content of 59%

Dry lens			Wet lens		
Back surface			**Back surface**		
Radius (mm)	**Diameter (mm)**		**Radius (mm)**	**Diameter (mm)**	
5.89	9.22		8.30	13.00	
6.45	10.07		9.10	14.20	
Power (D)			**Power (D)**		
+ 9.43			+ 5.25		
Centre thick (mm)			**Centre thick (mm)**		
0.199			0.280		
Front surface			**Front surface**		
Radius (mm)	**Diameter (mm)**	**Thickness (mm)**	**Radius (mm)**	**Diameter (mm)**	**Thickness (mm)**
5.38	5.67	0.12	7.59	7.99	0.17
5.91	9.36	0.11	8.33	13.20	0.16
6.28	10.07	0.09	8.85	14.20	0.13

constant. If a lens of BOZR 9 mm is fitted to a cornea of 8 mm radius the wrap factor is 9/8 which equals 1.125. Thus the new front surface radius is the old front surface radius divided by 1.125. Baron (1975) suggested that a change in back surface radius will be accompanied by an almost equal change in the front surface radius. The most useful work on this subject was published by Bennett (1976) who assumed that the lens volume remained unchanged; there is no redistribution of lens thickness, with the lens front surface remaining spherical when resting on a spherical cornea. He derived tables for both positive and negative lenses which led to the conclusion that the power change induced by flexure is independent of the initial power of the lens. The power changes are greatest for thick lenses and steep radii. As the lens steepens, its power becomes more negative. The fundamental principle is illustrated in **Figure 2.9** where a flat parallel plate is bent into a concentric meniscus lens. In these circumstances

$$r_1 = r_2 + t$$

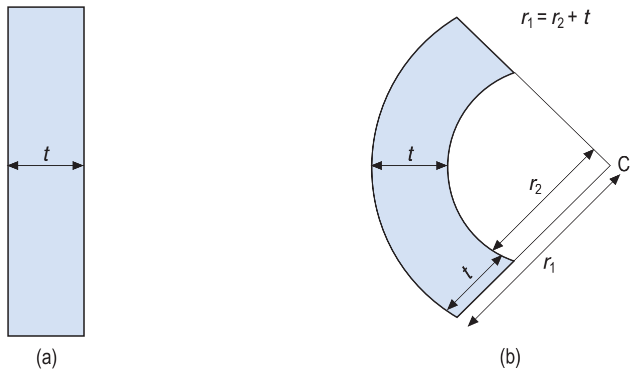

Figure 2.9 (a) A flat parallel plate. (b)The plate is bent to form a concentric meniscus lens. C is the centre of curvature of both surfaces

where r_1 is the radius of curvature of the front surface and r_2 is the radius of curvature of the back surface, with t representing the radial thickness of the material.

It will be recalled that for an afocal thick lens

$$r_1 = r_2 + \frac{(n-1)}{n}t \tag{2.2}$$

This means that the concentric lens has a front surface radius which is too long to maintain the afocal nature of the flat plate because $(n - 1/n)$ is inevitably less than unity. This excessive radius produces insufficient positive power for the front surface which results in a negative back vertex power. Thus, bending a lens to reduce the radii of curvature induces increasing negative power and the effect is more pronounced with thicker lenses. Bennett's investigation also considered what happens to a powered lens where the same changes were found to occur. He derived an equation to quantify the changes in powered lenses

$$\Delta F_v' = -300t\left\{\frac{1}{r_2'^2} - \frac{1}{r_2^2}\right\} \tag{2.3}$$

for a lens refractive index of 1.44, where $\Delta F_v'$ is the change in BVP, t is the centre thickness, r_2 is the original back surface radius and r_2' is the new back surface radius after bending the lens.

If we substitute typical values in the above equation we can get some idea of the degree of the power change. Let us assume that the soft lens has a central thickness of 0.3 mm, a BOZR of 8.80 mm and the anterior corneal surface radius is 7.80 mm. It is assumed that the contact lens back surface, on the eye, will take up the curvature of the cornea. Thus

$t = 0.3$ mm, $r_2 = 8.70$ mm, $r_2' = 7.60$ mm

then

$$\Delta F_v' = -\,300 \times 0.3\,(0.0132 - 0.0173)$$
$$= -\,0.37\text{ D}$$

The equation was found to be in close agreement with that derived by Wichterle (1967) by an entirely different route. We can therefore conclude that steepening a soft lens will induce an increase in negative power, owing to the flexure of the lens, and this increase is independent of the contact lens BVP but is greater for thick lenses than for thin. The flexure is of small consequence when performing a fitting with an over-refraction check. If, for example, a lens with a BOZR of 8.40 mm and central thickness 0.3 mm was resting on a cornea of radius 7.40 mm and the over-refraction suggested a BVP of – 2.00 D was required. Suppose this lens was removed and replaced by a lens of radius 8.70 mm. Both lenses are assumed to flex on the eye and take up the corneal curvature. The change in power produced by flexure is –0.37 D in the first lens and –0.45 D in the second. Thus the extra negative power induced by the greater flexure of the flatter lens is

$$0.37 - 0.45 = -0.08\text{ D}$$

Since the over-refraction will be measured to the nearest 0.25 D, it is unlikely that the effects of increased flexure in the second lens will be noticed.

During the time that a soft lens is worn, the lens temperature will increase from room to body temperature and this will encourage the lens to steepen. Also evaporation from the front surface of the lens will increase the refractive index of the lens material and again encourage the lens to steepen. This further steepening will serve to make the fluid lens anterior surface more positive. The refractive index increase leads to a more negative lens power for the myope and a more positive contact lens power for the hyperope. The partial dehydration, however, may increase the lens rigidity which will reduce the tendency towards flexure. Thus there are a number of interrelated factors which influence the final outcome of the power of the correction. In the clinical situation, it is good practice to perform the over-refraction using the lens expected BVP (BVP equal to the ocular refraction mean sphere) and a lens specification identical to that to be ordered for the eye in question.

2.8 Contact lens over-refraction using a lens with an inappropriate BVP

If the over-refraction is performed using a contact lens which has a BVP considerably different from that required for correction of the ametropia, then errors may arise as illustrated in the example below:

A PMMA scleral lens of refractive index 1.490, BVP of –12.00 D, central thickness 0.8 mm, a BOZR of 8.75 mm and a front optic zone radius (FOZR) of 11.40 mm is used for an over-refraction. The end point of the over-

refraction is reached when a +8.00 DS is interposed at a vertex distance of 12 mm from the contact lens. What BVP should be requested when the lens with a BOZR of 8.75 mm is ordered?

2.8.1 *Approximate calculation in clinical practice*

We would calculate the effective power of the +8.00 DS at the contact lens as follows.

$$\begin{array}{llcl}
 & \textbf{Vergence (D)} & & \textbf{Distance (mm)} \\
 & F_s = +8.00 \longrightarrow & \dfrac{1000}{8} \longrightarrow & 125 \\
\text{Effective power} & = +8.85 \longleftarrow & \dfrac{1000}{113} \longleftarrow & \dfrac{-12}{113}\ \text{(vertex dist.)}
\end{array}$$

We then add this effective power to the BVP of the contact lens giving

$+ 8.85 - 12.00 = - 3.15$ D

and this is the BVP that would be ordered.

2.8.2 *Accurate calculation*

In Figure 2. 10

front surface power of contact lens $F_{a0} = \dfrac{n_p - 1}{r_{a0}} = \dfrac{490}{11.4} = + 42.98$

back surface power of contact lens $F_0 = \dfrac{1 - n_p}{r_0} = \dfrac{-490}{8.75} = - 56.00$ D

$$\frac{t}{n_p} = \frac{0.8}{1.490} = 0.54 \text{ mm}$$

$$\begin{array}{lcl}
\textbf{Vergence (D)} & & \textbf{Distance (mm)} \\
L_1 = +\ 8.85 & & \\
F_{a0} = \underline{+42.98} & & \\
L_2 = +51.83 \longrightarrow & \dfrac{1000}{51.83} \longrightarrow & 19.29 = AD \\
 & & - 0.54 = t/n \\
L_3 = +53.33 \longleftarrow & \dfrac{1000}{18.75} \longleftarrow & 18.75 = BD \\
F_0 = \underline{-56.00} & & \\
L_4 = -\ 2.67 \text{ D} = \text{accurate BVP in air} & &
\end{array}$$

The clinical estimation of BVP was –3.15 D which is 0.48 D too negative. This error arises from the fact that the spectacle lens used in the over-

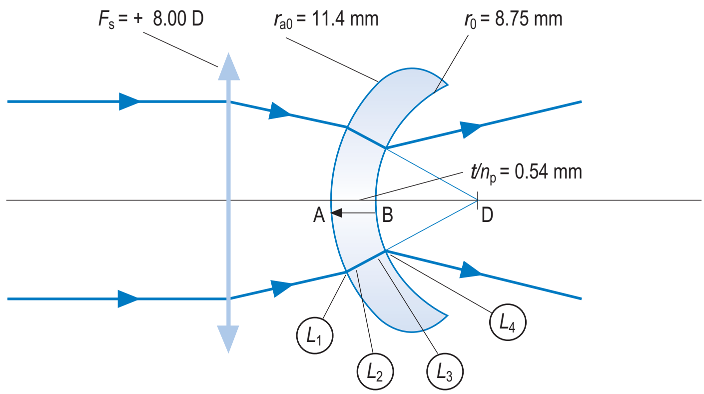

Figure 2.10 The over-refraction. Point D is the place where the point image is formed after refraction at the first contact lens surface only

refraction is of high power and the contact lens is a thick lens, with powerful surfaces. In the example illustrated above, we have a high plus spectacle lens at a finite vertex distance which serves to magnify the retinal image. When the contact lens returns with the correct BVP, much of this magnification is lost and the visual acuity may not be as good as that found at the time of the over-refraction.

2.9 Modifying an existing lens

It is not uncommon in scleral lens fitting to change the BOZR of a lens by modifying the lens worn by the patient. A modified BOZR not only changes the power of the anterior surface of the fluid lens but also alters the BVP of the contact lens.

The following example illustrates these changes.

A PMMA (n = 1.490) scleral contact lens of BOZR 8.25 mm, BVP – 3.00 D and central thickness 0.7 mm corrects the ametropia but the optic must be reground to a BOZR of 8.75 mm. What is the new front surface radius required if the lens is still to correct the eye? What is the front vertex power of this modified lens? The contact lens centre thickness is reduced to 0.6 mm by the modification. Assume that the fluid lens is a thin lens throughout. The refractive index of the material is 1.490.

2.9.1 Before modification

In Figure 2.11

vergence incident on the anterior surface of the fluid lens = – 3.00 D

power of the anterior surface of the fluid lens $F_{af} = \dfrac{336}{8.25} = +40.73$ D

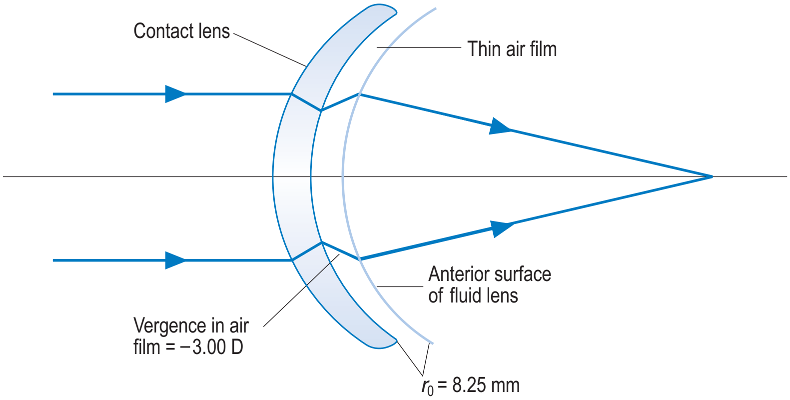

Figure 2.11 The lens before modification

Therefore

emergent vergence from the anterior surface of the fluid lens	= + 40.73 – 3.00 = + 37.73 D

This emergent vergence must remain unchanged after modification.

2.9.2 *After modification*

Figure 2.12 shows the lens after modification. If this lens is still to correct the refractive error, then vergence L_5 must be maintained at +37.73 D. We can therefore use the step along method to calculate r_{a0}.

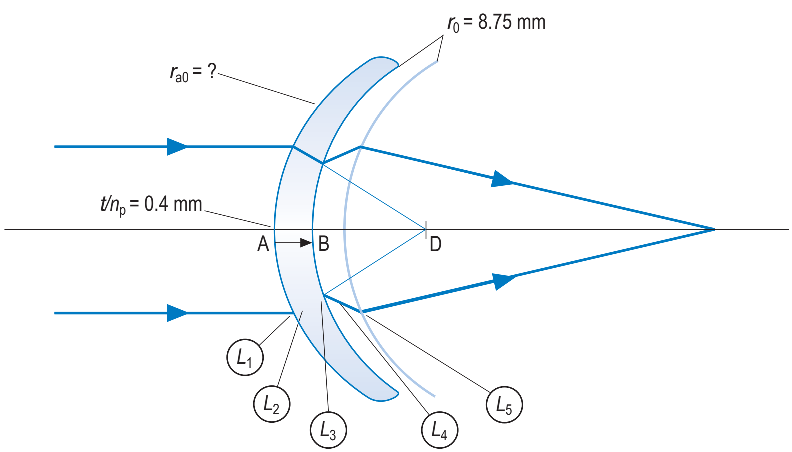

Figure 2.12 The lens after modification. L_5 must be +37.73 D

Power of anterior surface of fluid lens $F_{af} = \dfrac{336}{8.75} = +\ 38.40$ D

Power of back surface of contact lens $F_0 = \dfrac{-490}{8.75} = -\ 56.00$ D

Reduced thickness contact t/np $= \dfrac{0.6}{1.490} = 0.4$ mm for the lens

Vergence (D) **Distance (mm)**

L_5 must be +37.73

$$
\begin{array}{llcccl}
F_{af} = & \underline{+38.40} \text{ subtract} & & & & \\
L_4 = & -\ 0.67 = \text{BVP of contact lens in air} & & & & \\
F_0 = & \underline{-56.00} \text{ subtract} & & & & \\
L_3 = & +55.33 & \longrightarrow & \dfrac{1000}{55.33} & \longrightarrow & 18.07 = \text{BD} \\
 & & & & & \underline{00.4\ \ } = t/n \\
L_2 = & +54.14 & \longleftarrow & \dfrac{1000}{18.47} & \longleftarrow & 18.47 = \text{AD} \\
L_1 = & \underline{\ \ 0.00} \text{ subtract} & & & & \\
F_{a0} = & +54.14 & & & &
\end{array}
$$

where F_{a0} is the front surface power

$$r_{a0} = \frac{n-1}{F_{a0}} = \frac{490}{54.14} = 9.05 \text{ mm}$$

2.9.3 *Front vertex power (FVP)*

If we work through the system illustrated in **Figure 2.13** using the step along method we can calculate the front vertex power.

Vergence (D) **Distance (mm)**

$$
\begin{array}{llcccl}
L_1 & = \ \ \ 0.00 & & & & \\
F_0 & = \underline{-\ 56.00} & & & & \\
L_2 & = -\ 56.00 & \longrightarrow & \dfrac{1000}{-56} & \longrightarrow & -\ 17.86 = \text{BD} \\
 & & & & & -\ 00.4 = {}^{t}/_{np} \\
L_3 & = -\ 54.76 & \longleftarrow & \dfrac{1000}{-18.26} & \longleftarrow & -\ 18.26 = \text{AD} \\
F_{a0} & = \underline{+54.14} & & & & \\
L_4 & = -\ 0.62 & & & &
\end{array}
$$

Therefore front vertex power (FVP) is –0.62 D.

There is a computer program in Chapter 11 which calculates the FVP.

In the above example, we assumed no change in the fluid lens central thickness. The fluid lens was assumed to be thin throughout. A second example will be given to illustrate how the problem is treated when the fluid lens thickness changes.

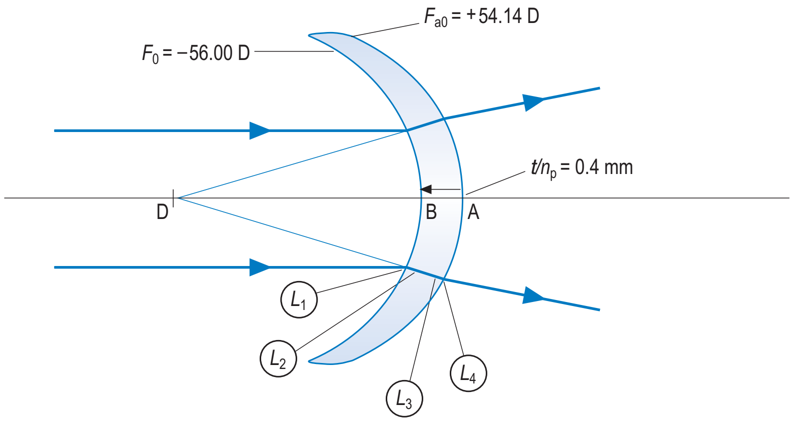

Figure 2.13 Front vertex power. L_4 is the FVP

A PMMA scleral contact lens has a BOZR of 8.25 mm, a BVP of –3.00 D and central thickness of 0.6 mm. This corrects the ametropia but slight apical touch persuades the practitioner to regrind the optic to a BOZR of 8.00 mm. This reduces the contact lens thickness to 0.5 mm and increases the fluid lens thickness from zero to 0.1 mm.

What is the new BVP required if the system is still to correct the ametropia? What front surface radius is required for the modified lens?

2.9.4 *Before modification*

The result is as in the previous example, i.e. the emergent vergence from the anterior surface of the fluid lens is +37.73 D. Since the fluid lens before modification has no thickness, this must also be the incident vergence on the posterior surface of the fluid lens. We conclude that this vergence must be maintained on this posterior surface.

2.9.5 *After modification*

After modification the contact lens/fluid lens system is as illustrated in Figure 2.14.

Power of anterior surface of fluid lens F_{af} is

$$\frac{n_f - 1}{r_0} = \frac{336}{8} = +42.00 \text{ D}$$

Reduced thickness of fluid lens is

$$\frac{t_f}{n_f} = \frac{0.1}{1.336} = 0.075 \text{ mm}$$

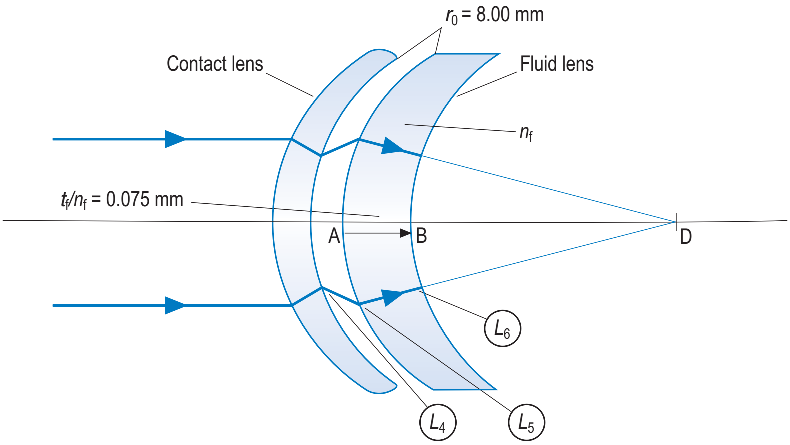

Figure 2.14 The system after modification. L_6 must be +37.73 D

Vergence (D)				**Distance (mm)**
$L_6 = +37.73$	$\longrightarrow$	$\frac{1000}{37.73}$	$\longrightarrow$	$26.50 = \text{BD}$
				$\underline{0.075} = tf/nf$
$L_5 = +37.63$	$\longleftarrow$	$\frac{1000}{26.575}$	$\longleftarrow$	$26.575 = \text{AD}$

$F_{af} = \underline{+42.00}$ subtract
$L_4 = -\ 4.37$ = BVP of contact lens in air

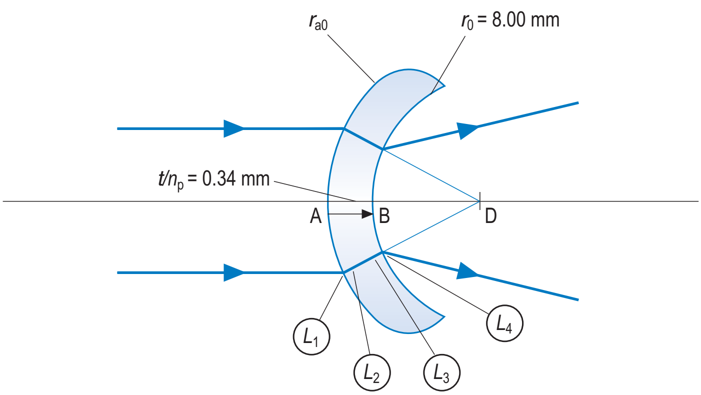

Figure 2.15 The modified contact lens. L_4 must be –4.37 D

In **Figure 2.15** the power of the back surface of the contact lens is

$$F_0 = \frac{1 - n\mathrm{p}}{r_0} = \frac{-490}{8} - 61.25\ \mathrm{D}$$

Reduced thickness of contact lens is

$$\frac{t}{n_\mathrm{p}} = \frac{0.5}{1.490} = 0.34\ \mathrm{mm}$$

$$\begin{array}{lllll}
 & \textbf{Vergence (D)} & & & \textbf{Distance (mm)} \\
L_4 & = -\ \ 4.37 & & & \\
F_0 & = \underline{-\ 61.25}\ \text{subtract} & & & \\
L_3 & = +56.88 \longrightarrow & \frac{1000}{56.88} & \longrightarrow & 17.58 = \mathrm{BD} \\
 & & & & 0.34 = t/n\mathrm{p} \\
L_2 & = +55.80 \longleftarrow & \frac{1000}{17.92} & \longleftarrow & 17.92 = \mathrm{AD} \\
L_1 & = \underline{\ \ 0.00}\ \text{subtract} & & & \\
F_{\mathrm{a}0} & = +55.80 = \text{power of the front surface of the contact lens} & & &
\end{array}$$

$$r_{\mathrm{a}0} = \frac{n_\mathrm{p} - 1}{F_{\mathrm{a}0}} = \frac{490}{55.8} = 8.78\ \mathrm{mm}$$

Thus the only difference between the first and second example of lens modification is that in the first example we maintained the emergent vergence from the anterior fluid surface but in the second a fluid lens thickness change results in the anterior surface changing position, and so we must maintain the vergence of light incident on the fluid lens posterior surface.

2.9.6 *Approximate rules*

There are two approximate rules that can be used clinically when modifying scleral lenses.

1. For each reduction of 0.05 mm in BOZR, +0.12 D must be added to the altered BVP of the contact lens in order to maintain correction by the contact lens/fluid lens system.
2. A reduction of the contact lens central thickness of 0.1 mm must be accompanied by a +0.25 D change in power. However, if this also increases the fluid lens central thickness by 0.1 mm then this will require a further power modification of –0.12 D. Thus in these circumstances the total power modification is +0.12 D.

It may therefore be advisable to order scleral lenses about 0.50 D more positive than the refraction result, to compensate for the optic grind outs (usually about 0.2 mm in total) required due to the settling of the contact lens.

2.10 The BVP of scleral lenses produced by the impression technique

One of the advantages of the impression technique is that the practitioner does not require expensive fitting sets that will be used on a small number of patients during her or his fitting career. The suggestion therefore that a fenestrated lens for optic measurement (FLOM) be used in the contact lens over-refraction for an impression scleral lens, is self-defeating. It is, however, worth considering the use of a rigid corneal lens as an aid to help determine the BVP to order.

The impression technique produces a contact lens which matches the scleral topography. The optic specification is usually ordered by asking for an optic radius 0.2–0.5 mm flatter than the flattest keratometry reading and specifying a central clearance from the cast. The central clearance for a first impression is usually 0.2 mm, which should result in a corneal apical clearance of about 0.1 mm. Let us suppose that the keratometry reading is 7.60 mm spherical and that the eye is 3.00 D myopic. The power of the fluid lens can be deduced using the approximate rule that a **0.1 mm radius change produces 0.50 D power change in the fluid lens**. In this example, suppose we used a back optic zone radius of 7.8 mm (0.2 mm flatter than the keratometry reading); then the fluid lens power is estimated as –1.00 D. The contact lens BVP ordered must therefore be –2.00 D to correct this 3.00 D myopic eye. It must be emphasized that this is an approximation which also ignores the effect of the fluid lens thickness. However, the BVP of the contact lens at worst will only need minor modification.

A more accurate BVP can be deduced just as easily using Heine's scale to deduce the surface powers of the fluid lens and adding them algebraically to give the fluid lens power. This still leaves the influence of the fluid lens thickness unaccounted for. However, if the ideal apical clearance of about 0.1 mm is achieved in the fitting, this will only add approximately +0.12 D to the power of the fluid lens.

One final possibility, already mentioned, is to determine the scleral lens BVP by using a corneal trial lens and performing an over-refraction. This gives a BVP to order which then requires alteration according to the difference between the BOZR of the trial lens and that of the optic of the scleral lens. Once again the approximate rule of thumb can be used but if there is a large difference in the BOZR of the trial lens and the scleral lens to be ordered then Heine's scale should be used to eliminate the approximation.

2.11 Heine's scale

In the early days of scleral contact lens fitting, a technique was used by Heine which involved correcting the ametropia with afocal contact lenses. This inevitably means that the fluid lens is used to correct the refractive error. We

can deduce the power of the fluid lens in air using the equation

$$F = \frac{n - 1}{r}$$

This allows us to calculate the surface powers of the fluid lens, and these can be simply added together to give total lens power, if we assume that the fluid lens is thin. The back surface of the fluid lens is determined by the corneal curvature and this can be measured using a keratometer.

Let us suppose that a 2.00 D myope has a keratometry reading of 7.40 mm.

$$\text{Power of posterior surface of fluid lens} = \frac{1 - n_f}{r_{pf}} = \frac{-336}{7.4} = -45.41 \text{ D}$$

the fluid lens is assumed to be thin and must have a BVP of –2.00 D to correct the myopia. Therefore

Power of anterior surface of the fluid lens = + 43.41 D

Therefore

$$\text{Radius of the anterior surface of fluid lens} = \frac{n_f - 1}{F_{af}} = \frac{336}{43.41}$$

$$= 7.74 \text{ mm}$$

Thus we require an afocal contact lens with a BOZR of 7.74 mm.

In a clinical situation all that is needed is a table or scale which converts power to radius, assuming a refractive index of 1.336. **Figure 2.16** shows a section of Heine's scale. The scale is used to convert radius to surface power or surface power to radius. The scale was originally calibrated for a refractive index of 1.332 and later modified to 1.336. The reader can construct such a scale or table easily by using the equation

$$F = \frac{n - 1}{r}$$

in radius steps of 0.05 mm

This allows accurate calculation of the fluid lens surface power changes with change of radius avoiding the errors arising from using the approximate rule.

Note that in the above example, if we applied the approximation that **0.1 mm radius difference produces a 0.50 D power change in the fluid lens,**

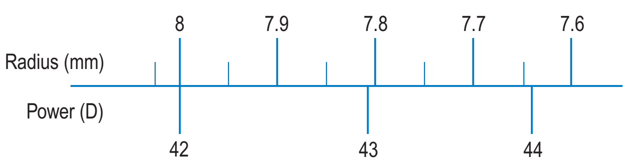

Figure 2.16 A small section of Heine's scale

we would order a contact lens with BOZR of 7.8 mm (the keratometry reading plus 0.4 mm, which is the approximate flattening required for a – 2.00 D fluid lens). This actually results in the fluid lens BVP being – 2.33 D. So where there is a large difference between the corneal radius and the contact lens BOZR or where you intend to make a large change to the lens BOZR, then the approximate rule will introduce unacceptable errors. In these circumstances you need to use

$$\text{power} = \frac{336}{\text{radius}} \quad \text{and} \quad \text{radius} = \frac{336}{\text{power}}$$

in order to work out the fluid lens surface power changes.

2.11.1 Calculation of fluid lens power where the fluid lens is thick

For average radii each 0.1 mm increase in fluid lens thickness adds around 0.12 D to the effective power of the system. Therefore, if the fluid lens is thick, the BOZR of the scleral contact lens that determines the front surface radius of the fluid lens should be increased an appropriate amount to adjust for this effect. The change required can be calculated by initially assuming a thin fluid lens then adjusting for thickness using $[(n - 1)/n]t$ derived in Section 2.4.

2.12 Prismatic effects with contact lenses

Prismatic effects in the case of rigid and soft contact lenses fitted on or near alignment will be small due to the relatively small movement of the contact lens. The prism at the pupil centre can be deduced using the Prentice rule

$$P = Fc$$

where P is the prism power in prism dioptres (Δ), F is the BVP of the contact lens in dioptres, and c is the decentration of the optical centre of the contact lens from the pupil centre in centimetres.

In an alignment fitting, the fluid lens undergoes no change in these circumstances since the back surface of the contact lens rotates about its own centre of curvature when the lens decentres.

A scleral lens, however, will rotate about the centre of curvature of the scleral portion upon decentration and this will displace the anterior surface of the fluid lens. The prism produced will therefore be due to both the contact lens decentration and the displacement of the anterior surface of the fluid lens.

Figure 2.17 illustrates a scleral lens that is riding low. The optical centre of the contact lens is decentred a mm from the visual axis. The centre of curvature of the anterior surface of the fluid lens is displaced from C_0 (its position when the lens is well centred) to C_0'.

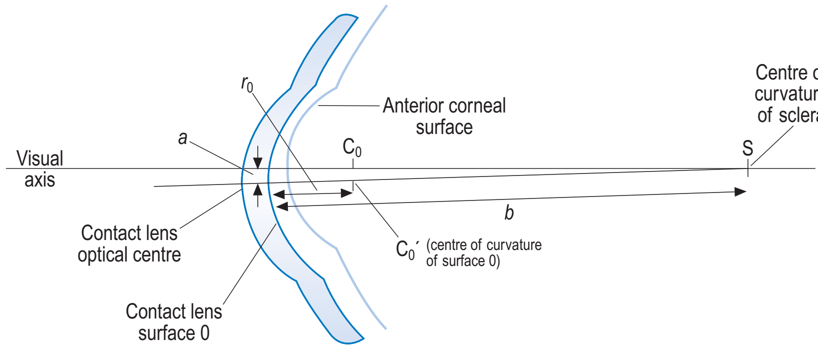

Figure 2.17 A low riding scleral contact lens inducing a vertical prism along the visual axis

The prism at the corneal vertex is due to

- the decentration of the contact lens.
- the displacement of the anterior surface of the fluid lens.

Figure 2.17 illustrates that the fluid lens is producing a base-down prism, which can also be deduced from the fact that the visual axis passes through the upper region of the fluid lens anterior surface which obviously always has a positive power.

2.12.1 *Prism due to the contact lens*

If the contact lens has a BVP of F D in air and is decentred a mm then

$$\text{the prism due to decentration} = \frac{aF}{10}$$

This will be base up for a negative or base down for a positive lens, when the lens rides low.

2.12.2 *Prism due to the fluid lens anterior surface*

Only the anterior surface of the fluid lens is changed by the lens decentration. The posterior surface which is shaped by the cornea obviously remains unchanged. It is therefore only necessary to consider the changes in the anterior surface of the fluid lens. A single refracting surface has no optical centre. The optical centre of a lens is the point which produces no deviation when a light ray passes through. The equivalent in a single refracting surface is the centre of curvature, in that any light ray directed to the centre of curvature will not be deviated as it crosses the surface because it strikes the surface at normal incidence. Therefore, if we can deduce the displacement of the centre of curvature from the visual axis (in cm) and multiply this by the

power of the surface, we will acquire the prism power due to the anterior surface displacement.

In **Figure 2.17**, by similar triangles

$$\frac{C_0C_0'}{a} = \frac{b - r_0}{b} = 1 - \frac{r_0}{b}$$

Therefore

$$C_0C_0' = a\left(1 - \frac{r_0}{b}\right) \text{ mm}$$

The power of the anterior surface of the fluid lens is

$$F_{af} = \frac{336}{r_0}$$

Therefore

$$\begin{aligned}\text{prism power in } \Delta &= \frac{C_0C_0' F_{af}}{10} \\ &= \frac{a(1 - r_0/b)(336/r_0)}{10} \\ &= a\left(\frac{33.6}{r_0} - \frac{33.6}{b}\right) \text{ base down}\end{aligned}$$

N.B. a, r_0 and b are all in millimetres.

2.12.3 Total prism

The total prism produced by decentration is obtained by adding together the prism due to the contact lens and the prism due to the fluid lens anterior surface.

2.13 Summary

1. **A 0.1 mm radius difference between the anterior cornea and posterior contact lens surface produces a fluid lens power of approximately 0.50 D**. This is a most useful rule of thumb which can be used to advantage many times in clinical work. Any calculation involving this rule can be made more precise by constructing a table based on the relationship:

$$F = \frac{1.336 - 1}{r}$$

Such a table constitutes a Heine's scale.

2. Avoid using a trial contact lens with a BVP that requires a high powered lens in the refractor head at the end point of the over-refraction.

3. When modifying scleral lenses, each reduction of 0.05 mm in BOZR must be accompanied by a power change of +0.12 D to the BVP of the modified lens. A reduction of the contact lens central thickness of 0.1 mm must be accompanied by a +0.25 D change in lens power. However, if this also increases the fluid lens central thickness by 0.1 mm then this will require a further power modification of –0.12 D, making the total power change +0.12 D.
4. The prismatic effect of a decentred contact lens will follow the Prentice rule $P = Fc$ when the contact lens lags and the back surface rotates about its own centre of curvature. Where this is not the case the prismatic effect of the decentred anterior fluid lens surface must also be taken into account.

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CHAPTER 3

Edge lift and lens thickness

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The calculation of edge lift and edge thickness requires the deduction of the contact lens overall sag which requires the use of the sag equation.

3.1 The sag equation

Figure 3.1 has the appearance of a bow and arrow with the bow pulled taut prior to firing. The vertical line in the diagram is the arrow and dimension s is called the sag, which is derived from the Latin word *sagitta* meaning arrow. The size of s will be determined by the radius of curvature r of the curved surface and the length of the chord $2y$.

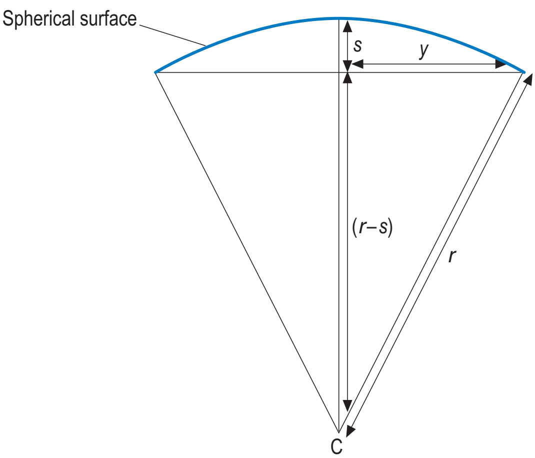

Figure 3.1 The sag of a circular arc. s is the sag, y is the semi-chord, r is the radius of curvature and C is the centre of curvature

In **Figure 3.1**, by Pythagoras' theorem,

$$r^2 = y^2 + (r - s)^2$$

Therefore

$$(r - s)^2 = r^2 - y^2$$

$$r - s = \sqrt{(r^2 - y^2)}$$

$$s = r - \sqrt{(r^2 - y^2)} \qquad (3.1)$$

3.2 Edge lift

The edge lift in a contact lens can be specified in an axial or radial direction. Axial edge lift (AEL) is defined as the distance, measured parallel to the lens axis, between a point on the back surface of a lens at a specified diameter and the continuation of the back central optic zone. Radial edge lift (REL) is the distance between the point on the back surface of the lens at a specified diameter and the continuation of the back central optic zone, measured along the radius of the latter. Thus radial edge lift is measured normal to the central curve.

3.2.1 Axial edge lift

The back surface specification of any contact lens is given by following the International standard ISO 83211–1991 *Optics and optical instruments – Contact Lenses – Part 1: Specification for rigid corneal and scleral lenses*.

The back surface is described by stating the back central optic zone radius (BOZR) followed by the back central optic zone diameter (BOZD). Each peripheral curve is then described in sequence by stating the back peripheral curve radius (BPR) followed by the back peripheral diameter (BPD) working from the centre of the lens to the lens periphery. The final diameter quoted will be the lens total diameter (TD).

A tricurve corneal lens will be used for purposes of illustration. The back surface specification is described as follows

C3 7.60:7.00/8.80:8.00/11.00:8.60

C3 indicates that the lens is a corneal lens with three back surface curves. The numbers represent

BOZR:BOZD/$\mathrm{BPR_1}$:$\mathrm{BPD_1}$/$\mathrm{BPR_2}$:TD

The internationally agreed symbols are

$r_0{:}\phi_0/r_1{:}\phi_1/r_2{:}\phi_T$

The peripheral curves will generate an edge lift as will be seen in the following example.

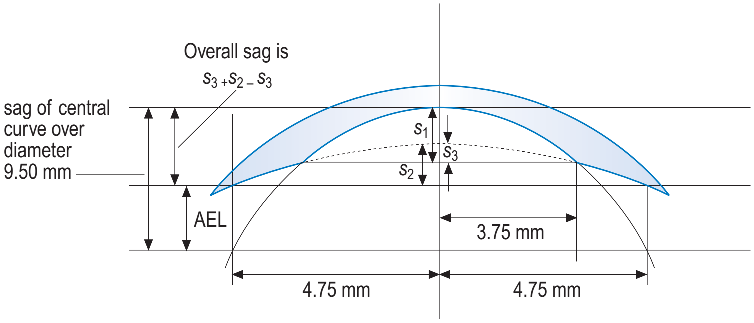

Figure 3.2 The axial edge lift (AEL) of the back surface of a bicurve contact lens

Calculate the axial edge lift at diameter 9.50 mm of the corneal bicurve lens:

C2 7.60:7.50/10.50:10.00

Figure 3.2 illustrates that the AEL is the sag of the central curve extended out to diameter 9.50 mm minus the overall sag of the lens back surface for the same diameter.

The overall sag is

$$s_1 + s_2 - s_3$$

For s_1

$$s_1 = 7.60 - \sqrt{(7.60^2 - 3.75^2)}$$
$$= 0.9896 \text{ mm}$$

For s_2

$$s_2 = 10.50 - \sqrt{(10.50^2 - 4.75^2)}$$
$$= 1.1358 \text{ mm}$$

For s_3

$$s_3 = 10.50 - \sqrt{(10.50^2 - 3.75^2)}$$
$$= 0.6925 \text{ mm}$$

overall sag = 0.9896 + 1.1358 − 0.6925
overall sag = 1.4329 mm

For sag of central curve extended to diameter 9.50 mm

$$s_0 = 7.60 - \sqrt{(7.60^2 - 4.75^2)}$$
$$= 1.6673 \text{ mm}$$

Axial edge lift (l_a) = 1.6673 − 1.4329
l_a = 0.234 mm

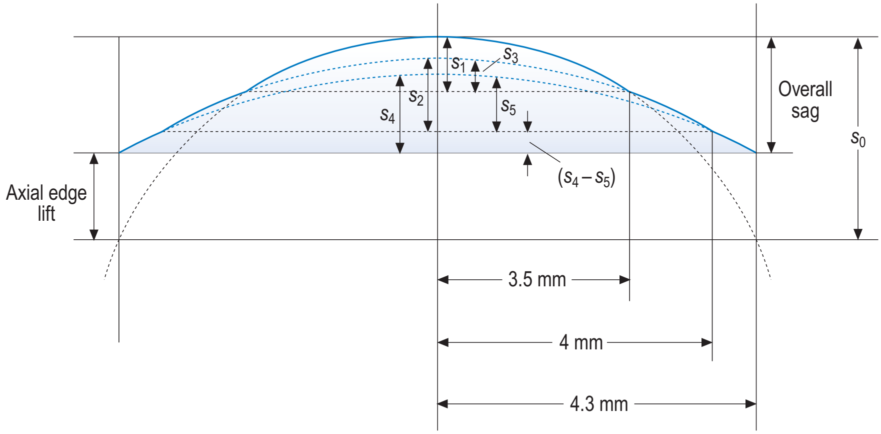

Figure 3.3 Axial edge lift for a tricurve back surface. The values 3.5 and 4 are the semi-diameters of the lens back surface transitions. The value 4.3 is the lens total semi-diameter

The addition of extra back surface curves to convert a bicurve lens into a tricurve, a tricurve to a quadracurve and so on is best illustrated by the next example.

Calculate the axial edge lift of the corneal lens:

C3 7.80:7.00/8.80:8.00/11.00:8.60

Note that we are not given a diameter for edge lift calculation. In these circumstances we must assume that the edge lift is required at the contact lens total diameter which is 8.60 mm in this example.

In **Figure 3.3** we can see that:

AEL $= s_0$ – overall sag

Also that the overall sag $= s_1 + s_2 - s_3 + s_4 - s_5$

This may be more obvious in **Figure 3.4** which is a non-scale diagram. It is suggested that the non-scale diagram is easier to draw and should be constructed when attempting this type of problem. Both diagrams illustrate that the third curve has added distance $(s_4 - s_5)$ to the overall sag when compared to that of a bicurve lens.

Figure 3.4 illustrates that the overall sag for the first two curves is given as before by

overall sag $= s_1 + s_2 - s_3$

The third curve adds a distance to the overall sag equal to $s_4 - s_5$.

For s_1

$$s_1 = 7.80 - \sqrt{(7.80^2 - 3.5^2)}$$
$$= 0.8293 \text{ mm}$$

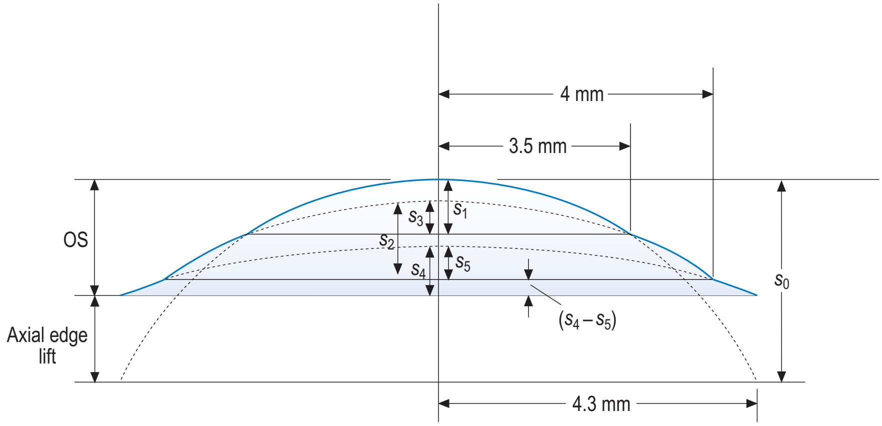

Figure 3.4 A non-scale diagram for a tricurve back surface indicating the sags required to deduce the surface overall sag and the axial edge lift

For s_2

$$s_2 = 8.80 - \sqrt{(8.80^2 - 4^2)} = 0.9616 \text{ mm}$$

For s_3

$$s_3 = 8.80 - \sqrt{(8.80^2 - 3.5^2)} = 0.7260 \text{ mm}$$

For s_4

$$s_4 = 11.00 - \sqrt{(11.00^2 - 4.3^2)} = 0.8753 \text{ mm}$$

For s_5

$$s_5 = 11.00 - \sqrt{(11.00^2 - 4^2)} = 0.7530 \text{ mm}$$

overall sag = 0.8293 +0.9616 – 0.7259 + 0.8753 – 0.7530
overall sag = 1.1872 mm

For s_0

$$s_0 = 7.80 - \sqrt{(7.80^2 - 4.3^2)} = 1.2923 \text{ mm}$$

$$\text{axial edge lift} = s_0 - \text{overall sag} = 1.2923 - 1.1872 = 0.105 \text{ mm}$$

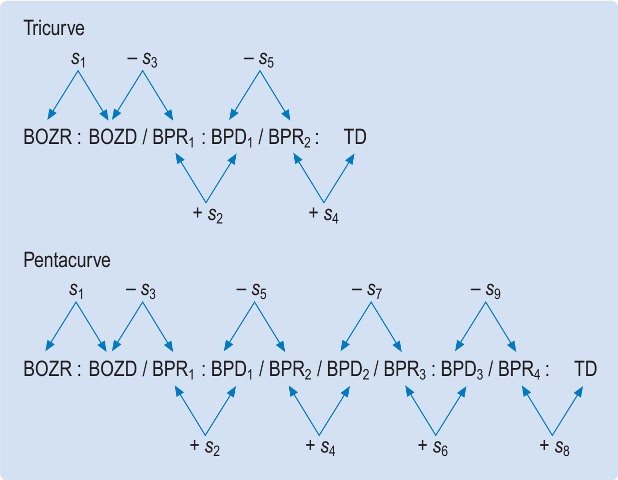

Figure 3.5 The radii and semi-diameters (semi-chords) to be selected for each of the sags to calculate the overall sag for a tricurve followed by a pentacurve back surface

To calculate the overall sag in a tricurve lens we have seen that the relationship between the sags requires

overall sag $= s_1 + s_2 - s_3 + s_4 - s_5$

Figure 3.5 indicates what radii and diameters are selected for the calculation of the sags for a tricurve and a pentacurve lens. When laid out like this, it is clear that, with the exception of s_1, we add all the even sags and subtract the odd sags.

The sequence repeats, following the pattern illustrated in **Figure 3.5**, no matter how many curves are present on the back surface of the contact lens For a tetracurve

overall sag $= s_1 + s_2 - s_3 + s_4 - s_5 + s_6 - s_7$

For a pentacurve

overall sag $= s_1 + s_2 - s_3 + s_4 - s_5 + s_6 - s_7 + s_8 - s_9$

and so on

3.2.2 *Radial edge lift*

The axial edge lift is a convenient parameter. However, it does not represent the shortest distance between the contact lens back surface and the extension of the central curve. The shortest distance is better represented by the radial edge lift (REL). The equation used to derive the REL is given below.

In **Figure 3.6**, the contact lens back surface is resting on a cylindrical pillar of diameter $2y$. Note that C_0 is the centre of curvature of the back optic zone which has a radius of r_0. **Thus REL (l_r) is measured normal to the back optic zone curve**.

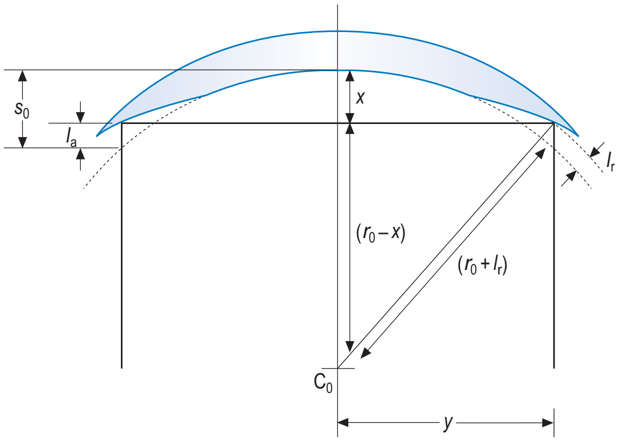

Figure 3.6 Measurement of the overall sag x to deduce the radial edge lift l_r and the axial edge lift l_a. C_0 is the centre of curvature of the back optic zone of radius r_0

From Pythagoras' theorem in **Figure 3.6**

$$(r_0 + l_r)^2 = (r_0 - x)^2 + y^2$$

where r_0 is the BOZR, l_r is the radial edge lift, and x is the overall sag measured on the cylindrical supporting pillar of semi-diameter y. Therefore

$$l_r = \sqrt{[(r_0 - x)^2 + y^2]} - r_0 \qquad (3.2)$$

Equation (3.2) indicates that for derivation of the radial edge lift we need to know the BOZR (r_0), the overall sag x and the semi-chord y.

We can use this equation to calculate the radial edge lift of the last example:

C3 7.80:7.00/8.80:8.00/11.00:8.60

The overall sag was calculated to be 1.1873 mm giving an AEL of 0.105 mm Substitution in Equation (3.2) gives

$$\text{REL} = l_r = \sqrt{[(7.80 - 1.1872)^2 + 4.3^2]} - 7.80 = 0.088 \text{ mm}$$

In practice an instrument similar to the sag measuring devices (like the lens measure used on spectacle lenses) is required for measurement of edge lift. Stone (1975) suggested the adaptation of the radiuscope (optical spherometer) for measuring the lens sag. Her suggested technique involves the use of a supporting cylindrical pillar of accurately known diameter $2y$ (see **Figure 3.6**). The radiuscope is focused on the flat top of the pillar. The contact lens is placed on the top of the pillar and the radiuscope is refocused on to the convex anterior lens surface. The overall sag (x) will be the movement of the radiuscope between these two positions, minus the lens centre thickness. We can measure the BOZR (r_o) using the same instrument. The diameter of the pillar (y) can be measured with a micrometer. We now have all the parameters required to deduce l_r.

The notion of using radial edge lift was suggested by Hodd (1966). **Figure 3.6** also illustrates that the axial edge lift (l_a) is the sag of the central curve (s_0)

over a diameter ($2y$) minus the overall sag (x) measured by the radiuscope. Thus

$$\text{axial edge lift } l_a = s_0 - x \tag{3.3}$$

s_0 can be easily calculated, using equation (3.1), if the BOZR is measured and y is known.

The above technique can be used for checking the edge lift of any back surface design including multicurve, offset or aspheric surfaces. The only difference between the different designs is the way that the overall sag is calculated. In the above measurements, the overall sag is measured in exactly the same way for all designs.

The accuracy of the technique is adversely affected by poor centration of the lens on the pillar. Douthwaite and Hurst (1998a, b) devised a collar for the pillar to ensure that the lens was always accurately centred on the pillar. They examined multicurve lenses and their results indicated that edge lift can be measured to an accuracy sufficiently high to ensure that the ISO tolerances on peripheral radius are met. The technique was also validated on aspheric back surfaces by Dietze *et al*. (2003). Thus the measurement of edge lift provides us with an indirect means of assessing the peripheral curve radii and diameters or the asphericity of a conicoidal surface. If any of these are outside the established tolerances then the edge lift measurement will also be in error. We therefore have a convenient clinical technique to assess the accuracy of the manufacture of the lens periphery.

It is worth noting that, for lens checking purposes, the deduction of edge lift is not necessary. If we wish to check the accuracy of the peripheral radii and diameters of an actual lens, then all we need to do is measure the lens overall sag. This measurement can then be compared with the calculated result. Any difference between the measured and the calculated overall sag indicates inaccuracy of one or more of the contact lens back surface peripheral dimensions.

The relationship between REL and AEL is not exactly constant. However, the ratio varies little over typical contact lens parameter variation. The radial-to-axial ratio is around 0.8 for an axial edge lift of 0.175 mm. A constant axial edge lift gives more radial lift on flat lenses; however, the increase in radial lift with increase in BOZR typically amounts to something only around 0.016 mm from the flattest to the steepest lens.

One final point that should be made on the topic of edge lift is that, as the BOZR of a lens is increased, the degree of flattening of the peripheral curve or curves must be increased to maintain a particular edge lift. Thus:

C3 7.20:6.50/7.70:7.50/8.20:8.50

gives an axial edge lift of 0.078 mm. Note that this is produced by flattening the peripheral curves in 0.5 mm steps.

C3 8.20:6.50/8.70:7.50/10.00:8.50

also gives an axial edge lift of 0.078 mm. The mid peripheral curve is once again 0.5 mm flatter than the BOZR but the third curve is 1.3 mm flatter than the mid curve.

3.2.3 *Calculation of the back peripheral radius required to give a specified axial edge lift*

One possible approach to contact lens design is to decide on the axial edge lift required and then to calculate the back peripheral radius or radii needed to achieve this. In the case of a bicurve design, we would know the BOZR, BOZD, TD and AEL and would need to calculate the BPR.

Figure 3.7 illustrates the principles involved in this problem. We wish to determine the back peripheral radius (distance BC_{1}) having been given distance l_a. We would know the BOZR, BOZD, TD and AEL.

s_1 = sag of the BOZR over the BOZD ($2y_0$)
s_0 = sag of BOZR over the lens total diameter ($2y_1$)
x = overall sag of the lens back surface
l_a = AEL
$x = s_0 - l_a$

In triangle BDE

$$BE = x - s_1$$
$$DE = y_1 - y_0$$
$$\angle MC_1A = \theta$$

because line C_1M bisects the chord BD and $\angle C_1MD$ is a right angle

$$\angle BDE = \theta$$
$$\tan\theta = \frac{BE}{DE}$$
$$= \frac{x - s_1}{y_1 - y_0}$$

We now know θ.

In triangle BDE, if M bisects BD then P bisects ED. Therefore

$$DP = PE = \frac{y_1 - y_0}{2}$$
$$MC_1 = \frac{y_0 + PE}{\sin\theta}$$
$$\cos\theta = \frac{DP}{DM}$$
$$DM = \frac{DP}{\cos\theta}$$
$$DM = MB$$

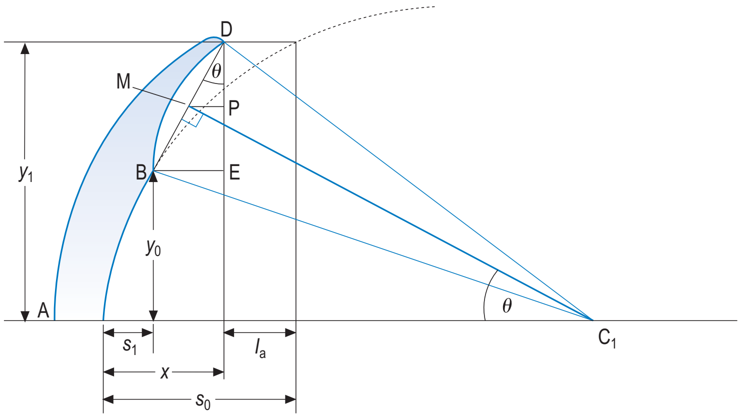

Figure 3.7 Deduction of the back peripheral radius (BC_1) given the axial edge lift (l_a)

In triangle BC_1M

$$BC_1^2 = MB^2 + MC_1^2$$

We now know the radius of curvature for the peripheral curve BC_1.

We can now take a numerical example.

A corneal bicurve contact lens is to be manufactured with a BOZR of 7.90 mm and a BOZD of 6.00 mm. The total diameter will be 9.50 mm. What back peripheral radius is required to give an axial edge lift of 0.08 mm at a diameter of 9 mm?

$$s_1 = 7.9 - \sqrt{(7.9^2 - 3^2)} = 0.5918 \text{ mm}$$

$$s_0 = 7.9 - \sqrt{(7.9^2 - 4.5^2)} = 1.4069 \text{ mm}$$

$$x = s_0 - z = 1.4069 - 0.08 = 1.3269 \text{ mm}$$

In triangle BDE

$$BE = 1.3269 - 0.5918 = 0.7351 \text{ mm}$$

$$DE = 4.5 - 3 = 1.5 \text{ mm}$$

$$\angle BDE = \theta$$

$$\tan\theta = \frac{BE}{DE} = \frac{0.7351}{1.5} = 0.4901$$

$$\theta = 26.1079°$$

In triangle BDE, if M bisects BD then P bisects ED. Therefore

$$DP = PE = \frac{1.5}{2} = 0.75$$

$$MC_1 = \frac{3 + 0.75}{\sin 26.1079} = 8.5215 \text{ mm}$$

$$\cos\theta = \frac{DP}{DM}$$

$$DM = \frac{0.75}{\cos 26.1079} \qquad = 0.8352 \text{ mm}$$

$$DM = BM$$

In triangle BC_1M

$$BC_1^2 = MB^2 + MC_1^2 = 0.8352^2 + 8.5215^2 = 73.3135 \text{ mm}$$
$$BC_1 = 8.5623 \text{ mm}$$
$$BPR_1 = 8.56 \text{ mm for an AEL of 0.08 mm at a diameter of 9.00 mm}$$

3.2.4 The sharing of the AEL in multicurve lenses

The calculation of AEL for a bicurve lens, as above, is straightforward. The practitioner will have decided on a BOZR. The dimensions that require a decision from the lens designer are BOZD, TD and BPR_1 to produce the AEL required. Where there is more than one peripheral curve, the designer must decide on how much of the edge lift will be shared by each individual curve.

Before that decision is made it is necessary to decide on what is meant by the edge lift of an intermediate curve. There are two possibilities.

The first is to consider the edge lift of an intermediate curve where it finishes, as illustrated in **Figure 3.8(a)**. This has been called the 'bandwidth' method by Rabbetts (1993). So that for the specification

C3 7.90:7.80/9.10:8.60/11.90:9.00

the AEL of the 9.10 mm radius curve will be 0.041 mm at diameter 8.60 mm.

The AEL provided by the 11.90 mm radius curve at diameter 9.00 mm is calculated by assuming a back surface specification of

C2 7.90:8.60/11.90:9.00

The AEL for this specification is 0.055 mm. Thus the total AEL for the total diameter of 9.00 mm is 0.041 + 0.055 = 0.096 mm.

The second approach is illustrated in **Figure 3.8(b)**. Rabbetts (1993) has called this the 'step-by-step' approach. Here the AEL of the mid curve is calculated as the AEL that is produced if the mid curve is extended out to the lens total diameter. We therefore see the influence of the mid curve at the edge of the lens and the term axial **edge** lift is more appropriate. In other words we have considered a specification of

C2 7.90:7.80/9.10:9.00

This gives the AEL of the mid curve at diameter 9.00 mm.

In our example the 9.10 mm radius curve produces an AEL of 0.065 mm at the TD of 9.00 mm.

To calculate the AEL produced by the second peripheral curve we consider the back surface to be

C2 9.10:8.60/11.90:9.00

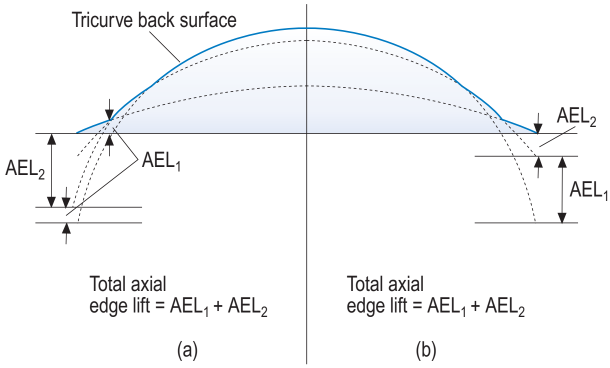

Figure 3.8 (a) The 'band width' and (b) the 'step-by-step' approaches to the relative contribution of peripheral curves to the total axial edge lift

The 11.90 mm radius curve produces an AEL of 0.031 mm at the TD of 9.00 mm
This gives a total AEL of 0.065 + 0.031 = 0.096 mm.

Therefore the two approaches result in the same total AEL but the relative contribution of the two peripheral curves is assessed in a different manner. The step-by-step approach produces a larger AEL for the mid peripheral curve and a smaller AEL for the third curve when compared with the bandwidth approach. It is obviously important to be clear which system is being adopted when the topic of AEL involves multicurve lenses. We will subsequently use the step by step approach which calculates the influence of every curve out at the specified extreme diameter.

We have seen that it is not necessary to specify the AEL at the very edge of the lens. It can be specified at any diameter. The advantage of specifying the AEL at a diameter slightly less than the lens TD is that this avoids the influence of the edge taper and edge polish on the finished lens and in consequence this AEL could be checked using Janet Stone's method described in Section 3.2.2.

Having arrived at the way in which the contribution of each peripheral curve is calculated, we must decide how much of the total AEL is to be produced by each curve. If the component peripheral curves share the AEL equally (step-by-step) then the design finishes up with very flat outer peripheral curves. The most peripheral curve starts close to the lens edge. It must, therefore, be very flat if it is to make a substantial contribution to the AEL. This type of exercise lends itself to computerization. There are two computer programs in Chapter 11 that calculate AEL, REL and BPRs for multicurve lenses (C2 to C5).

Table 3.1 indicates the contribution of each peripheral curve in the default setting of the computer programs. The bicurve has only one peripheral curve and this is therefore responsible for the whole of the AEL. In the tricurve lens, the mid curve produces two-thirds of the AEL leaving the third curve with a one-third contribution. In the C4 the first peripheral curve produces $\frac{1}{2}$, the

Table 3.1 Distribution of the share of the axial edge lift in the multicurve lenses in the computer programs (step-by-step approach)

Lens type	BPR1	BPR2	BPR3	BPR4
C2	1.00			
C3	0.67	0.33		
C4	0.50	0.33	0.17	
C5	0.40	0.30	0.20	0.10

second $\frac{1}{3}$ and the fourth $\frac{1}{6}$. In the C5 the contribution is $\frac{4}{10}$, $\frac{3}{10}$, $\frac{2}{10}$ and $\frac{1}{10}$ respectively for the four peripheral curves. These designs give respectable tear layer profiles when matched against typical corneal curves. Having a computer program to calculate the radii, like those in Chapter 11, makes it very easy to modify the way in which the AEL is shared. There are programs to calculate the axial and radial edge lift given the back surface radii and programs to calculate the back surface radii given the AEL in any given lens. The default relative contributions of the peripheral curves can be changed if the user wishes to do this.

3.3 Edge and centre thickness

The deduction of either the centre or the edge thickness of a contact lens, when the other contact lens surface parameters are known, once again revolves around the use of the sag equation (equation (3.1)). The edge thickness can be calculated as an axial or a radial edge thickness. The calculation of edge thickness can be applied to the calculation of lens thickness at any point on the lens. Thus a lens thickness profile can be constructed. Lens thickness variations do not only influence the fitting and comfort of a lens but also influence the gas transmission of the material. A thin lens is not only likely to be comfortable but will also give the best oxygen transmission for a given material permeability. Many modern soft lens designs incorporate lenticulation in order to allow minimum thickness with minimum thickness variation.

3.3.1 Axial edge thickness

The axial edge thickness (t_{ak}) is measured from the lens back surface to the lens front surface in a direction parallel to the optical axis of the lens at a specified diameter. A typical axial edge thickness example is illustrated below.

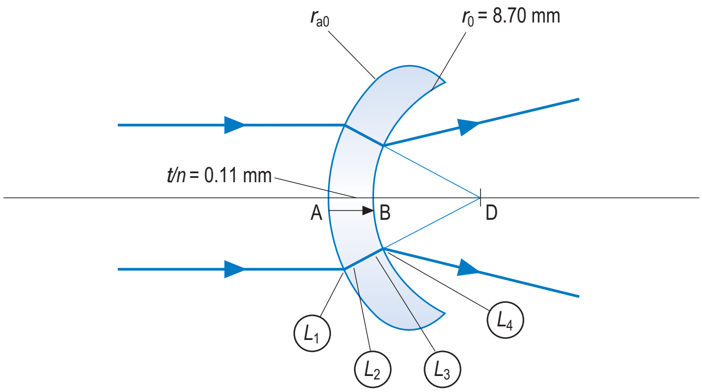

Figure 3.9 The vergences and distances required to calculate the front surface radius (r_{ao})

What is the axial edge thickness ignoring any effects of an edge bevel at diameter 14.30 mm. Also calculate the axial thickness at diameter 12.40 mm in the following soft contact lens

8.70:12.50/9.80:14.40 BVP –5.00 D
refractive index 1.385, centre thickness 0.15 mm

We first need to calculate the front surface radius. **Figure 3.9** illustrates the vergences.

$$t/n = 0.11 \text{ mm}, \quad F_0 = \frac{-385}{8.70} = -44.25 \text{ D}, \quad L_4 = -5.00 \text{ D}$$

Vergence (D)		**Distance (mm)**
$L_4 = -\ 5.00$		
$F_0 = \underline{-\ 44.25}$ subtract		
$L_3 = +39.25 \longrightarrow$	$\dfrac{1000}{39.25} \longrightarrow$	$25.48 = \text{BD}$
		$0.11 = t/n$
$L_2 = +39.08 \longleftarrow$	$\dfrac{1000}{27.70} \longleftarrow$	$25.59 = \text{AD}$
$L_1 = \underline{\ \ 0.00}$ subtract		
$F_{a0} = +39.08$		

$$r_{a0} = \frac{n-1}{F_{a0}} = \frac{385}{39.08} = 9.8516 \text{ mm}$$

$$r_{a0} = 9.85 \text{ mm}$$

Now we have the front surface radius, we need to draw a diagram to deduce the edge thicknesses. **Figure 3.10** illustrates how we calculate the edge thickness at diameter 14.30 mm.

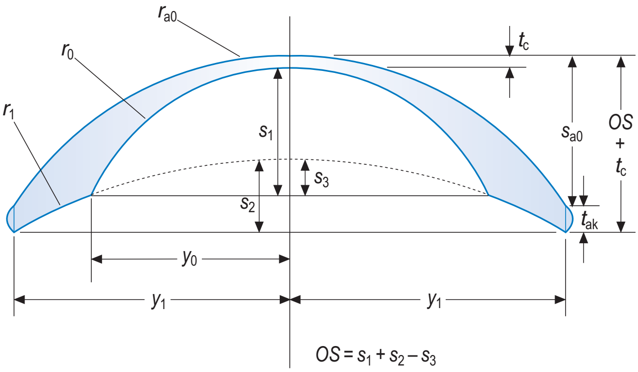

Figure 3.10 The sags of the contact lens back surface. t_c is the centre thickness and t_{ak} is the axial edge thickness. $t_{ak} = (OS + t_c) - s_{a0}$

The axial edge thickness (t_{ak}) equals the overall sag (OS) plus the centre thickness (t_c) minus the sag of the front surface (s_{a0}). The overall sag is seen to be $s_1 + s_2 - s_3$ as before for a bicurve lens. All that remains therefore is to calculate sags s_1, s_2, s_3 and s_{a0}.

For s_1

$$\begin{aligned} s_1 &= 8.7 - \sqrt{(8.7^2 - 6.25^2)} \\ &= 2.6479 \text{ mm} \end{aligned}$$

For s_2

$$\begin{aligned} s_2 &= 9.8 - \sqrt{(9.8^2 - 7.15^2)} \\ &= 3.0979 \text{ mm} \end{aligned}$$

For s_3

$$\begin{aligned} s_3 &= 9.8 - \sqrt{(9.8^2 - 6.25^2)} \\ &= 2.2517 \text{ mm} \\ t_c &= 0.15 \end{aligned}$$

Therefore

$$s_1 + s_2 - s_3 + t_c = 3.6441 \text{ mm}$$

Front surface sag

$$\begin{aligned} s_{a0} &= 9.85 - \sqrt{(9.85^2 - 7.15^2)} \\ &= 3.070 \text{ mm} \end{aligned}$$

Axial edge thickness $t_{ak} = 3.6441 - 3.0750 = 0.5691$ mm
$t_{ak} = 0.570$ mm

To calculate the axial thickness at diameter 12.40 mm requires the use of the same relationships. The only difference is that the back surface out to diameter 12.40 mm contains only one curve and so the overall sag is the sag of this curve.

Overall sag = s_1

$$s_1 = 8.7 - \sqrt{(8.7^2 - 6.2^2)}$$
$$= 2.5967 \text{ mm}$$
$$t_c = 0.15$$

Therefore

$$s_1 + t_c = 2.7467 \text{ mm}$$

Front surface sag

$$s_{a0} = 9.85 - \sqrt{(9.85^2 - 6.2^2)}$$
$$= 2.1961 \text{ mm}$$

$$\text{Axial thickness } t_{ak} \text{ at diameter 12.40 mm} = 2.7467 - 2.1961$$
$$= 0.5506$$
$$t_{ak} = 0.551 \text{ mm}$$

Thus this negative lens has an axial edge thickness of 0.551 mm at diameter 12.40 mm. The flat peripheral back surface radius which starts at diameter 12.50 mm serves to minimize any further increase in thickness all the way out to the lens edge. The above example illustrates one of the problems of soft lens design that the lens is quite thick in the transition region of the lens compared to the centre thickness. The underlying area of the cornea will receive a poor oxygen supply due to poor oxygen transmission arising from the increased lens thickness. Lenses can be made thinner by lenticulation.

3.3.2 *Lenticulation*

3.3.2.1 Lenticulation of negative power lenses

The previous example was a lens that is thick in the more peripheral regions compared to the centre thickness. This excessive thickness can be reduced by providing the negative lens with a positive carrier. Let us assume that we decide to lenticulate this lens with a FOZD (ϕ_{a0}) of 8.10 mm. The cross-section of the optic portion of the lenticulated lens will be as shown in **Figure 3.11** which illustrates that the junction thickness (t_{a0}) is the centre thickness (t_c) plus the sag of the central back surface curve (s_0) minus the sag of the central front surface curve (s_{a0}).

$$s_0 = 8.7 - \sqrt{(8.7^2 - 4.05^2)} = 1.0002 \text{ mm}$$
$$s_{a0} = 9.85 - \sqrt{(9.85^2 - 4.05^2)} = 0.8711 \text{ mm}$$
$$t_c = 0.15 \text{ mm}$$
$$t_{a0} = 1.0002 + 0.15 - 0.8711 = 0.2791 \text{ mm}$$

Figure 3.12 illustrates the lenticulated lens where it can be seen that the axial edge thickness (t_{ak}) is equal to the centre thickness (t_c) plus the overall sag of the back surface (OS) minus the overall sag of the front surface (OS_a). In this

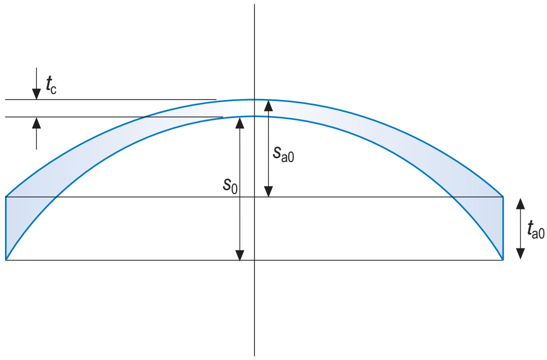

Figure 3.11 Finding the junction thickness (t_{a0}) at the edge of the optic zone of a lenticulated lens. $t_{a0} = (s_0 + t_c) - s_{a0}$

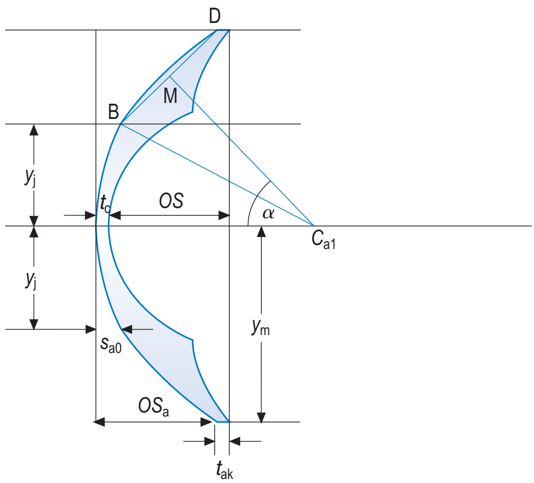

Figure 3.12 The cross-section of a negative lenticulated soft lens. BD is the front peripheral curve with its centre of curvature at C_{a1}

case, both the front and back surfaces are of bicurve construction so the overall sag of both surfaces will be derived using

overall sag = $s_1 + s_2 - s_3$

The lens designer will need to calculate what front peripheral radius is required to achieve a requested axial edge thickness.

Let us suppose that we want an axial edge thickness (t_{ak}) of 0.08 mm at diameter 14.30 mm. The lens back surface specification is

8.70:12.50/9.80:14.40 BVP –5.00, $n_p = 1.385$, $t_c = 0.15$ mm,
FOZD = 8.10 mm

Calculating the peripheral radius

The overall sag of the back surface OS is $s_1 + s_2 - s_3$.

$$s_1 \quad = 8.7 - \sqrt{(8.7^2 - 6.25^2)} = 2.6479 \text{ mm}$$
$$s_2 \quad = 9.8 - \sqrt{(9.8^2 - 7.15^2)} = 3.0979 \text{ mm}$$

$$s_3 = 9.8 - \sqrt{(9.8^2 - 6.25^2)} = 2.2517 \text{ mm}$$
$$OS = 3.4941 \text{ mm}$$

From Figure 3.12

$$OS_a = OS + t_c - t_{ak} = 3.4941 + 0.15 - 0.08$$
$$OS_a = 3.5641 \text{ mm}$$
$$s_{a0} = 9.85 - \sqrt{(9.85^2 - 4.05^2)} = 0.8711 \text{ mm}$$

In Figures 3.12 and 3.13

$$\tan\theta = \frac{y_m - y_j}{OS_a - s_{a0}} = \frac{3.1}{2.6930} = 1.1511$$

$$\theta = 49.0180°$$

$$BD = \frac{OS_a - s_{a0}}{\cos\theta} = 4.1063 \text{ mm}$$

$$BM = \frac{BD}{2} = 2.0532 \text{ mm}$$

$$MP = BM \cdot \sin\theta = 1.5500 \text{ mm}$$
$$\angle\alpha = 90 - \theta = 90 - 49.018 = 40.982°$$
$$MC_{a1} = \frac{y_j + MP}{\sin\alpha} = \frac{4.05 + 1.5500}{0.6558} = 8.5389 \text{ mm}$$

In ΔBMC_{a1}

$$BC_{a1}{}^2 = BM^2 + MC_{a1}{}^2 = 4.2156 + 72.9128 = 77.1284$$
$$BC_{a1} = 8.7823 \text{ mm}$$
$$r_{a1} = 8.78 \text{ mm}$$

The lenticulation has reduced the lens thickness from the FOZD outwards compared to the non-lenticulated lens. The non-lenticulated thickness at diameter 12.40 mm was 0.551 mm. This is reduced to 0.303 mm in the lenticulated lens. The axial edge thickness at diameter 14.30 mm non-lenticulated was 0.569 mm. This is reduced to 0.08 mm in the lenticulated lens.

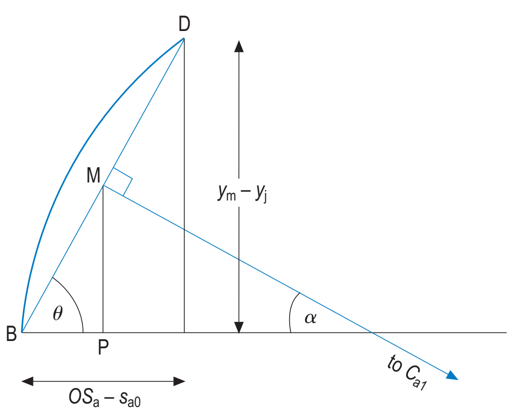

Figure 3.13 An enlarged view of the front peripheral curve region from Figure 3.12

The lenticulated lens has a steeper front surface peripheral curve producing a carrier region that is thicker at the FOZD junction (t_{a0} = 0.279 mm) than at the lens edge (t_{ak} = 0.08 mm). This is called a positive carrier. It will help to discourage both soft and RGP lenses from riding high when worn on the eye. The calculation of peripheral radius for an RGP lens would be dealt with in exactly the same way as in the above example.

3.3.2.2 Lenticulation of positive power lenses

A contact lens with a high positive power will possess a centre of gravity which is well forward, and this encourages a soft or RGP lens to ride low on the eye. Some improvement in position may be achieved by reducing the central thickness of the lens, which usually means that the lens must be made in lenticular form as illustrated in **Figure 3.14** which shows that the carrier zone (i.e. the lens periphery) should display a slightly increasing thickness towards the lens periphery in order to encourage the upper lid to hold the lens in a higher position. This is called a negative carrier. The axial thickness of the junction should not be less than around 0.15 mm. This form of lenticulation once again reduces the lens thickness profile overall. However, the major gain in gas transmission will occur at the lens centre because the lenticulation allows a substantial reduction in lens centre thickness.

The problem with positive lenses is determining the value of the lens central thickness required for a given junction thickness. Calculations involved in determining the lens parameters are illustrated in the following example.

A soft lens of the following specification

8.90/14.00 BVP + 8.00 D

is a monocurve back surface design that is lenticulated with a FOZD of 8.00 mm. The junction axial thickness must be 0.15 mm. The contact lens refractive index is 1.380.

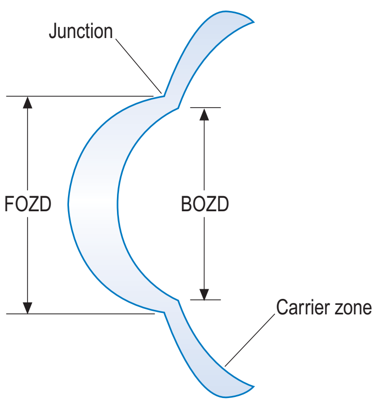

Figure 3.14 A lenticulated positive power contact lens with a negative carrier

(a) What is the centre thickness required for this lens?
(b) What is the front peripheral radius if the design requires an axial edge thickness of 0.25 mm at diameter 14.00 mm neglecting the effect of an edge bevel?

In this example the junction thickness represents an optimum value, in that it is as thin as possible without putting the lens durability at risk. The axial edge thickness requested will ensure that there is a negative carrier although this represents an edge thickness that is greater than the optimum.

The centre thickness

Figure 3.15 illustrates that the centre thickness t_c can be determined if we know the junction thickness t_{a0}, the sag of the front surface s_{a0} and the overall sag of the back surface OS for the junction semi-diameter y_j.

$$t_c = s_{a0} + t_{a0} - OS \tag{3.4}$$

to find OS.

The overall sag of the monocurve back surface is determined using equation (3.1)

$$OS = 8.9 - \sqrt{(8.9^2 - 4^2)} = 0.9495 \text{ mm}$$

The sag of the front surface

$$s_{a0} = r_{a0} - \sqrt{(r_{a0}{}^2 - y_j{}^2)}$$

$$t_c = r_{a0} - \sqrt{(r_{a0}{}^2 - y_j{}^2)} + t_{a0} - OS \quad \text{from (3.4)}$$

To calculate t_c, the front optic zone radius, r_{a0}, must be known. This is the radius which gives the lens its required BVP.

From Section 2.4

$$L_4 = \frac{F_{a0}}{1 - (t/n)\,F_{a0}} + F_0 \tag{3.5}$$

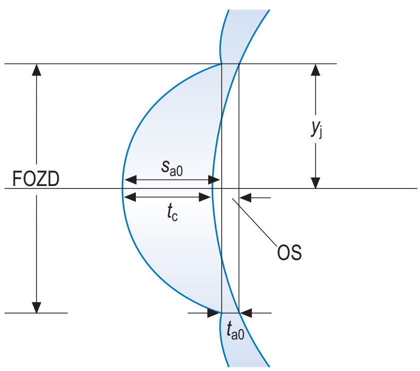

Figure 3.15 The central optic region of a positive powered lenticulated contact lens

where L_4 is the emergent vergence when light of zero incident vergence is refracted by surface powers F_{a0} (front) and F_0 (back) separated by thickness t in a lens of refractive index n.

Substituting

$$F_{a0} = \frac{n-1}{r_{a0}} \quad \text{and} \quad F_0 = \frac{1-n}{r_0}$$

into equation (3.5) and rearranging gives

$$r_{a0} = \frac{(n-1)\, r_0}{L4 \cdot r_0 + (n-1)} + \frac{t(n-1)}{n} \tag{3.6}$$

putting this in terms of the lens parameters

$$\text{FOZR } r_{a0} = \frac{1000(n-1)\text{BOZR}}{\text{BVP} \cdot \text{BOZR} + 1000(n-1)} + \frac{t_c(n-1)}{n} \tag{3.7}$$

The problem which arises here is that the front optic zone radius, r_{a0}, and the central thickness, t_c, are interdependent in that if the central thickness changes then the front optic zone radius will need to change in order to maintain the back vertex power.

To simplify matters we can group some of the constants together.

$$\text{Let K} = \frac{1000(n-1)\text{BOZR}}{\text{BVP} \cdot \text{BOZR} + 1000(n-1)} \tag{3.8}$$

$$\text{Let N} = \frac{n}{n-1} \tag{3.9}$$

$$\text{Let M} = t_{a0} - OS \tag{3.10}$$

Thus

$$r_{a0} = \text{K} + t_c/\text{N} \quad \text{from (3.7)}$$

Therefore

$$t_c = (r_{a0} - \text{K})\text{N} \tag{3.11}$$

Also

$$t_c = r_{a0} - \sqrt{(r_{a0}{}^2 - y_j{}^2)} + \text{M} \quad \text{from (3.4) and (3.10)}$$

Therefore

$$\begin{aligned}
r_{a0} - \sqrt{(r_{a0}{}^2 - y_j{}^2)} + \text{M} &= (r_{a0} - \text{K})\text{N} \\
r_{a0} + \text{M} - (r_{a0} - \text{K})\text{N} &= \sqrt{(r_{a0}{}^2 - y_j{}^2)} \\
r_{a0} + \text{M} - \text{N}r_{a0} + \text{KN} &= \sqrt{(r_{a0}{}^2 - y_j{}^2)} \\
(1-\text{N})r_{a0} + (\text{KN} + \text{M}) &= \sqrt{(r_{a0}{}^2 - y_j{}^2)} \\
[(1-\text{N})r_{a0} + (\text{KN} + \text{M})]^2 - (r_{a0}{}^2 - y_j{}^2) &= 0 \\
(1-\text{N})^2 r_{a0}{}^2 + 2(1-\text{N})(\text{KN} + \text{M})r_{a0} + (\text{KN} + \text{M})^2 - r_{a0}{}^2 + y_j{}^2 &= 0 \\
[(1-\text{N})^2 - 1]\text{r}_{a0}{}^2 + 2(1-\text{N})(\text{KN} + \text{M})r_{a0} + (\text{KN} + \text{M})^2 + y_j{}^2 &= 0 \\
(\text{N}^2 - 2\text{N})r_{a0}{}^2 + 2(1-\text{N})(\text{KN} + \text{M})r_{a0} + (\text{KN} + \text{M})^2 + y_j{}^2 &= 0
\end{aligned}$$

This is a quadratic equation of the form

$$Ar_{a0}^2 + Br_{a0} + C = 0$$

where

$$A = N^2 - 2N$$
$$B = 2(1 - N)(KN + M)$$
$$C = (KN + M)^2 + y_j^2$$

which is solved by the standard form

$$r_{a0} = \frac{-B - \sqrt{(B^2 - 4AC)}}{2A}$$

In our example

$$K = \frac{(380)(8.9)}{(8)(8.9) + 380} = 7.4956 \quad \text{from (3.8)}$$

$$N = 1.380/0.380 = 3.6316 \quad \text{from (3.9)}$$
$$M = 0.15 - 0.9495 = -0.7995 \quad \text{from (3.10)}$$

Hence:

$$A = 5.9253$$
$$B = -139.0617$$
$$C = 714.0968$$

$$r_{a0} = 7.5893 = 7.59 \text{ mm}$$

from (3.7)

$$t_c = (r_{a0} - K)N \tag{3.11}$$
$$t_c = 0.3403 = 0.34 \text{ mm}$$

Therefore the lens central thickness is 0.34 mm.

To cross check the result we have:

from (3.4)

$$t_{a0} = t_c + OS - s_{a0}$$
$$s_{a0} = 7.5893 - \sqrt{(7.5893^2 - 4^2)} = 1.1397$$

Thus

$$t_{a0} = 0.3403 + 0.9495 - 1.1397$$

Junction thickness

$$t_{a0} = 0.15 \text{ mm}$$

The peripheral radius

In **Figure 3.16**, t_{ak} is the axial edge thickness, OS is the overall sag of the back surface out to the specified diameter for edge thickness $2y_m$, t_c is the centre

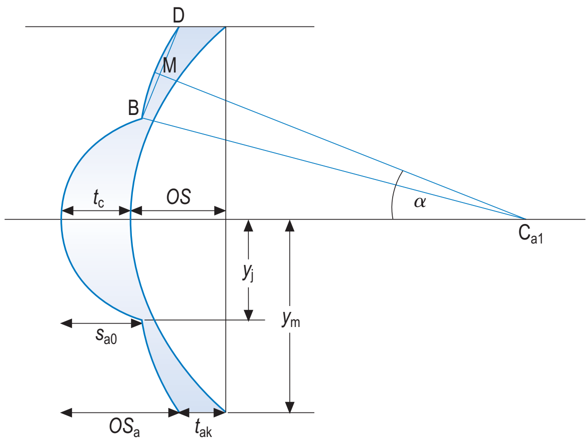

Figure 3.16 The cross-section of a positive power lenticulated lens. BD is the front peripheral curve with its centre of curvature at C_{a1}

thickness, s_{a0} is the sag of the front surface over the front optic zone diameter and y_j is the front optic zone semi-diameter.

The overall sag OS for the back surface is derived using the standard sag equation (3.1) as before:

$$OS = 8.9 - \sqrt{(8.9^2 - 7^2)} = 3.4036 \text{ mm}$$

$$OS_a = OS + t_c - t_{ak}$$

$$OS_a = 3.4036 + 0.3403 - 0.25 = 3.4939 \text{ mm}$$

$$s_{a0} = 7.5893 - \sqrt{(7.5893^2 - 4^2)} = 1.1397 \text{ mm}$$

In Figure 3.17

$$\tan\theta = \frac{y_m - y_j}{OS_a - s_{a0}} = \frac{3}{2.3542}$$

$$= 1.2743$$

$$\theta = 51.8771°$$

$$\text{BD} = \frac{OS_a - s_{a0}}{\cos\theta} = \frac{2.3542}{0.6174}$$

$$= 3.8131$$

$$\text{BM} = \frac{\text{BD}}{2} = 1.9066$$

$$\sin\theta = \frac{\text{MP}}{\text{BM}}$$

$$\text{MP} = \text{BM} \cdot \sin\theta = 1.9066 \times 0.7867$$

$$= 1.4999 \text{ mm}$$

$$\angle\alpha = 90 - \theta = 90 - 51.8771$$

$$= 38.1229°$$

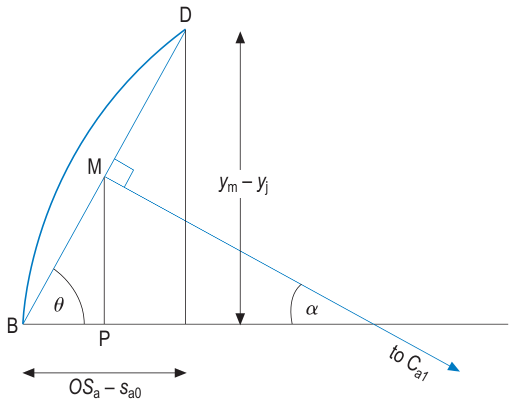

Figure 3.17 An enlarged view of the front peripheral curve BD from Figure 3.16

In Figures 3.16 and 3.17

$$\begin{aligned} MC_{a1} &= \frac{y_j + MP}{\sin \alpha} \\ &= \frac{5.4999}{0.6174} \\ &= 8.9082 \text{ mm} \end{aligned}$$

In $\Delta BC_{a1}M$

$$\begin{aligned} BC_{a1}{}^2 &= BM^2 + MC_{a1}{}^2 \\ &= 1.9066^2 + 8.9082^2 \\ &= 82.9912 \\ BC_{a1} &= 9.1100 \text{ mm} \\ r_{a1} &= 9.11 \text{ mm} \end{aligned}$$

where r_{a1} is the front peripheral radius.

A computer program is included in Chapter 11 which follows the method described above in order to calculate central thickness and FPR_1 for both positive and negative lenses with back surfaces consisting of multicurves up to C5s, offset bicurves and aspherics.

3.3.3 *Radial thickness*

In the case of RGP and soft lenses an assessment of the oxygen transmission by the lens can only be made if the thickness of the lens is calculated. Axial thickness is inappropriate for dealing with the problems of gas flowing through the material. If we assume that the flow is via the shortest route then we need to calculate the radial thickness. This produces a dilemma because the flow cannot be radial to both surfaces. If we express radial thickness in relation to an extension of the lens BOZR then this results in an inappropriate measurement for the periphery of a multicurve or aspheric back surface

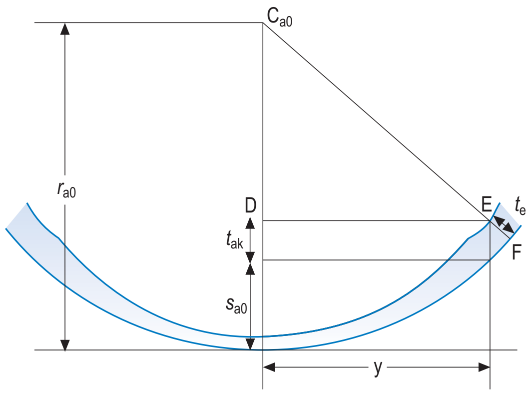

Figure 3.18 The axial edge thickness (t_{ak}) and the radial edge thickness (t_e) of a bicurve design. C_{a0} is the centre of curvature of the front surface

contact lens, since the direction of measurement is not normal to either front or back peripheral surfaces.

The best compromise is probably to measure the lens thickness normal to the front surface of the contact lens and this is the recommendation in the *International Standard 8320 – 1986 and the British Standard 3521: part 3: 1988* which deal with the terminology for contact lenses. **Thus radial thickness is measured normal to the front surface**.

We can use the axial edge thickness as a starting point to calculate the radial thickness. Figure 3.18 illustrates the relationships where r_{a0} is the radius of curvature of the front surface, t_{ak} is the axial edge thickness and s_{a0} is the sag of the front surface over semi-diameter y.

In the triangle $C_{a0}DE$

$$C_{a0}E^2 = C_{a0}D^2 + DE^2$$
$$C_{a0}E^2 = (r_{a0} - s_{a0} - t_{ak})^2 + y^2$$
$$C_{a0}F = r_{a0}$$
$$t_e = C_{a0}F - C_{a0}E$$
$$t_e = r_{a0} - C_{a0}E$$

where t_e is the radial edge thickness measured normal to the front surface **from a specified semi-diameter on the back surface**.

Take the example in Section 3.3.1 where we considered a soft lens of the following specification

8.70:12.50/9.80:14.40 BVP –5.00 D
refractive index 1.385, centre thickness 0.15 mm

We now wish to know the radial edge thickness (t_e) at diameter 14.30 mm.
The front optic zone radius (r_{a0}) was calculated to be 9.85 mm.
The sag of the front surface will be

$$s_{a0} = 9.85 - \sqrt{(9.85^2 - 7.15^2)} = 3.0750 \text{ mm}$$

The axial edge thickness (t_{ak}) at diameter 14.30 mm was calculated to be 0.569 mm.

The semi-chord y is 7.15 mm

$$
\begin{aligned}
C_{a0}\,E^2 &= (9.85 - 3.075 - 0.569)^2 + 7.15^2 \\
&= 89.6369 \\
t_e &= r_{a0} - C_{a0}E \\
&= 9.85 - 9.4677 \\
&= 0.3823 \\
t_e &= 0.382 \text{ mm}
\end{aligned}
$$

To calculate the radial thickness at diameter 12.40 mm requires the use of the same relationships.

The front optic zone radius (r_{a0}) was calculated to be 9.85 mm.

The sag of the front surface will be

$$s_{a0} = 9.85 - \sqrt{(9.85^2 - 6.2^2)} = 2.1961 \text{ mm}$$

The axial edge thickness (t_{ak}) at diameter 12.40 mm was calculated to be 0.551 mm.

The semi-chord y is 6.2 mm

$$
\begin{aligned}
C_{a0}\,E^2 &= (9.85 - 2.1961 - 0.551)^2 + 6.2^2 \\
&= 88.8912 \\
t_e &= r_{a0} - C_{a0}E \\
&= 9.85 - 9.4282 \\
&= 0.4218 \\
t_e &= 0.422 \text{ mm}
\end{aligned}
$$

3.3.4 *Mean thickness*

One approach to assessing the mean thickness of a contact lens is to measure its thickness at a series of uniformly spaced points across the section of a single meridian and calculate the average value. Obviously the more points considered, the more precise the assessment. In the case of calculated values of thickness, a computer can be used to provide the calculated thickness at a large number of points across the section and will do this very quickly. If n thicknesses are measured or calculated, the assessment can be written as

$$t_{av} = \frac{(t_1 + t_2 + t_3 + \ldots + t_n)}{n} \tag{3.12}$$

which simplifies to

$$t_{av} = \frac{\Sigma(t)}{n} \tag{3.13}$$

Unfortunately this is an inadequate assessment for the contact lens and the treatment that follows is a summary of the original thoughts of Henry Burek.

What does t_{av} tell us about the lens? It is the average thickness of the lens cross-section. In fact dividing the cross-sectional area by the total diameter also gives an evaluation of t_{av}. Is this the same as the average thickness of the lens as a whole? Unfortunately not.

If a lens is considered as a solid of revolution of its cross-section, it must be obvious that a greater proportion of the lens is created by the parts furthest from the axis of rotation than by the parts closest to it. So, thickness measurements, as they approach the lens periphery, should have a progressively greater weighting in the assessment of mean thickness. The lack of such weighting in the calculation of t_{av} underestimates the average thickness of negative lenses and overestimates the average thickness of positive lenses. Rather than evaluating average thickness with respect to cross-sectional area we should be looking at average thickness in relation to lens volume.

3.3.4.1 Compensated mean thickness

Each thickness measurement at a given diameter actually represents the thickness of a cylindrical band of lens material as illustrated in **Figure 3.19**.

If we consider that this band consists of thickness t and semi-diameter y to the middle of the annulus which has a width w, then the cylinder is as illustrated in **Figure 3.20**.

$$\text{Base area } = \pi\left(y + \frac{w}{2}\right)^2 - \pi\left(y - \frac{w}{2}\right)^2$$

$$= \pi y^2 + \frac{\pi wy}{2} + \frac{\pi wy}{2} + \frac{\pi w^2}{4} - \pi y^2 + \frac{\pi wy}{2} + \frac{\pi wy}{2} - \frac{\pi w^2}{4}$$

$$\text{base area } = 2\pi wy$$

$$\text{volume of the cylinder} = 2\pi wyt$$

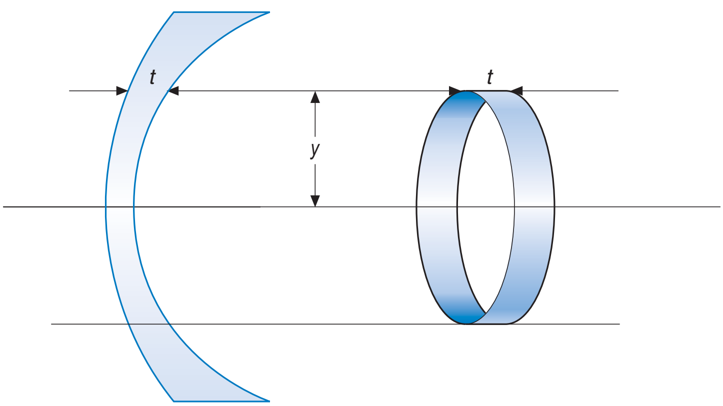

Figure 3.19 A contact lens is a rotationally symmetrical body. Thus a thickness measurement at a point on the lens represents the thickness of a cylindrical band of lens material

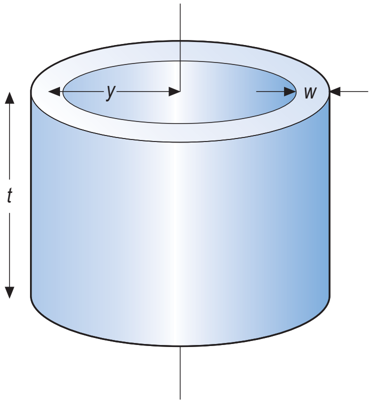

Figure 3.20 The annular zone cut from the contact lens seen in Figure 3.19

Now consider a lens approximated by many such bands, each of width w with no intermediate gaps. The total area is the sum of the individual areas.

$$\text{Total area } A = 2\pi w y_1 + 2\pi w y_2 + 2\pi w y_3 + \ldots + 2\pi w y_n$$
$$A = 2\pi w(y_1 + y_2 + y_3 + \ldots + y_n) \tag{3.14}$$

and total volume is

$$V = 2\pi w y_1 t_1 + 2\pi w y_2 t_2 + 2\pi w y_3 t_3 + \ldots + 2\pi w y_n t_n$$
$$V = 2\pi w(y_1 t_1 + y_2 t_2 + y_3 t_3 + \ldots + y_n t_n) \tag{3.15}$$

We wish to determine a mean thickness value t_m such that if each cylindrical band had that thickness, the total volume of the lens would remain the same.

Putting $t_1 = t_m,\ t_2 = t_m,\ t_3 = t_m,$ etc. into equation (3.15) gives

$$V = 2\pi w(y_1 t_m + y_2 t_m + y_3 t_m + \ldots + y_n t_m)$$
$$V = 2\pi w t_m(y_1 + y_2 + y_3 + \ldots + y_n) \tag{3.16}$$

Equating equations (3.15) and (3.16) and re-arranging we acquire

$$t_m = \frac{2\pi w(y_1 t_1 + y_2 t_2 + y_3 t_3 + \ldots + y_n t_n)}{2\pi w(y_1 + y_2 + y_3 + \ldots + y_n)} \tag{3.17}$$

or

$$t_m = \frac{\Sigma(yt)}{\Sigma(y)} \tag{3.18}$$

Equation (3.18) has taken into consideration the fact that more peripheral thickness values lie on a circle of greater circumference. Thus the compensated mean thickness t_m gives a mean for the whole lens, not a mean for the cross-section.

From equations (3.14), (3.15) and (3.17)

$$t_m = \frac{V}{A} \tag{3.19}$$

What does t_m tell us about the lens? If the material of a contact lens could be re-distributed so that the thicker parts were transferred to the thinner parts (within the same total diameter) then this would produce a meniscus shell of uniform axial thickness. The thickness of that shell would represent the mean axial thickness of the lens.

3.3.4.2 Harmonic mean thickness

For the purpose of assessing the transmissibility of a contact lens for a gas such as oxygen, it is desirable to establish its equivalent thickness. Fatt (1979) states that 'For every lens of non-uniform thickness that transmits dissolved gas, water or heat, there is another lens of the same material but of uniform thickness that would have the same transmission properties under the same conditions.' The thickness of that uniform lens is the equivalent thickness of the original lens.

Unfortunately the compensated mean thickness (t_m) of a lens is not the same as its equivalent thickness. Sammons (1980, 1981) pointed out that since gas transmissibility is inversely proportional to lens thickness, it is more appropriate to evaluate the harmonic mean rather than the 'batting average' mean in the evaluation of equivalent thickness.

The harmonic mean thickness of the lens section t_h can be evaluated as

$$\frac{1}{t_h} = \frac{(1/t_1 + 1/t_2 + 1/t_3 + \ldots + 1t_n)}{n} \tag{3.20}$$

$$= \frac{\Sigma(1/t)}{n}$$

Therefore

$$t_h = \frac{n}{\Sigma(1/t)} \tag{3.21}$$

Sammons (1981) showed that the harmonic mean thickness gave a true evaluation of the equivalent thickness with respect to the lens cross-section. However, the lens is a solid of revolution and the equivalent thickness of the lens must allow for the relative weighting of thickness measurements in proportion to their diameter of measurement.

3.3.4.3 Compensated harmonic mean thickness

The same arguments, as those applied to the mean thickness in Section 3.3.4.1, can be developed to give the compensated harmonic mean thickness (t_{hc}). By considering an analogue of a lens in which the cylindrical bands have a thickness of $1/t$ instead of t, the same arguments can be developed from equation (3.20) as were developed from equation (3.12) which lead to the

conclusion that the compensated harmonic mean thickness can be evaluated as

$$\frac{1}{t_{hc}} = \frac{(y_1/t_1 + y_2/t_2 + y_3/t_3 + \ldots y_n/t_n)}{(y_1 + y_2 + y_3 + \ldots y_n)}$$

$$t_{hc} = \frac{\Sigma(y)}{\Sigma(y/t)} \tag{3.22}$$

t_{hc} is the compensated harmonic mean thickness for the whole lens where t_h is the harmonic mean thickness of the cross-section.

3.3.4.4 Validity of the equations

At this stage it is worthwhile to consider a numerical example to test the validity of the equations.

We will consider a model consisting of stepped concentric cylinders where the width and height of each step increases by one unit. This is illustrated in Figure 3.21.

The dimensions of this contact lens are given in Table 3.2.

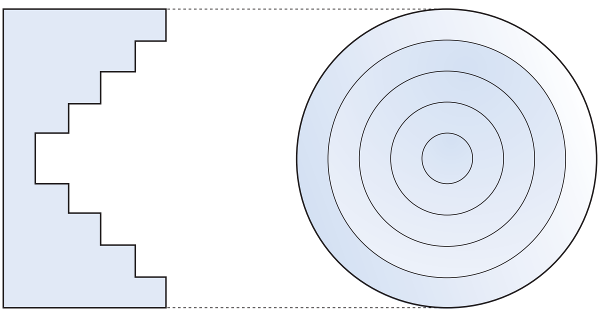

Figure 3.21 The plan view and cross-section of a lens design that consists of a series of stepped sections

Table 3.2 The dimensions of the model contact lens illustrated in Figure 3.21

Semi-diameter (units)	Thickness (units)	Base area (units^2)	Volume (units^3)
From 0 to 1	1	$\pi \times 1^2 = 1\pi$	$\pi \times 1 = 1\pi$
From 1+ to 2	2	$2\pi^2 - \pi^2 = 3\pi$	$3\pi \times 2 = 6\pi$
From 2+ to 3	3	$3\pi^2 - 2\pi^2 = 5\pi$	$5\pi \times 3 = 15\pi$
From 3+ to 4	4	$4\pi^2 - 3\pi^2 = 7\pi$	$7\pi \times 4 = 28\pi$
From 4+ to 5	5	$5\pi^2 - 4\pi^2 = 9\pi$	$9\pi \times 5 = 45\pi$

Now, gas flux, Q is proportional to area divided by thickness

$$Q = \frac{k.A}{t} \tag{3.23}$$

where k is a constant dependent on the gas permeability of the material and on the difference in oxygen tension across the lens.

Since the total flux is the sum of the flux through the individual portions

$$\begin{aligned} \text{total } Q &= k\pi + k\pi\,\frac{3}{2} + k\pi\,\frac{5}{3} + k\pi\,\frac{7}{4} + k\pi\,\frac{9}{5} \\ &= k\pi\left(1 + \frac{3}{2} + \frac{5}{3} + \frac{7}{4} + \frac{9}{5}\right) \\ &= 7.7167k\pi \end{aligned} \tag{3.24}$$

What we are looking for is a value of thickness t_{eq} which, if each individual portion of the lens had that thickness, would result in a lens of uniform thickness having the same total gas flux.

From equation (3.23)

$$t_{eq} = \frac{k(\text{total area})}{\text{total } Q} \tag{3.25}$$

The total base area is the sum of the individual base areas.

$$\text{Total base area} = (1\pi + 3\pi + 5\pi + 7\pi + 9\pi) = 25\pi \text{ units}^2 \tag{3.26}$$

Substituting from (3.24) and (3.26) into (3.25)

$$\begin{aligned} t_{eq} &= \frac{25\pi k}{7.7167\pi k} \\ &= \mathbf{3.2397\ units} \end{aligned}$$

Now that we know what the answer should be, we can assess which evaluation from sets of discrete thickness measurements gives the most realistic assessment of the equivalent thickness t_{eq}.

We have derived equations for:

the average thickness of a lens section $t_{av} = \dfrac{\Sigma t}{n}$ (3.13)

the average thickness of the contact lens $t_m = \dfrac{\Sigma(yt)}{\Sigma(y)}$ (3.18)

the harmonic mean thickness of the section $t_h = \dfrac{n}{\Sigma(1/t)}$ (3.21)

the harmonic mean thickness of the lens $t_{hc} = \dfrac{\Sigma(y)}{\Sigma(y/t)}$ (3.22)

Considering the semi-diameters in one unit steps, we can construct Table 3.3.

From equations (3.13), (3.18), (3.21) and (3.22)

Table 3.3 The values of y and t in the model lens

	y	t	1/t	y.t	y/t
	1	1	1	1	1
	2	2	1/2	4	1
	3	3	1/3	9	1
	4	4	1/4	16	1
	5	5	1/5	25	1
Totals (n = 5)	15	15	137/60	55	5

the cross-section average thickness $t_{av} = 3.0000$
the lens average thickness $t_m = 3.6667$
the cross-section harmonic mean thickness $t_h = 2.1898$
the lens harmonic mean thickness $t_{hc} = 3.0000$

All these results are wide of the mark but this is based on only five thickness measurements. If we made measurements at semi-diameters 0.5 to 5 in 0.5 mm steps on the same lens the revised table would look like **Table 3.4**.

The data set in **Table 3.4** produces

The cross-section average thickness $t_{av} = 3.0000$
The lens average thickness $t_m = 3.7273$
The cross-section harmonic mean thickness $t_h = 2.1898$
The lens harmonic mean thickness $t_{hc} = 3.1044$

The results have changed and this encourages us to investigate the effect of smaller increments of semi-diameter on the evaluations. **Table 3.5** illustrates the changes in average thickness as the increments are decreased thereby increasing the number of thickness measurements from which the averages are extracted.

Table 3.5 illustrates that the values of t_{av} and t_h remain unchanged irrespective of the number of thickness measurements considered. The constancy of these values is an artefact of the stepped nature of the model. The important point to note is that neither evaluation gives an assessment of the equivalent thickness t_{eq}.

The value of t_m is seen to approach 3.8 which is larger than the 3.2397 calculated for the equivalent thickness. Only t_{hc} gives a result close to 3.2397. Thus the equivalent axial thickness t_{eq} of a lens can be found by evaluating its compensated harmonic mean thickness t_{hc}.

The above treatment has dealt with axial thickness. An evaluation derived from radial thickness measurements would be more appropriate for assessing gas transmission through a contact lens.

Table 3.4 The values of y and t in the model lens

	y	t	1/t	y.t	y/t
	0.5	1	1	0.5	1/2
	1	1	1	1	1
	1.5	2	1/2	3	3/4
	2	2	1/2	4	1
	2.5	3	1/3	7.5	5/6
	3	3	1/3	9	1
	3.5	4	1/4	14	7/8
	4	4	1/4	16	1
	4.5	5	1/5	22.5	9/10
	5	5	1/5	25	1
Totals (n = 10)	27.5	30	274/60	102.5	1063/120

Table 3.5 The effect of the number of thickness measurements on the average thickness

Increment	Number of thickness measurements	t_{av}	t_m	t_h	t_{hc}
1.0	5	3.0000	3.6667	2.1898	3.0000
0.5	10	3.0000	3.7273	2.1898	3.1044
0.1	50	3.0000	3.7843	2.1898	3.2096
0.05	100	3.0000	3.7921	2.1898	3.2244
0.01	500	3.0000	3.7984	2.1898	3.2366
0.005	1000	3.0000	3.7992	2.1898	3.2382
0.001	5000	3.0000	3.7998	2.1898	3.2394

ISO 8320–1986 and *BS 3521: part 3: 1988* indicate that radial edge thickness must be measured normal to the front surface of the contact lens. It would, therefore, be more appropriate to apply equations (3.18) and (3.22) to the situation where the various values of thickness t are in fact radial rather than axial thicknesses. Radial thickness measurements or calculations should give the most suitable assessment of equivalent thickness for the purpose of assessing gas transmission.

From the standpoint of rigorous theory, simple substitution of radial values into the equations derived from axial values cannot be supported. However, such substitution will give numerical results which are very good approximations.

Pearson (1986) has reviewed fully the equations employed for both axial and radial thicknesses which include axial/radial conversions and a discussion of the various approaches to the problem of defining average contact lens thickness.

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CHAPTER 4

Aspherical surfaces

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The term ‘aspherical surface’ simply describes a surface which is not spherical. In the contact lens context, the term is most frequently used to describe ellipsoidal surfaces. An ellipse can be produced by cutting an oblique section through a circular cone. If two straight lines intersect each other at a point (but are not perpendicular), a right circular cone is formed at the surface of revolution of one (the generator) about the other (the axis). The point at which the generator crosses the axis is the vertex of the cone. Except when the plane passes through the vertex (producing a degenerate conic), the shape of the curve obtained at the intersection of plane and cone will be a circle, ellipse, parabola or hyperbola (see **Figure 4.1**). If the section is cut parallel to the base of the cone then a circle is produced. If the section is parallel to one edge of the cone, the section takes the form of a parabola.

The actual shape of the curve depends on two factors

a) The angle the generator makes with the axis (the vertex angle of the cone).
b) The angle the intersecting plane makes with the axis.

The conic sections illustrated in **Figure 4.1** show a circle, an ellipse, a parabola and a hyperbola. On being rotated about their axis of symmetry, each gives rise respectively to a sphere, an ellipsoid, a paraboloid and a hyperboloid. These solid figures are collectively termed conicoids. The typical corneal section is considered to be a prolate (flattening) ellipse, consisting of

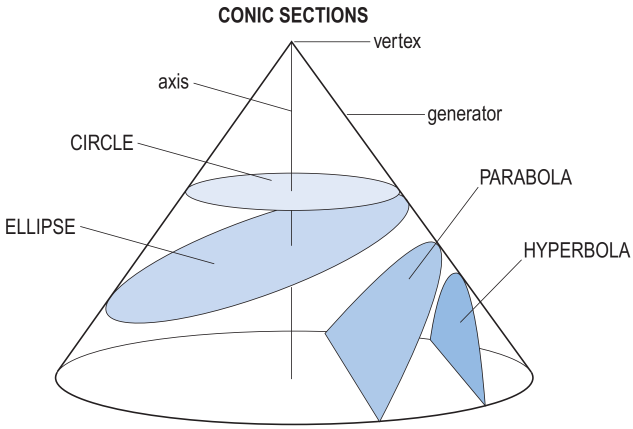

Figure 4.1 Sections cut through a right circular cone

a steep central (apical) portion with progressive flattening of the curve as the limbus is approached. It must be noted, however, that some researchers have found corneas where the form is that of an oblate (steepening) ellipsoid.

Strictly speaking, the terms 'oblate' and 'prolate' can only be applied to ellipsoids and not to ellipses. In an ellipse, the region of greatest curvature at either end of the major axis is the prolate region. The oblate region is the region of least curvature at either end of the minor axis. As all ellipses have both major and minor axes, there is no such thing as a prolate or oblate ellipse. However, these terms are encountered in the literature to indicate whether the ellipse is flattening or steepening with displacement from the apical region.

4.1 Conic sections

There is a simple method whereby the circle, ellipse, parabola and hyperbola can all be treated in the same way. This was suggested by T. Y. Baker in 1943. **Figure 4.2** shows all the various conic sections plotted on a Cartesian system with their common apex at the origin O. The x-axis is the axis of revolution and all the conics have the same apical radius, r_o with the centre of curvature at C_0.

The general equation to all the conic sections is given by Baker's equation as

$$y^2 = 2r_o x - px^2 \tag{4.1}$$

where y is the semi-chord, r_o is the apical radius and x is the sagitta of the section. (See **Figure 4.3**.)

The value p is a quantity that has been described as either the shape factor or the p-value of the surface. It gives an indication of the degree of flattening or steepening of the surface from the apex to the periphery and this is pre-

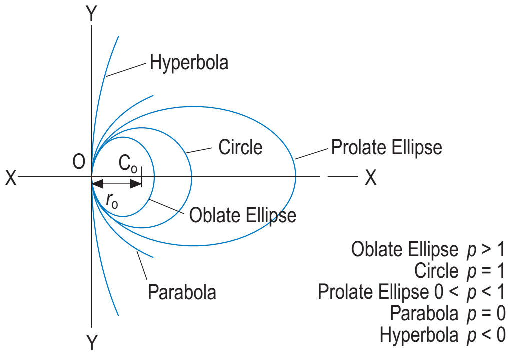

Figure 4.2 Conic sections. All sections illustrated have an apical radius r_o. This is the radius of curvature at the origin of the X, Y co-ordinates

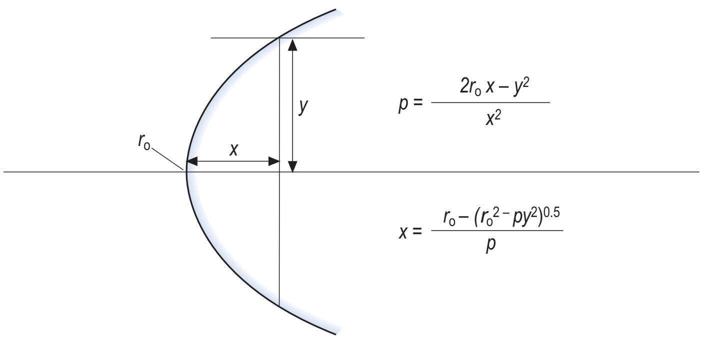

Figure 4.3 Baker's equation for a conic section. r_o is the apical radius, x is the sagitta for the semi-chord y

sumably why the term *shape factor* was adopted although, strictly speaking, it is not a factor but a parameter. Unfortunately this term is also used in thick lens theory to describe the magnification produced by altering the lens form (see Section 1.5.3.1). It is obviously undesirable to use the term *shape factor* when it already has a distinct meaning, completely unrelated to the shape of aspheric surfaces. It is, therefore, preferable to use the term *p-value*. The p-value quantifies the degree of asphericity. A circle has a p-value of unity. A parabola has a p-value of zero.

Bennett (1988) used Baker's equation to deduce equations for the apical radius and the p-value using the major and minor semi-axes of the ellipse. These equations are as follows

$$r_o = \frac{b^2}{a} \tag{4.2}$$

$$p = \frac{b^2}{a^2} \tag{4.3}$$

where a represents the horizontal semi-axis and b represents the vertical semi-axis of the ellipse as illustrated in Figure 4.4.

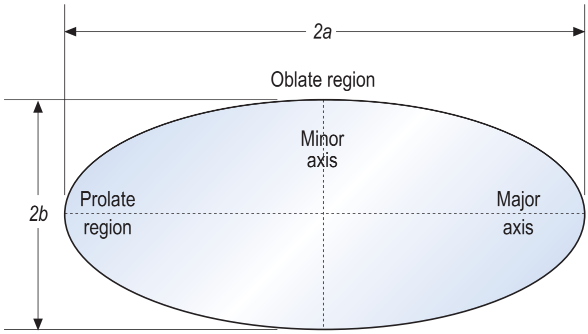

Figure 4.4 The major and minor axes of an ellipse

Equation (4.3) illustrates that in the circle (a limiting case of the ellipse) $b = a$ and so $p = 1$. The parabola is the other limiting case in which a extends to infinity with the result that $p = 0$. Thus the prolate (flattening) ellipses are a family of curves, where the major axis coincides with the x-axis, b is less than a and so the corresponding range of p-values must lie between 0 and 1. As the p-value increases, the shape of the ellipse becomes less elongated.

Figure 4.2 illustrates the oblate (steepening) ellipse where $p > 1$. In this case, the ellipse has its minor axis along the x-axis and so the surface becomes progressively steeper from the apex outwards.

In the equations listed above the quantity a must always be regarded as the horizontal semi-axis and b as the vertical.

4.2 Eccentricity

An alternative means of expressing the degree of flattening of a conic section is by using the term eccentricity (e). A conic section can be defined as the locus of a point which moves so that its distance from a fixed point (the focus) bears a constant ratio e to its perpendicular distance from a fixed straight line (the directrix). In **Figure 4.5**, F is the focus and DD the directrix. B, C and E are points on the curve and BG, CH and EJ their respective perpendicular distances from the directrix. If the curve is a true conic section

$$\frac{\mathrm{BF}}{\mathrm{BG}} = \frac{\mathrm{CF}}{\mathrm{CH}} = \frac{\mathrm{EF}}{\mathrm{EJ}} = e$$

It will be seen from the distances in the diagram that in this case ($e = 2/3$). The curve is an ellipse and this will be the case whenever $e < 1$ but > 0.

For a circle $e = 0$. For a parabola $e = 1$. For a hyperbola $e > 1$.

Bennett (1968, 1969) showed how the directrix/focus construction could lead to the general expression

$$y^2 = 2r_o\, x - (1 - e^2)\, x^2 \tag{4.4}$$

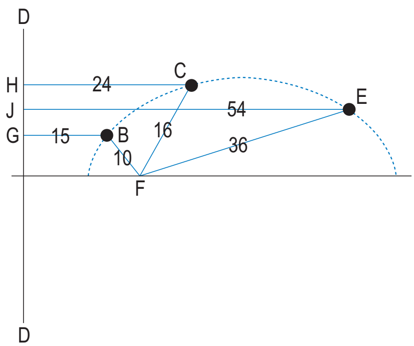

Figure 4.5 A conic section considered as the locus of a point which moves so that its distance from a focus F bears a constant ratio to its perpendicular distance from the directrix DD (after Bennett)

Comparison with equation (4.1) now shows that

$$p = 1 - e^2 \tag{4.5}$$

and

$$e = \sqrt{(1 - p)} \tag{4.6}$$

Bennett discards the use of *e* for two reasons. Firstly he feels that it is more difficult to visualize the relationship between a particular value of *e* and the conic which it represents. Secondly, with *e* as the parameter, the general scheme cannot accommodate the ellipse being rotated about its minor axis to produce an oblate ellipsoid. However, it could be argued that *e* is a less confusing representation of a conic section because the circle possesses an eccentricity of 0 and the parabola an eccentricity of 1. Thus the eccentricity increases with the degree of flattening in the case of the prolate ellipsoid. However, the fact that it cannot be used in the case of the oblate ellipsoid limits its usefulness.

There is some work in the literature which may cause some confusion. Townsley (1970) resorted to what he described as a mathematically improper but conceptually useful convention of adopting e^2 as the term describing the degree of flattening or steepening of the surface. This term was called the shape factor. Townsley's shape factor thus produces a value of zero for a sphere and a value of unity for a paraboloid. Therefore the shape factor is a measure of asphericity whereas the p-value is a measure of non-paraboloidal shape. Townsley's work was the foundation for the development of the Wesley–Jessen System 2000 Corneal Analysis and Photoelectric Keratometer (PEK) which measures and records e^2 (see Chapter 5).

Another approach to describing the asphericity of a conicoid is to use *Q* as the asphericity parameter (Kiely *et al.*, 1982). This, like Townsley's shape factor, results in *Q* being zero for a sphere and unity for a paraboloid.

4.3 Conic parameters

A circle can be described in terms of a single parameter which could be the radius, the diameter or the area, for example. A conic section requires two parameters. The ellipse can be described by specifying the major and the minor axes, but this is not a particularly convenient description. In the optometric discipline, the most useful approach is achieved by specifying the apical radius and the p-value. The apical radius indicates how steep or flat is the curvature of the section. The p-value indicates how rapidly the section flattens (or steepens) with displacement from the apex and thus indicates the degree to which an aspheric curve differs from the spherical form.

4.4 The cornea

Various aspects of corneal topography are of interest in optometry. For some purposes, such as evaluation of paraxial corneal astigmatism, it is sufficient to consider that the two principal meridians possess circular sections. Applications concerned with the inter-relationship between contact lenses and the cornea require consideration of most of the corneal surface. In the absence of exact and complete topographical data from actual corneas, a schematic or mathematical model is required. Despite its shortcomings, a simplified model based on experimental data is a better guide than that based on the spherical corneal surface of the schematic eye. The general cornea, as well as being convex, displays apical toricity coupled with progressive curvature change towards the limbus. The apical curvature, the rate of curvature change and the degree of toricity all vary between individual corneas. The principal meridians are assumed to be mutually perpendicular, and it has been established that the sections of the principal meridians can be acceptably modelled by ellipses (Douthwaite, 2003).

4.4.1 Tangential and sagittal radii

If we assume that the cornea is free from corneal astigmatism then the anterior corneal surface can be considered to be ellipsoidal. The vertical section of this surface is illustrated in **Figure 4.6** with a small sagittal section of the surface drawn at point P which is perpendicular distance y from the major axis of the ellipse. The tangential meridian is in the plane of the paper and the sagittal meridian is perpendicular to this, so that the small portion of the sagittal curve drawn in **Figure 4.6** is actually starting from beneath the page and finishing at a point above it. This diagram illustrates that the radius of curvature of an ellipsoid, at any point on the surface, can be expressed as a tangential radius (PC_t) or a sagittal radius (PC_s). At all points, apart from the apex, these two radii will differ. The relationship between the two is given by the following equation

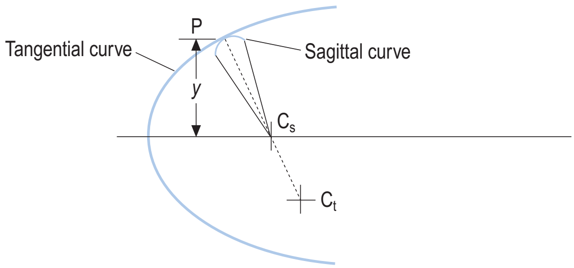

Figure 4.6 The relationship between the tangential and the sagittal centres of curvature on a prolate ellipsoid. PC_t is the tangential, PC_s is the sagittal radius of curvature for the point P which is perpendicular distance *y* from the major axis of the ellipse

$$r_s^{\,3} = r_t \cdot r_o^{\,2} \tag{4.7}$$

where r_s is the sagittal radius, r_t is the tangential radius and r_o is the apical radius. This poses the interesting question as to whether the keratometer measures the tangential or the sagittal radius. This is discussed in Chapter 5.

4.5 Aspherical contact lens surfaces

Contact lens manufacturers now produce aspherical back surfaces and claim this to be an improvement in contact lens back surface design. The back surface is sometimes specified by stating the apical radius and the eccentricity or p-value. An alternative is to specify the apical radius and the axial or radial edge lift.

Bennett (1968) suggested using the concept of a z-value as a method of specifying the departure of an aspherical surface from that of a spherical one which has the same radius of curvature at the apex. The z-value has since been described as the axial edge lift and is illustrated in **Figure 4.7**.

Suppose we required a lens with a central apical radius r_o of 7.60 mm with an axial edge lift l_a of 0.1 mm at a total diameter $2y$ of 9.00 mm. In **Figure 4.7** we can determine the sag (s_0) of the spherical surface of radius r_o. If we then subtract the AEL from this we acquire the overall sag x. We must then use the conic section equation

$$p = \frac{2r_o\,x - y^2}{x^2} \tag{4.8}$$

in order to deduce p which quantifies the asphericity of the conic section.

For the sphere in Figure 4.7

$$\begin{aligned} s_0 &= r_0 - \sqrt{(r_o^{\,2} - y^2)} \qquad (3.1)\\ &= 7.6 - \sqrt{(7.6^2 - 4.5^2)}\\ s_0 &= 1.48 \text{ mm} \end{aligned}$$

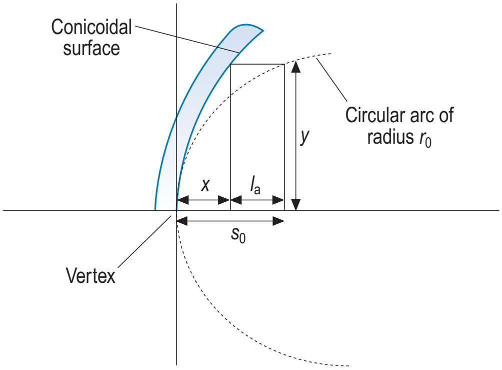

Figure 4.7 Determination of the conicoid required to give a specific AEL (l_a). s_0 is the sag of the circular arc, x is the overall sag of the conicoidal surface, l_a is the AEL and y is the semi-chord for the sags. The conicoidal surface possesses an apical radius r_o

From Figure 4.7

$$x = s_0 - l_a$$
$$= 1.48 - 0.1$$
$$x = 1.38 \text{ mm}$$

For the conicoid

$$p = \frac{2r_o\,x - y^2}{x^2} \qquad (4.8)$$

$$= \frac{2 \times 7.6 \times 1.38 - 4.5^2}{1.38^2}$$

$$p = 0.38$$

The p-value (p) describes the asphericity of the surface. It is greater than one for an oblate (steepening) ellipsoid, is equal to one for a sphere, lies between zero and one for a prolate ellipsoid, is zero for a paraboloid and is negative for a hyperboloid. Thus in the above example the conicoid required is a prolate (flattening) ellipsoid with a p-value equal to 0.38.

If we needed to check this surface then we must refer again to equation (4.8).

The p-value depends on r_o, y and x. If the apical radius r_o is found to be correct and the semi-diameter y has been fixed, then a measurement of x can be used to check the accuracy of the back surface specification. The checking can be performed using a modified radiuscope as outlined in Section 3.2.2. We need to measure the value of x over a specific diameter $2y$. The best accuracy can be achieved with a large diameter, but from a practical point of view this would have to be limited to something less than the total diameter of the contact lens to eliminate inaccuracies arising from the edge taper. In order to check our lens, we must measure the value x and compare this with the value deduced from the relationship

$$x = s_0 - l_a$$
$$x = 1.48 - 0.1 = 1.38 \text{ mm for a diameter of 9.00 mm}$$

The radial edge lift can be determined using

$$l_r = \sqrt{[(r_o - x)^2 + y^2]} - r_o \tag{3.2}$$

4.5.1 Determination of the overall sag x

To calculate x we can use the expression

$$x = \frac{r_o - \sqrt{(r_o^{\,2} - py^2)}}{p} \tag{4.9}$$

In the case of a sphere, where $p = 1$, the equation is simplified to

$$x = r_o - \sqrt{(r_o^{\,2} - y^2)}$$

which is the familiar sag equation (3.1) for a spherical surface. For a parabola, where $p = 0$, the conic section equation (4.1) reduces to

$$y^2 = 2r_o\, x$$

therefore

$$x = \frac{y^2}{2r_o}$$

Alternatively, if the apical radius is known, allowing calculation of s_0, we can calculate x from the simple relationship

$$x = s_0 - l_a$$

for any given axial edge lift at any particular diameter.

Numerical substitution in the above equations indicates that the AEL decreases with increase in apical radius for a paraboloid or an ellipsoid of fixed p-value. Therefore an aspherical contact lens fitting set that is limited to a single p-value across the range of apical radii will not be a constant edge lift design.

4.6 Offset continuous bicurve lenses

It is not an easy matter to produce a conicoidal surface. The surface production can be made easier by generating a surface which approximates to a conicoid. This can be achieved as illustrated in **Figure 4.8**.

The central region of the lens is spherical with a suitable BOZR (r_0), a centre of curvature C_0 and BOZD (back optic zone diameter $2y_0$). The peripheral section has a BPR (back peripheral radius r_1) with the centre of curvature C_1 lying offset from the optical axis AC_0. Since these two curves have a common normal at B there will be no visible transition, as occurs in a conventional bicurve design. The central curve will run tangentially into the peripheral curve. The centre of curvature C_1 is offset to the opposite side of the axis, giving rise to the term *contralateral offset continuous bicurve*. The peripheral

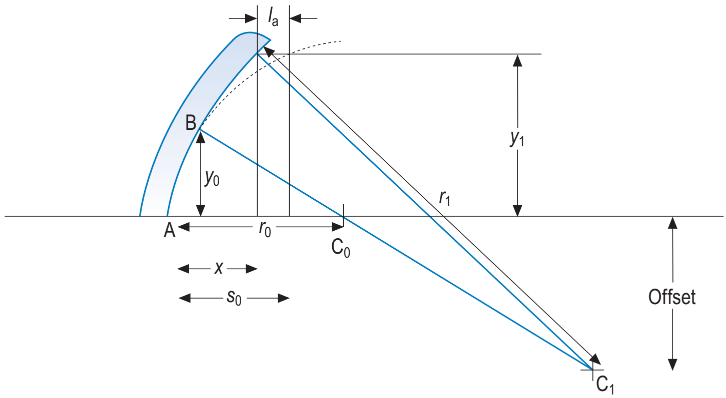

Figure 4.8 The contralateral offset continuous bicurve contact lens. Note that the centres of curvature for the central and peripheral back surface curves lie on the line BC_0C_1. Thus the peripheral curve runs tangentially into the central curve

surface which results from this type of construction is an eccentric zone of a barrel toric. The AEL is determined by the peripheral radius and the offset. Inspection of **Figure 4.8** should lead to the conclusion that these are interdependent since C_1 must always lie on the line BC_0C_1.

In order to check that the axial edge lift is as requested, we can resort to a similar inspection to that for the conicoid. In **Figure 4.8** we can see that a measurement of the overall sag of the lens over an aperture $2y_1$ (which must be larger than the BOZD but smaller than the total diameter) provides us with the value x. The sag s_0 can be calculated from the sag equation

$$s_0 = r_0 - \sqrt{(r_0^2 - y_2^2)} \tag{3.1}$$

and the AEL deduced from the relationship

$$l_a = s_0 - x$$

Once again the radial edge lift can be found using

$$l_r = \sqrt{[(r_0 - x)^2 + y_2^2]} - r_0 \tag{3.2}$$

Alternatively a radius could be given for the peripheral curve and a calculation of the AEL would be required in order to compare this with the result acquired by measurement using the method just described. Thus we can check the BPR indirectly by checking the AEL.

4.6.1 *Calculation of the AEL when the BPR is given*

A contralateral offset continuous bicurve lens is made to the following specification: BOZR 7.60 mm; BOZD 6.00 mm; BPR 12 mm; TD 9.50 mm. What is the AEL at a diameter of 9.00 mm?

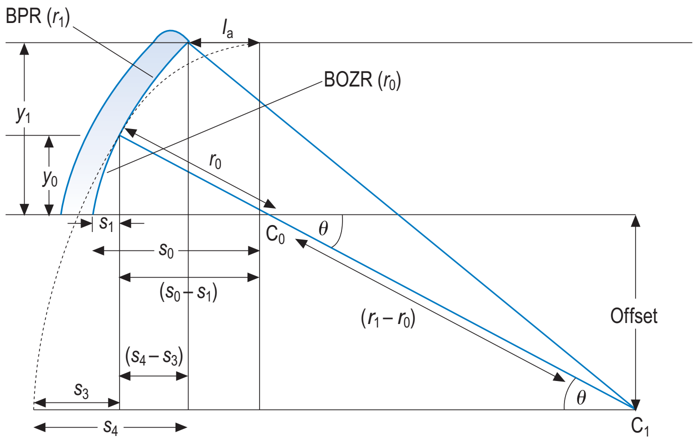

Figure 4.9 The sags of the offset lens required to calculate the AEL (l_a)

From **Figure 4.9** we can see that

$$l_a = (s_0 - s_1) - (s_4 - s_3)$$
$$s_1 = 7.60 - \sqrt{(7.60^2 - 3^2)} = 0.617 \text{ mm}$$
$$s_0 = 7.60 - \sqrt{(7.60^2 - 4.5^2)} = 1.475 \text{ mm}$$
$$s_0 - s_1 = 0.858 \text{ mm}$$

For s_3 and s_4 we must first calculate semi-diameters y_2 and y_3 which must include the offset

$$y_2 = y_0 + \text{offset} \quad y_3 = y_1 + \text{offset}$$

From **Figure 4.9**

$$\sin\theta = \frac{\frac{1}{2}\text{BOZD}}{\text{BOZR}} = \frac{3.00}{7.60} = 0.3947$$

But

$$\sin\theta = \frac{\text{offset}}{r_1 - r_0}$$

Therefore

$$\text{offset} = \sin\theta.(r_1 - r_0) = 1.737 \text{ mm}$$

We can now calculate s_3 and s_4
For s_3

$$y_2 = 3 + 1.737 = 4.737 \text{ mm}$$

For s_4

$$
\begin{aligned}
y_3 &= 4.5 + 1.737 && = 6.237 \text{ mm} \\
s_3 &= 12 - \sqrt{(12^2 - 4.737^2)} && = 0.975 \text{ mm} \\
s_4 &= 12 - \sqrt{(12^2 - 6.237^2)} && = 1.748 \text{ mm} \\
s_4 - s_3 &= 0.773 \text{ mm} \\
l_a &= (s_0 - s_1) - (s_4 - s_3) \\
l_a &= 0.858 - 0.773 \\
\text{AEL} &= 0.085 \text{ mm}
\end{aligned}
$$

4.6.2 *Calculation of the peripheral radius for a given axial edge lift*

Figure 4.10 illustrates the principles involved in calculating the back peripheral radius required to give a specific axial edge lift.

In Figure 4.10

s_1 = the sag of the BOZR (r_0) over the BOZD ($2y_0$)
s_0 = the sag of the BOZR over the lens total diameter ($2y_1$)
x = the overall sag of the lens back surface over $2y_1$
l_a = the axial edge lift
BM = half the chord BD which spans the peripheral band
BC_0 = the BOZR (r_0)
BC_1 = the **back peripheral radius**

$$
\begin{aligned}
\text{sag } s_1 &= r_0 - \sqrt{(r_0^2 - y_0^2)} \\
\text{sag } s_0 &= r_0 - \sqrt{(r_0^2 - y_1^2)} \\
\text{sag } x &= s_0 - l_a
\end{aligned}
$$

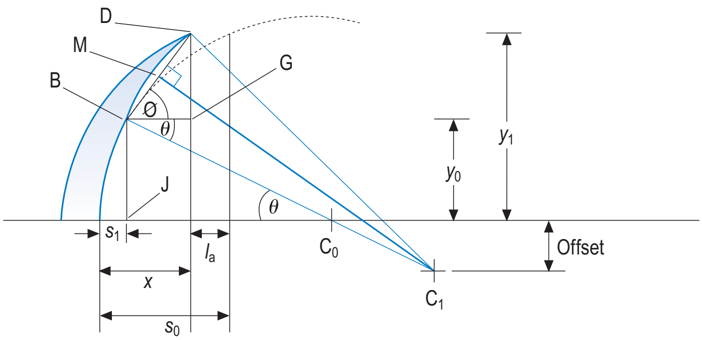

Figure 4.10 Illustrating the principles involved in calculating the back peripheral radius required to give a specified AEL. Distance BC_1 is the back peripheral radius of this lens

In triangle BDG

$$\mathrm{BD}^2 = (x - s_1)^2 + (y_1 - y_0)^2$$

$$\mathrm{BM} = \frac{\mathrm{BD}}{2}$$

$$\tan \phi = \frac{y_1 - y_0}{x - s_1}$$

In triangle $\mathrm{BJC_0}$

$$\sin \theta = \frac{y_0}{r_0}$$

In triangle $\mathrm{BMC_1}$

$$\cos(\phi + \theta) = \frac{\mathrm{BM}}{\mathrm{BC_1}}$$

$$\mathrm{BC_1} = \frac{\mathrm{BM}}{\cos(\phi + \theta)} = \mathrm{BPR}$$

Let us now take the previous example and find the back peripheral radius given the axial edge lift.

A contralateral offset continuous bicurve lens is required with the following specification: BOZR 7.60 mm, BOZD 6.00 mm, TD 9.00 mm and AEL of 0.085 mm at a diameter of 9.00 mm. What is the back peripheral radius for this lens?

$$\text{sag } s_1 = r_0 - \sqrt{(r_0^2 - y_0^2)} = 7.60 - \sqrt{(7.60^2 - 3^2)} = 0.6172$$
$$\text{sag } s_0 = r_0 - \sqrt{(r_0^2 - y_1^2)} = 7.60 - \sqrt{(7.60^2 - 4.5^2)} = 1.4755$$
$$\text{sag } x = s_0 - l_a = 1.4755 - 0.085 = 1.3905$$

In triangle BDG

$$\mathrm{BD}^2 = (x - s_1)^2 + (y_1 - y_0)^2 = (1.3905 - 0.6172)^2 + 1.5^2 = 2.848$$
$$\mathrm{BD} = 1.6876$$

$$\mathrm{BM} = \frac{\mathrm{BD}}{2} = 0.8438$$

$$\tan \phi = \frac{(y_1 - y_0)}{(x - s_1)} = \frac{1.5}{(1.3905 - 0.6172)} = 1.9397$$

$$\phi = 62.7274°$$

In triangle $\mathrm{BJC_0}$

$$\sin \theta = \frac{y_0}{r_0} = \frac{3}{7.6} = 0.3947$$

$$\theta = 23.2496°$$
$$\phi + \theta = 62.7274 + 23.2496 = 85.977°$$

In triangle BMC_1

$$\cos(\phi + \theta) = \frac{BM}{BC_1}$$

$$BC_1 = \frac{BM}{\cos(\phi + \theta)} = \frac{0.8438}{0.0702} = 12.02 \text{ mm}$$

Therefore

$$r_1 = BPR = 12.02 \text{ mm}$$

4.7 Polynomial surfaces

We have seen that a contact lens back surface can achieve the peripheral flattening required to give a specified edge lift by using a conicoidal surface, or more specifically a prolate ellipsoidal, a paraboloidal or a hyperboloidal surface. Plus powered contact lenses suffer from positive spherical aberration. A flattening conicoid will exacerbate this aberration. Also aspheric lenses will suffer from the aberration of coma when the lens decentres on the eye. The contralateral offset lens has the optical advantage of a spherical central zone for the lens back surface. However, greater flexibility in determining the shape of the back surface can be achieved by introducing more variables affecting the value of x (the sag) in relation to y (the semi-chord). One way of achieving this is to equate x into a series of even powers of y, each with its own independent coefficient. Thus, the sagitta x of the aspherical surface is expressed in terms of the semi-chord y by

$$x = Ay^2 + By^4 + Cy^6 + Dy^8 \tag{4.10}$$

This is a polynomial equation. The designer is free to include as many terms as s/he thinks appropriate and to decide on the values for the coefficients A, B, C, D etc. If the number of points to be specified on the lens back surface was n, then the polynomial equation must contain n terms after Ay^2.

From the polynomial equation we see that $x = 0$ when $y = 0$, so the vertex of the curve coincides with the Cartesian origin of coordinates. The coefficient A can be made to represent any desired conic with any specified apical radius r_o. The coefficients B, C, D, etc. can then be used to modify this basic conic in a predetermined manner as y increases.

Baker's equation for the conicoid (4.1) can be transformed into the expression

$$x = \frac{y^2}{r_o + \sqrt{(r_o^2 - py^2)}} \tag{4.11}$$

$$x = Ay^2 + By^4 + Cy^6 + Dy^8, \text{ etc.}$$

When

$$B = C = D = 0$$
$$x = Ay^2 \tag{4.12}$$

Therefore

$$A = \frac{x}{y^2}$$

Substituting in (4.11)

$$A = \frac{1}{r_o + \sqrt{(r_o^2 - py^2)}} \tag{4.13}$$

Thus the basic conic is specified by the Ay^2 term and the curve can then be modified in order to pass through any number of predetermined points.

In the hope of clarifying the picture, let us take an example.

Suppose that the cornea under consideration has an apical radius of 7.7 mm and is assumed to be ellipsoidal with a p-value of 0.8. Using equation (4.9) we can deduce the sagitta x for this cornea at any specified diameter. It is intended that we design a back surface with a **spherical central zone** which will fit the cornea with around 0.02 mm of apical clearance. The lens will rest on the corneal mid periphery and will then give an axial edge clearance from the cornea of 0.1 mm at the contact lens total diameter of 9.40 mm. **Figure 4.11** illustrates the fitting relationship between the contact lens and the cornea. Note that the calculations that follow are displayed to four decimal places. The numerical values were acquired working to the maximum number of decimal places available to the calculator.

4.7.1 The sags

We decide that the central spherical region will occupy a diameter of 6 mm. The bearing zone will lie between diameters 6 and 8 mm. This leaves the region from 8 mm out to the lens total diameter to provide the edge clearance from the cornea. We therefore need to know the corneal sags at these three diameters. They are (using equation (4.9)) as follows

Corneal diameter (mm)	Corneal sag (mm)
6	0.6033
8	1.1021
9.4	1.5610

We now consider the back surface of the contact lens. The sag equation for spherical surfaces (equation (3.1)) can be manipulated to give

$$r = \frac{y^2 + s^2}{2s}$$

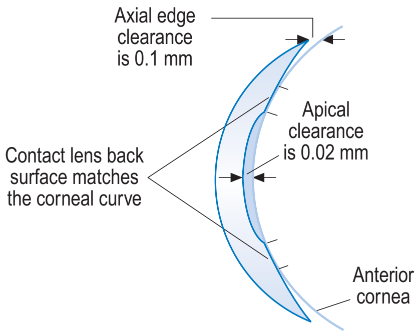

Figure 4.11 The fitting relationship between the cornea and the contact lens with the proposed polynomial back surface

The apical clearance of the contact lens is 0.02 mm, so the contact lens sag for diameter 6 mm will be 0.6233 mm.

$$r = \frac{3^2 + 0.6233^2}{2 \times 0.6233} = 7.5313 \text{ mm}$$

This indicates that a sag of 0.6233 mm is produced by a BOZR of 7.5313 mm when a BOZD of 6 mm is being considered. This sag will provide an apical clearance from the cornea of 0.02 mm. If we consider a contact lens BOZR of 7.5 mm our apical clearance will be 0.0228 mm which should be acceptable. This produces a sagitta of 0.6261 mm for the 6 mm chord.

The contact lens sag (x) must be 0.6261 mm at diameter ($2y$) 6 mm

Between diameters 6 and 8 mm, the increase in sagitta for the cornea and the lens must be equal for the contact lens to bear on this part of the cornea. The increase in the sag of the cornea between the 6 and 8 mm rings is 0.4988 mm. This means that for the contact lens back surface the sag must be

$$0.6261 + 0.4988 = 1.1249$$

The contact lens sag (x) must be 1.1249 mm at diameter ($2y$) 8 mm

Between diameters 8 and 9.4 mm, the increase in sag of the contact lens must be 0.1 mm less than the increase in sag of the cornea if the axial edge clearance from the cornea is to be 0.1 mm. The increase in sag of the cornea is 0.4589. This means that, for the contact lens back surface, the sag must be

$$1.1249 + 0.3589 = 1.4838 \text{ mm}$$

The contact lens sag (x) must be 1.4838 mm at diameter ($2y$) 9.4 mm

We have decided that the basic conic is to be a sphere ($p = 1$) of radius r_o = 7.5 mm.

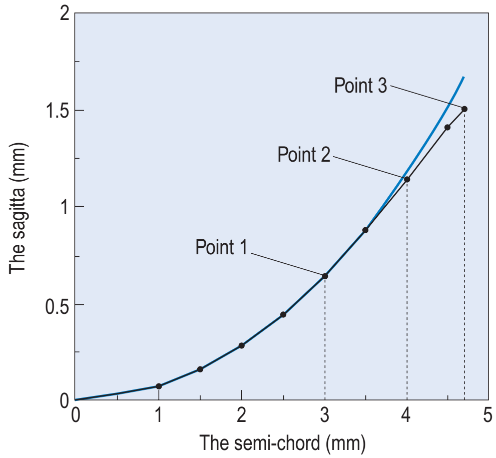

Figure 4.12 The shape of the polynomial back surface plotted using the *x*–*y* coordinates of Table 4.1. Note that *x* occupies the vertical axis and *y* occupies the horizontal. The blue curve is a plot of a spherical surface of radius 7.5 mm. The dotted vertical lines mark the semi-diameters used to determine the three sags which allowed derivation of the coefficients of the polynomial equation

From equations (4.11), (4.12) and (4.13)

$$x = \mathrm{A}y^2 = \frac{y^2}{7.5 + \sqrt{(56.25 - y^2)}}$$

It has been decided that the curve must pass through the three points which are given below by their x and y coordinates.

	Semi-chord (y mm)	Sagitta (x mm)
Point 1	3	0.6261
Point 2	4	1.1249
Point 3	4.7	1.4838

These three points are illustrated in **Figure 4.12** which shows the shape of the polynomial curve. Point 1 will lie on a circular arc of radius 7.5 mm and the surface will be indistinguishable from a spherical surface over this central 6 mm diameter. The curve must then undergo some degree of flattening between $y = 3$ and $y = 4$ mm. The curve will then undergo a more dramatic change in curvature between $y = 4$ and $y = 4.7$ mm.

4.7.2 The polynomial equation

From the above coordinate pairs we can acquire three simultaneous equations from which the coefficients B, C and D can be determined.

4.7.2.1 Simultaneous equations

A reminder of how we deal with simultaneous equations might be appropriate before we continue with our polynomial surface.

Suppose, for purposes of illustration we had the following three simultaneous equations

$$
\begin{aligned}
2B + 6C + 5D &= 80 && (1)\\
10B + 4C + 8D &= 116 && (2)\\
45B + 15C + 40D &= 525 && (3)
\end{aligned}
$$

Consider equations (1) and (2). Multiply equation (1) by 5 (10/2)

$$
\begin{array}{lrl}
\text{Equation (1) becomes} & 10B + 30C + 25D & = 400 \\
\text{Subtract equation (2)} & \underline{10B + \ \ 4C + \ \ 8D} & \underline{= 116} \\
 & 26C + 17D & = 284 \quad (4)
\end{array}
$$

Consider equations (2) and (3). Multiply equation (2) by 4.5 (45/10)

Equation (2) becomes $45B + 18C + 36D = 522$

Subtract this new equation from equation (3)

$$
\begin{array}{lrl}
 & 45B + 15C + 40D & = 525 \quad (3) \\
\text{Subtract} & \underline{45B + 18C + 36D} & \underline{= 522} \\
 & -3C + \ \ 4D & = 3 \qquad (5)
\end{array}
$$

Considering equations (4) and (5)

Multiply (5) by 4.25 (17/4) giving $-12.75C + 17D = 12.75$

Subtract this new equation from (4)

$$
\begin{array}{lrl}
 & 26C + 17D & = 284 \quad (4) \\
\text{Subtract} & \underline{-12.75C + 17D} & \underline{= \ \ 12.75} \\
 & 38.75C & = 271.25 \\
 & C & = \ \ 7
\end{array}
$$

Substituting for C in equation (5) we have

$$
\begin{aligned}
-3C + 4D &= 3\\
-21 + 4D &= 3\\
4D &= 24\\
D &= 6
\end{aligned}
$$

Substituting for C and D in equation (1) we have

$$
\begin{aligned}
2B + 6C + 5D &= 80\\
2B + 42 + 30 &= 80\\
2B &= 80 - 72 = 8\\
B &= 4
\end{aligned}
$$

We now know B, C and D.

4.7.2.2 For our contact lens polynomial surface

For point 1 (y =3)

For the basic conic

$$x = Ay^2 \tag{4.12}$$

and

$$By^4 + Cy^6 + Dy^8 \qquad = 0$$

Substituting for y

$$81B + 729C + 6561D \ = 0 \qquad (4.14)$$

Point 2 (y = 4)

$$Ay^2 = \frac{16}{7.5 + \sqrt{(56.25 - 16)}} \quad \text{from (4.11), (4.12) and (4.13)}$$

$$Ay^2 = 1.1557$$
$$x \quad = 1.1249$$

From equation (4.10)

$$x = 1.1249 = 1.1557 + 256B + 4096C + 65\,536D$$

Therefore

$$256B + 4096C + 65\,536D = -\,0.0308 \qquad (4.15)$$

Point 3 (y = 4.7)

From equations (4.11), (4.12) and (4.13)

$$Ay^2 = \frac{22.09}{7.5 + \sqrt{(56.25 - 22.09)}}$$

$$Ay^2 = 1.6553$$
$$x \quad = 1.4838$$

From equation (4.10)

$$x = 1.4838 = 1.6553 + 487.9681B + 10\,779.2153C + 238\,112.8666D$$

Therefore

$$487.9681B + 10\,779.2153C + 238\,112.8666D = -\,0.1715 \qquad (4.16)$$

Solution of the simultaneous equations whose components are equations (4.14), (4.15) and (4.16) yields the following values

$$81B + 729C + 6561D = 0 \qquad (4.14)$$

$$256B + 4096C + 65\,536D = -\,0.0308 \qquad (4.15)$$

$$487.9681B + 10\,779.2153C + 238\,112.8666D = -\,0.1715 \qquad (4.16)$$

For equations (4.14) and (4.15) multiply (4.14) by 3.160 493 827

(4.14) becomes $\quad 256B + 2304C + 20\,736D = 0$
(4.15) is $\quad 256B + 4096C + 65\,536\,D = -\,0.0308$

Subtract (4.14) from (4.15). This gives (4.17)

$$1792C + 44\,800D = -\,0.\,0308 \tag{4.17}$$

For equations (4.15) and (4.16) multiply (4.15) by 1.906 125 391

(4.15) becomes $487.9681B + 7807.4896C + 124\,919.8336D = -\,0.0588$
(4.16) is $487.9681B + 10\,779.2153C + 238\,112.8666D = -\,0.1715$

Subtract (4.15) from (4.16). This gives (4.18)

$$2\,971.7257C + 113\,193.033D = -\,0.1127 \tag{4.18}$$

For equations (4.17) and (4.18) multiply (4.17) by 1.658 329 074

(4.17) becomes $2971.7257C + 74\,293.1425D = -\,0.0512$
(4.18) is $2971.7257C + 113\,193.033D = -\,0.1127$

Subtract (4.17) from (4.18). This gives (4.19)

$$38\,899.8905D = -\,0.0615 \tag{4.19}$$

Therefore

$$D = -\,1.5836 \times 10^{-6}$$

Substitute in equation (4.17) gives

$$\begin{aligned} 1792C - 0.0709 &= -\,0.0308 \\ 1792C &= 0.0401 \end{aligned}$$

Therefore

$$C = 2.2382 \times 10^{-5}$$

Substitute in equation (4.14) gives

$$\begin{aligned} 81B + 0.0163 - 0.0104 &= 0 \\ 81B &= -\,0.0059 \\ B &= -\,7.3172 \times 10^{-5} \end{aligned}$$

Therefore

$$\begin{aligned} B &= -\,7.3172\,.\,10^{-5} \\ C &= 2.2382\,.\,10^{-5} \\ D &= -\,1.5836\,.\,10^{-6} \end{aligned}$$

The complete equation for this polynomial surface is therefore

$$x = \frac{y^2}{7.5 + \sqrt{(7.5^2 - y^2)}} - 7.3172\,.\,10^{-5}.\,y^4$$
$$+\,2.2382\,.\,10^{-5}.\,y^6 - 1.5836\,.\,10^{-6}.\,y^8 \tag{4.20}$$

Table 4.1 uses equation (4.20) to illustrate the influence of each coefficient on the final value of x for increasing values of y. This table was used to construct the curve in **Figure 4.12**.

Table 4.1 The contribution of each term in the polynomial equation (4.20) to the total value (*x*) for increasing values of *y*. All units in the table are in mm

y	Ay^2	By^4	Cy^6	Dy^8	Sum (*x*)
1.0	0.0670	–0.0001	0	0	0.0669
1.5	0.1515	–0.0004	0.0003	0	0.1514
2.0	0.2716	–0.0012	0.0014	–0.0004	0.2714
2.5	0.4289	–0.0029	0.0055	–0.0024	0.4291
3.0	0.6261	–0.0059	0.0163	–0.0104	0.6261
3.5	0.8668	–0.0110	0.0411	–0.0357	0.8612
4.0	1.1557	–0.0187	0.0917	–0.1038	1.1249
4.5	1.5	–0.0300	0.1859	–0.2663	1.3896
4.7	1.6553	–0.0357	0.2413	–0.3771	1.4838

Inspection of **Table 4.1** illustrates that the polynomial curve shows no notable departure from the spherical curve out to semi-diameter 3.0 mm. The sag of the spherical curve is given in the Ay^2 column and this does not differ significantly from the Sum in the right-hand column for y values up to 3. The larger semi-diameters display an overall sag (x) which shows an increasing disparity with the sag of the spherical surface as the lens edge is approached. Thus the lens will provide the edge clearance from the cornea which was requested. **Figure 4.13** shows a plot of the polynomial surface matched with a plot of the cornea. The cornea is positioned on the ordinate to give an apical clearance of 0.0228 mm.

A closer examination of the fitting relationship between the polynomial surface and the anterior cornea indicates that the bearing zone is actually the region where the polynomial curve crosses the corneal boundary as shown in **Figure 4.14** in an exaggerated manner.

This means that if a contact lens was manufactured with the polynomial back surface that has been derived above, the TLT and AEC would, in fact, be a little greater than the values assumed at the outset. There is a computer program in Chapter 11 that can be used to calculate the theoretical values for TLT and AEC. The computer program indicates that the TLT will be 0.03 mm rather than the 0.0228 mm requested and the AEC will be 0.107 mm instead of the requested value of 0.1 mm. The computer assumes that the lens will be in contact with the cornea at a single point and the contact diameter is calculated as 7.22 mm. A real lens on a real cornea will be in contact over a

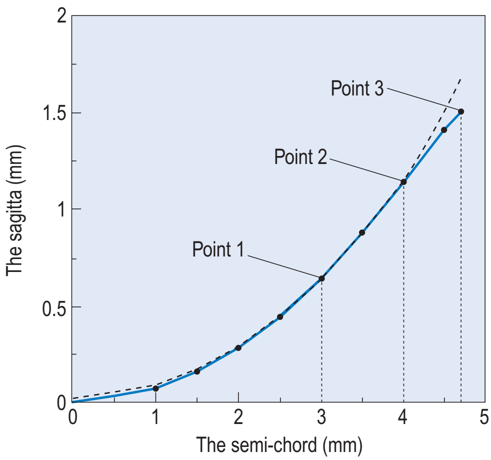

Figure 4.13 An identical plot of the polynomial surface to that in Figure 4.12 but the blue curve is a plot of the cornea which is positioned to give an apical clearance of 0.0228 mm. This graph gives some indication of the actual extent of the region of contact between the lens and the cornea

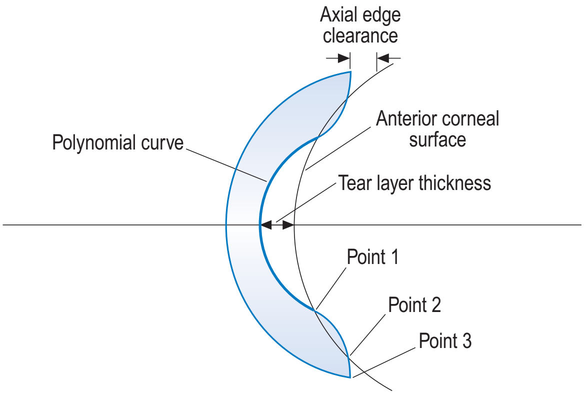

Figure 4.14 The fitting relationship between the polynomial curve and the cornea with the corneal clearances greatly exaggerated

region which will serve to decrease the clearances from those given by the computer, so that the actual TLT and AEC will lie between the values assumed at the outset and those calculated by the computer. This problem is not unique to the polynomial surface. The TLT and AEC will be less than the computer calculated values for all lens designs due to the lens bedding in at the point of contact thereby extending the point into an area. The problem peculiar to the polynomial, arising from the surface crossing the corneal boundary, can be minimized by requesting a very narrow bearing zone.

The polynomial above was derived to fulfil the original requirements of an apical clearance of 0.02 mm and an axial edge clearance of 0.1 mm with the central 6 mm of the back surface maintaining a spherical form. There are many other options open to the lens designer. The polynomial surface could be completely aspheric or it could have an aspheric centre with a spherical periphery. The polynomial provides us with a very flexible design that is capable of achieving almost anything that we are likely to request. The

polynomial surface promises flexibility for future contact lens aspheric or combined spherical and aspheric back surface designs.

4.8 Contact lens and ocular aberrations

The topic of aspherical surfaces is linked to the problem of aberrations in the sense that aspherical optical surfaces can be used to minimize aberrations in both spectacle and contact lenses. The problem for the spectacle lens designer is to minimize the effects of oblique astigmatism and field curvature in order to give a good lens performance when the eye is directed to the peripheral regions of the spectacle lens. For the contact lens designer this is not the main problem because the lens moves with the eye in all directions of gaze. The main troublesome aberrations for the contact lens designer are spherical aberration and coma-like aberrations that occur when the contact lens decentres due to lens lag.

4.8.1 Wavefront aberrations

The quality of vision will be influenced by the aberrations induced by the optics of the eye and its refractive correction. The shallow surface curves of single vision spectacle lenses generate on-axis aberrations that are negligible compared to the ocular aberrations. Contact lenses, with their small surface radii of curvature, will change the aberrations of the contact lens/eye system. This change in aberration could be an increase or a decrease dependant on the individual optical characteristics of the eye being corrected. Refractive surgery may also induce significant changes in ocular aberrations. The correction of ocular aberrations may provide superior visual performance for the subject and allow very high resolution images of the fundus to be acquired.

An ideal optical system will produce point images from point objects for all point objects within the field. Aberrations produce departures from this ideal. They cause rays from object points to depart from the paths predicted by paraxial optics with the effect that the images are no longer focused at single points.

Abberations could be classified in two ways:

1. *Longitudinal aberration*
   For an axial point object, the longitudinal aberration of a ray is the distance along the optical axis between the intersection point of the ray and optical axis compared to the intersection point of the non-aberrated ray.
2. *Transverse aberration*
   Transverse aberration is the difference in position between the aberrated and non-aberrated ray measured perpendicular to the optical axis.

These two aberrations are shown in **Figure 4.15**.

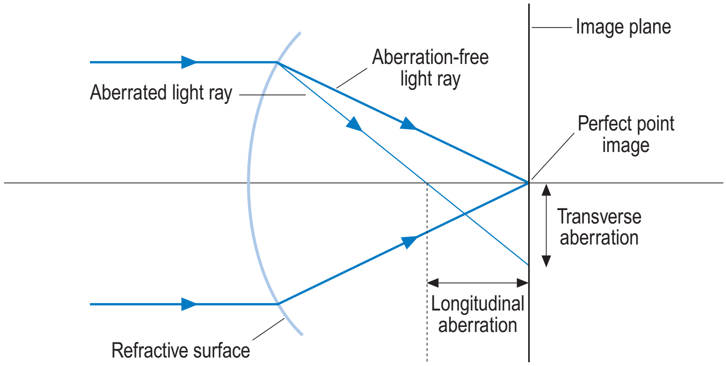

Figure 4.15 The longitudinal and transverse aberration of a ray that passes through one particular region of the pupil

If we consider light rays diverging from a point object and we measure the same specified distance on each ray and mark a point in that position then all the points will coincide with a wavefront. In this case the wavefront will be spherical in form. It is also obvious that the light ray will be normal to the wavefront. After refraction by an aberration-free lens the light rays will be either divergent or convergent but in both cases the wavefront will be spherical with the centre of curvature coincident with the perfect point image (see **Figure 4.16**).

The wavefront aberration is the optical path length difference, measured along a ray, between the actual wavefront and the reference sphere (the ideal theoretical spherical wavefront). The wavefront aberration is given as the deviation of the refracted wave with respect to the reference sphere in a specified plane, for example, the exit pupil. For clarity, the radius of the reference sphere is often set to infinity. We can deduce image space wavefronts by measuring transverse aberrations directly or indirectly in image space using a cross-cylinder aberroscope or by psychophysical procedures. The image space in the human eye is not directly accessible and so the principle of the reversibility of the path of light rays is utilized by taking a point object on the retina and examining the emergent light rays as shown in **Figure 4.17**.

In **Figure 4.17** the wavefront aberration is given as the deviation of the refracted wavefront from a plane reference sphere (infinite radius).

Many optical calculations assume paraxial optics where angles are small. Paraxial theory is sometimes known as first-order theory because the sine terms are replaced with only the first-order term of their mathematical expansion. If aberrations are going to be described then it is necessary to introduce more precise relationships. Detailed ray tracing will reveal the presence of aberrations. von Seidel showed that an assessment of aberrations can be achieved simply by retaining the third-order term as well as the first. The aberrations resulting from this analysis are known as primary, third-order or Seidel aberrations. These are the familiar

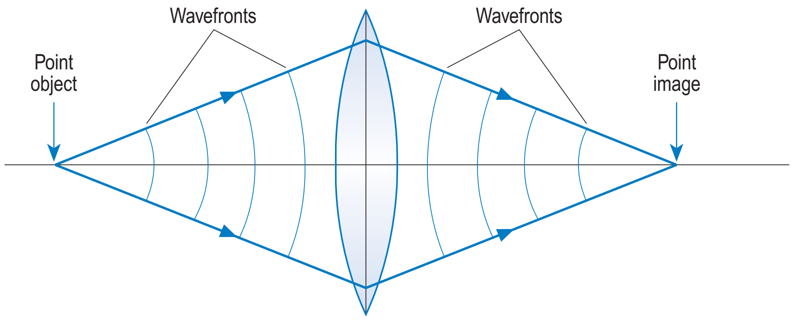

Figure 4.16 The perfect point object produces spherical wavefronts. The perfect point image is produced by spherical wavefronts with their centre of curvature at the image point

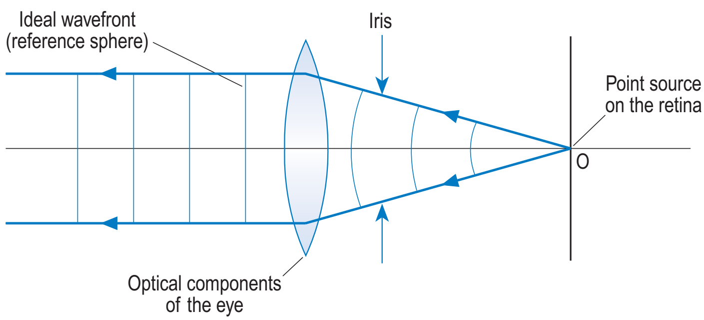

Figure 4.17 The principle of measuring wavefront aberrations. Light rays diverge from the point object O on the retina. The blue arcs indicate the spherical wavefronts. The emergent light rays are parallel in this perfect system so the blue wavefronts become straight lines

(a) spherical aberration
(b) coma
(c) oblique astigmatism
(d) field curvature
(e) distortion

These aberrations can be in one of two groups. The first is for those that occur for axial object points and only spherical aberration falls into this category. The second group is for those aberrations that only occur for off-axis objects.

(a) Spherical aberration

Figure 4.18 illustrates a contact lens with positive spherical aberration. The paraxial emergent rays intersect with the axis at a point further from the lens than is the case for the more peripheral rays. In this case the emergent wavefront will have an aspherical shape. This aberration will exhibit rotational symmetry for a rotationally symmetrical system. The lens in Figure

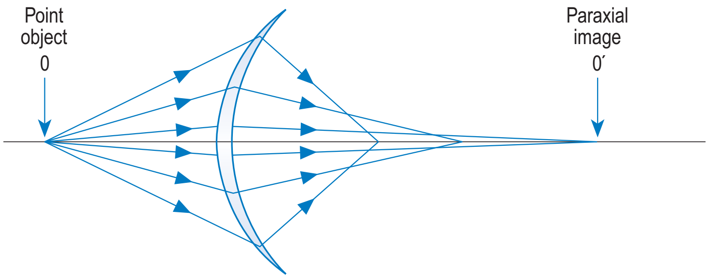

Figure 4.18 Light rays passing through a lens with positive spherical aberration

4.18 illustrates that the aberration increases for light rays passing through the more peripheral regions.

(b) Coma

Figure 4.19 illustrates coma produced by an off-axis point object. The paraxial light rays will form an image at the paraxial image point but the more peripheral rays will be responsible for the image being stretched out to give a comet-like appearance. The brightest part of the image will be around the region of the paraxial rays.

A contact lens wearer will only possess good retinal/neural resolution abilities in the foveal region which implies that we will only be concerned with the on-axis spherical aberration. However coma-like aberrations will be present possibly due to asymmetry in the eye's optical system and due to decentration in a contact lens correction.

(c) Oblique astigmatism

Oblique astigmatism will develop for very large field angles (objects and images well off axis) only and it is unlikely to interfere with vision because of the decline of neural resolution when moving from the fovea to the peripheral retina.

(d) Curvature

The aberration of field curvature results in an image of a plane object being formed on a curved image surface rather than a plane. This aberration is not significant for foveal vision. It can only affect peripheral retinal image points where neural resolution is low.

(e) Distortion

Distortion is the third aberration that requires a large field angle to become significant. Again this is unlikely to produce a noticeable effect in the peripheral region where neural resolution is low.

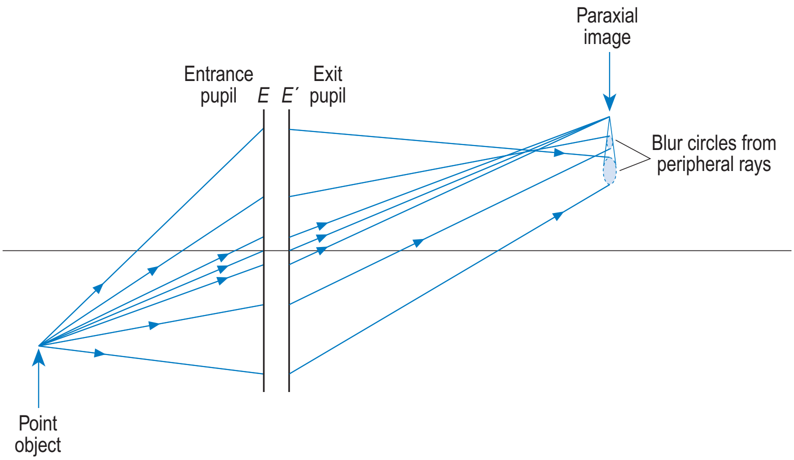

Figure 4.19 The aberration of coma produced by an oblique bundle of light rays. The aberration increases for light rays passing through the more peripheral regions

4.8.2 *Assessment of image quality*

Image quality can be assessed by one, or a combination of the following

(a) point spread function
(b) line spread function
(c) optical transfer function
(d) variance of the wavefront aberration
(e) Strehl intensity ratio
(f) Maréchal criterion

(a) Point spread function (PSF)

This is the distribution of illuminance or irradiance in the image of a point object. In an aberration free system the PSF corresponds to the Airy Disc irradiance distribution.

(b) Line spread function (LSF)

This is defined as the irradiance distribution in an image formed from a line object. The LSF is the sum of a line of PSFs

(c) Optical transfer function (OTF)

This is a measure of the reduction in contrast and the change in phase of a sinusoidal pattern as a function of spatial frequency, imaged by an optical system. The contrast attenuation alone gives the modulation transfer function (MTF). This is the ratio of the image contrast to object contrast as a function of spatial frequency. Spherical aberration will produce a decrease in contrast but the resulting symmetrical PSF will leave the phase of the grating

unchanged. Asymmetrical aberrations like coma will produce a decrease in contrast and a phase shift that varies with spatial frequency in a complex fashion.

(d) Variance of the wavefront aberration

This is also known as the mean squared wavefront deviation because it describes the average amount of deviation of the wavefront (when squared) from a reference surface.

(e) Strehl intensity ratio

This is the ratio of the peak of the PSF to the peak of the PSF of an aberration-free system.

(f) Maréchal criterion

This states that an optical system is well corrected for aberrations if the square root of the variance of the wavefront aberration does not exceed $\lambda/14$.

The ocular aberrations we are most interested in are spherical and coma-like aberrations. Due to the complexity and asymmetry of the eye's optical system it may be necessary to consider higher-order aberrations whose analysis requires greater accuracy than even the third-order approach.

4.8.3 Measurement of aberrations

Ocular spherical aberration was measured over 200 years ago using Young's Coincidence Optometer. Subsequently refractive techniques, applied to restricted regions of the pupil, demonstrated the asymmetrical nature of ocular aberrations.

4.8.3.1 The crossed-cylinder aberroscope

Howland and Howland (1976, 1977) developed the crossed-cylinder aberroscope. This consisted of a point source of light positioned 1m from the eye. A 4×4 or 5×5 grid was placed 27 mm from the eye. The optical components of this device consisted of two 5.00 D crossed cylinders with their axes mutually perpendicular. Light from the source passed through the grid which was drawn on the crossed-cylinder lenses. The shadow of the grid was projected onto the subject's retina. The deviation of the subject's impression of the grid from a reference grid was then used to calculate the coefficients of the Taylor polynomial. Zernike polynomials were calculated from the Taylor polynomials.

4.8.3.2 The objective aberroscope

Walsh *et al.* (1984) and Walsh and Charman (1985) modified the instrument to allow an objective method for measuring the grid displacement photo-

graphically. A beam splitter was used to allow the reflected retinal image to be photographed. The use of a suitable light source with filters restricted the light to wavelengths between 550 and 660 nm. They used 6 × 6 and 7 × 7 grids with 0.9 × 0.9 mm squares on the grid pattern. The grid image was photographed. The grid intersections and the pupil centre were located. This allowed calculation of the Taylor coefficients. Wave aberration contours could then be plotted to show the variation in aberration across the pupil. Speed and accuracy of the method were improved by Walsh and Cox (1995) who used a laser source, replaced the photographic camera with a CCD camera and programmed a computer to analyse the image.

4.8.3.3 The Hartmann–Shack wavefront sensor

This consists of a series of lenslets that divide the wavefront into a series of small regions which are focused individually in the focal planes of the lenslets. An ideal plane wavefront incident on the lenslets would produce a regular pattern of spot foci with each spot image lying on the optical axis of its lenslet. The presence of aberration will displace these spots. The magnitude of the displacements (the transverse aberration) will be due to the deviations of the light rays and thus the local slopes of the wavefront. The optical system of the Hartmann–Shack aberrometer is illustrated in a simplified form in Figure 4.20.

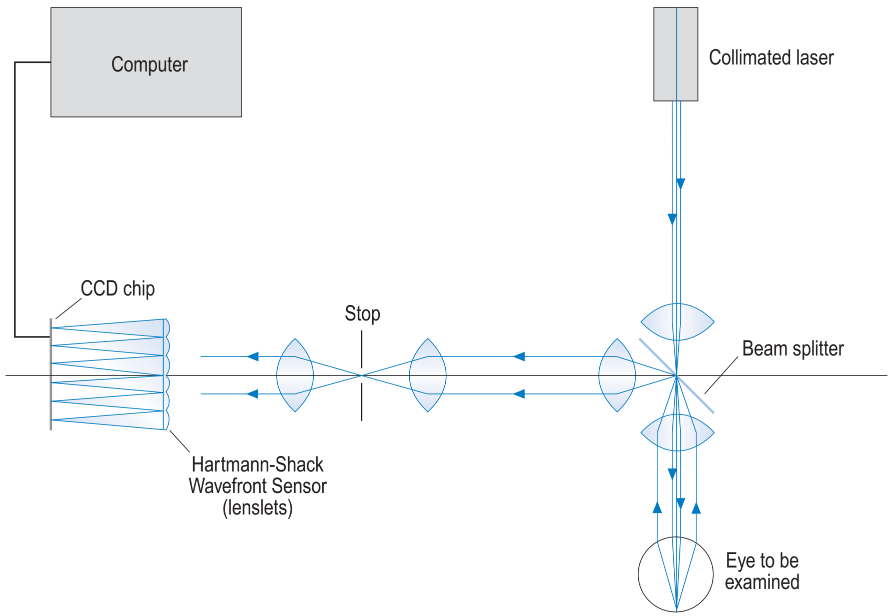

Figure 4.20 Simplified diagram of the Hartmann–Shack aberrometer. The plane of the lenslets is conjugate with the exit pupil of the eye (for light travelling into the eye, this is the entrance pupil)

The effect of an aberrated wavefront on the displacement of the point images is shown in **Figure 4.21**. Each lenslet images a local section of the light onto the CCD chip. Each lenslet can be located by its X–Y coordinates. The corresponding location of the lenslet optical axis intersection with the CCD chip can be given by the X′–Y′ coordinates. For simplicity, the diagram illustrates aberrations along the Y′ axis only. The transverse aberration is noted for a single lenslet. The transverse aberration for every lenslet allows derivation of the wavefront aberration across the pupil.

In **Figure 4.20**, light from a narrow beam laser source is focused to form a point image at the fovea of the eye. This is the point object for the aberroscope. The emergent light will display a distorted wavefront when aberrations are present. This distorted wavefront will be imaged on the CCD chip via the beamsplitter and the wavefront sensor lenslets. The wavefront distortion will result in lenslet images displaced from their individual on axis positions as shown in **Figure 4.21**. This displacement can be measured by the CCD chip and computer and is a direct measurement of transverse aberration of a light ray representing the average of all rays in the bundle defined by the aperture of the lenslet. From this ray, the local slope of the wavefront can be derived. There are typically around 150 to 200 lenslets across the image of the pupil. Thus the aberration is measured around 150 to 200 regions of the pupil.

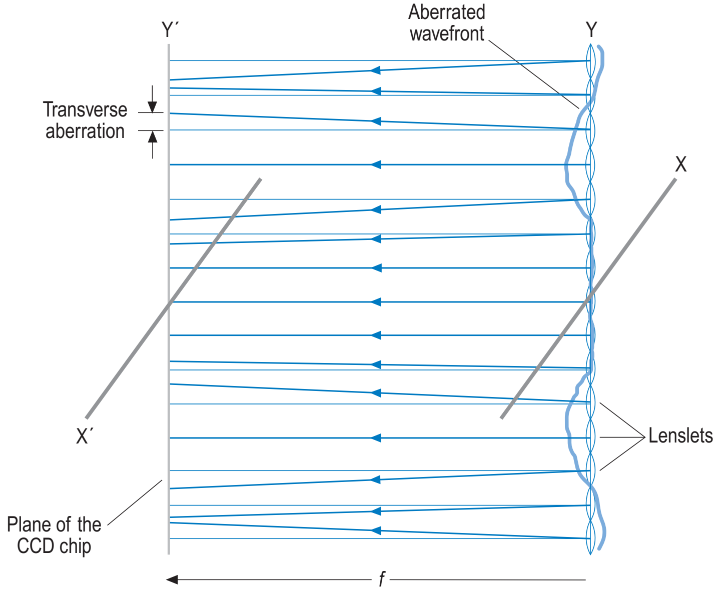

Figure 4.21 An aberrated wavefront imaged by the Hartman–Shack wavefront sensor. The lenslets are illustrated along the Y axis only. In reality they form a matrix in both the X and Y axes. *f* is the focal length of the lenslets. The dotted horizontal lines represent the optical axes of the lenslets. The solid light rays coincide with the optical axes when no transverse aberration is present

These transverse aberrations are then mathematically matched to provide coefficients for the Taylor polynomial that describes the wavefront aberration across the pupil aperture. The Taylor coefficients are then transformed into Zernike coefficients.

4.8.4 Polynomial representation of wavefront aberration

4.8.4.1 Seidel expressions

The power series related to the terms by von Seidel applies to rotationally symmetrical systems

$$W(\varphi, X, Y) = {}_0W_{0,0} + {}_0W_{2,0}(X^2 + Y^2) + {}_0W_{4,0}(X^2 + Y^2)^2 + {}_1W_{3,1}\,\varphi(X^2 + Y^2)Y + {}_2W_{2,0}\,\varphi^2(X^2 + Y^2) + {}_2W_{2,2}\varphi^2 Y^2 + {}_3W_{1,1}\varphi^3 Y + \text{higher order terms} \tag{4.21}$$

$W(\varphi, X, Y)$ is the wavefront aberration in micrometers, X and Y are the co-ordinates of the light ray in the pupil plane and φ is the angular displacement (field angle) of the object from the optical axis.

The first two terms in this expansion stand for phase error and spherical defocus, the next five terms are known as monochromatic primary aberrations, where each term represents a primary aberration as follows

${}_0W_{4,0}(X^2 + Y^2)^2$	spherical aberration
${}_1W_{3,1}\,\varphi(X^2 + Y^2)Y$	coma
${}_2W_{2,0}\,\varphi^2(X^2 + Y^2)$	field curvature
${}_2W_{2,2}\varphi^2 Y^2$	oblique astigmatism
${}_3W_{1,1}\varphi^3 Y$	distortion

The Seidel expression above is of limited value because it cannot deal with tilt, decentration and other asymmetries. It lacks the descriptive power to represent the types of wavefront encountered in the human eye. Also the measurement of ocular aberrations is usually made along the line of sight where the field angle φ is zero. More suitable expansions are the power series established by G. I. Taylor and F. Zernike.

4.8.4.2 Taylor expressions

Equation (4.22) is a two-dimensional Taylor polynomial equation that can be used to describe the ocular wavefront aberration.

$$W(X, Y) = w_0 + w_1X + w_2Y + w_3X^2 + w_4XY + w_5Y^2 + w_6X^3 + w_7X^2Y + w_8XY^2 + w_9Y^3 + w_{10}X^4 + w_{11}X^3Y + w_{12}X^2Y^2 + w_{13}\,XY^3 + w_{14}Y^4 \tag{4.22}$$

$W(X, Y)$ is the wavefront aberration in micrometres, and X and Y are the co-ordinates of the light ray in the pupil plane. Equation (4.22) contains powers up to the fourth order, each order representing aberrations of a different nature.

The term w_0 is a constant. The first-order terms w_1 and w_2 signify the tilting of the axis of a spherical wavefront which leads to a lateral displacement of the image across the focal plane in much the same way that a prism displaces an image. The second-order terms w_3, w_4 and w_5 cause a change in the curvature of the wavefront which is very close to spherical along each meridian. These terms represent a change in curvature of the wavefront leading to a change in the paraxial image distance as in spherical and astigmatic defocus. The coefficients that we are most interested in are those that lead to changes in the sphericity of the wavefront as a function of pupil size. In other words the wavefront changes induced by aberrations. These are the third- and fourth- order terms w_6 to w_{14}. The coefficients w_6 to w_9 represent rotationally asymmetrical wavefront aberrations and are therefore coma-like aberrations of the optical system. The remaining fourth-order terms (w_{10} to w_{14}) define a symmetrical change in curvature along any given meridian. This type of wavefront is characteristic of a spherical-like aberration.

4.8.4.3 Zernike polynomials

Zernike polynomials are a series of polynomial equations that are orthogonal over a circular space. Some Zernike terms are closely related to given Seidel aberrations. This is not true of the Taylor polynomial where individual aberrations are described by combinations of terms. Equation (4.23) is a Zernike polynomial expressed up to the fourth order. The order of each term corresponds to the power of the radial pupil coordinate, ρ and θ the angle from the right horizontal axis.

$$\begin{aligned} W(\rho, \theta) = {} & C_0 + C_1\rho\, sin(\theta) + C_2\rho\, cos(\theta) + C_3\rho^2 sin(2\theta) + C_4(2\rho^2 - 1) \\ & + C_5\rho^2 cos(2\theta) + C_6\rho^3 sin(3\theta) + C_7(3\rho^3 - 2\rho)\, sin(\theta) + C_8(3\rho^3 - 2\rho) \\ & cos(\theta) + C_9\rho^3 cos(3\theta) + C_{10}\rho^4 sin(4\theta) + C_{11}(4\rho^4 - 3\rho^2)\, sin(2\theta) \\ & + C_{12}(6\rho^4 - 6\rho^2 + 1) + C_{13}(4\rho^4 - 3\rho^2)\, cos(2\theta) + C_{14}\rho^4 cos(4\theta) \\ & + \text{higher order terms} \end{aligned} \tag{4.23}$$

Experiments attempting to measure aberrations in human eyes have indicated the following.

1. Only third- and fourth-order aberrations need be considered with pupil diameters around 3.5 mm. However, the higher-order aberrations become important with increasing pupil size. It was also noted that aberrations vary between subjects but there is usually similarity between the two eyes of any individual.
2. The results of various aberroscope studies show that individual aberrations are highly variable.
3. Spherical aberration appears to be less prominent than was thought but coma-like aberrations are more prominent.
4. The asymmetry of the aberrations suggests that the eye is not a rotationally symmetrical co-axial optical system.

5. Eyes that display positive spherical aberration present in the unaccommodated eye then tend toward negative spherical aberration under increasing accommodative effort.
6. Third-order coma-like aberrations predominate at most accommodation levels and with most pupil sizes. Coma changes with accommodation but the direction and magnitude of the change is subject dependant.
7. Correction of ocular aberrations using adaptive optics (a deformable mirror) produced significant improvements in contrast sensitivity measurements particularly for larger pupil diameters. Retinal images acquired through these adaptive optics were of higher contrast and quality to the point where it was possible to resolve individual cone cells in all parts of the retina.
8. Radial keratotomy causes ocular aberrations to increase but post-operative corneal changes cause a partial recovery.
9. Photorefractive keratectomy also causes an increase in aberrations especially coma immediately after surgery but this decreases over a twelve-month period. This appears to be due to peripheral corneal changes.
10. LASIK may also affect higher-order aberrations.
11. Spherical aberration may increase with age but coma-like aberrations appear to be similar in young and old groups of subjects but it must be noted that the large inter-individual variation in aberrations makes this type of comparison suspect.

The Hartmann–Shack wavefront sensor provides a fine sampling pattern combined with modern imaging and computer technology allowing rapid retrieval of precise wavefront data. Robustness, accuracy and speed have resulted in this becoming the standard clinical method for assessing wavefront aberrations.

4.8.5 Contact lens correction with soft lenses

Theoretical studies agree that spherical contact lenses in air produce significant levels of spherical aberration, but what happens to this aberration when a soft lens is placed on the eye is less clear. Some workers claim that most of the spherical aberration is eliminated on the eye but others have claimed that most of the spherical aberration remains. Soft lenses will steepen their surface curvatures when placed on the eye producing a fluid lens power less than 0.25 D. Negative lenses tend to retain their paraxial power on the eye but positive lenses lose around 0.50 D of paraxial power.

Dietze and Cox (2003) examined Bausch and Lomb Soflens 66 lenses over a power range of –8.50 to + 4.50 D. They found that spherical aberration was very similar on and off the eye. This supports the notion that 'aberration free' lens designs that claim to eliminate the contact lens spherical aberration are likely to maintain their aberration free status when placed on the eye. However these lenses do not correct the ocular aberration. Dietze and Cox

(2003) found that most subjects in their study exhibited positive ocular spherical aberration. Thus, for the myope, the positive ocular spherical aberration will be partially eliminated by correcting the eye with a negative power soft contact lens with spherical surfaces which produce negative spherical aberration in the contact lens. The aberration free design will leave the ocular spherical aberration unchanged and so the aspherical contact lens wearer is now at a disadvantage. In the case of the hyperope, a positive powered contact lens with spherical surfaces will possess positive spherical aberration and this will be added to the positive ocular spherical aberration. Clearly the hyperope would be at an advantage when wearing an aberration free lens design. This conclusion is supported by contrast sensitivity measurement studies where myopic contrast sensitivity did not improve with aberration-corrected soft contact lenses (deBrabander *et al.*, 1998).

Dietze and Cox (2004) measured the wavefront aberration and the visual performance using

- a standard (spherical surfaces) soft lens.
- an aberration free soft lens design (correcting its own spherical aberration in air).
- a custom made soft lens that corrected the total spherical aberration of the lens and eye in combination. This lens type had a spherical back and an aspheric front surface.

The soft lenses were made of Biogel 60 material (Filcon 4 60% water content) with a spherical back surface of BOZR 8.70 mm, a refractive index of 1.404 (hydrated) and a centre thickness of 0.1 mm (for BVP – 3.00 DS). Most of the lenses had BVPs in the region from – 4.00 to – 7.75 D. The BVP required for each lens was determined using standard clinical over-refraction procedures. High contrast log MAR visual acuity and contrast sensitivity were measured with each lens in place and also with the correcting spectacle lens.

Their results suggested that:

1. the ocular spherical aberration can be successfully corrected using customized aspherical front surface soft lenses.
2. the negative spherical aberration produced by negatively powered standard soft contact lenses partially cancels the (on average) positive ocular spherical aberration in myopes.
3. soft contact lenses designed to be aberration free in air, increase the spherical aberration of the eye-contact lens system when compared to standard soft lenses with spherical surfaces.

The typical cornea is tilted in the horizontal meridian in relation to the line of sight with the corneal apex displaced from a central position. An aspheric lens resting in a tilted and displaced position will induce non-rotationally symmetrical (coma-like) aberrations. For example a – 6.00 D front aspheric soft lens with a p-value of 0.5 tilted by a typical angle of three degrees with

the apex displaced by 0.5 mm in the same direction, more than doubles the horizontal coma produced by an equally tilted and displaced spherical soft lens for a 6 mm pupil diameter. The coma-like aberration will be further increased by any lens decentration on the cornea.

Under natural viewing conditions Dietze and Cox (2004) found that the wavefront aberration present in all lenses produced less image disturbance than that provided by 0.25 D of spherical defocus.

The standard soft lens manufacturer recommendation is to use a spherical soft lens to correct eyes with astigmatism up to 1.00 DC. This, it is suggested, provides a retinal image acceptable to most patients despite the fact that all or most of the astigmatism will not be corrected. Clinical experience suggests that this recommendation is acceptable for most patients. In these circumstances, the wearing of spherical soft contact lenses produces more retinal image degradation due to the small uncorrected astigmatism than that caused by all higher-order aberrations combined. It is worth noting that the higher-order aberrations will produce more image degradation when the pupil diameter increases. Photopic activity will take place with relatively small pupils. Thus Dietze and Cox (2004) found no improvement in high contrast log MAR visual acuity but did find improved contrast sensitivity with the custom made soft lens.

They concluded that coma-like aberrations arising from tilt/displacement, the effects of chromatic aberration, the effects of lens flexure, manufacturing tolerances, the dynamics of the contact lens on the eye, and changes of ocular aberrations with accommodation and age may all limit the potential benefit from aberration correcting soft contact lenses.

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CHAPTER 5

Measurement of the cornea

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5.1 The keratometer

The keratometer is an instrument which measures the radius of curvature of the central region of the anterior corneal surface. Keratometers are usually provided with a secondary scale which predicts the refracting power of the cornea. The instrument utilizes an internally illuminated object (the mire) to produce an image by reflection in the cornea. If we know the size and position of this object and the size and position of the image formed by reflection at the surface of a convex mirror (the cornea), we can deduce the radius of curvature of that reflecting surface. Thus the keratometer is essentially a device for measuring image size.

5.2 The keratometer equation

In **Figure 5.1** we can see that an internally illuminated target (the mire of the keratometer) can be used as an object which produces an image by reflection at the anterior surface of the cornea. This arrangement allows us to derive the radius of curvature of the cornea (r) as follows.

In **Figure 5.1** by similar triangles

$$\frac{h_1'}{h_1} = \frac{FO}{FB} = \frac{f}{-x}$$

but for a convex mirror

$$f = \frac{r}{2}$$

Therefore

$$\frac{h_1'}{h_1} = \frac{\frac{r}{2}}{-x} = \frac{r}{-2x}$$

Therefore

$$r = -2\frac{h_1'}{h_1}x \qquad (5.1)$$

If we were designing a keratometer we would need to decide on a value for the mire size h_1, and we would measure the mire image size h_1'. The only other value required to determine the radius of the anterior cornea is the distance x from the focal plane F to the mire B. If we knew the position of the

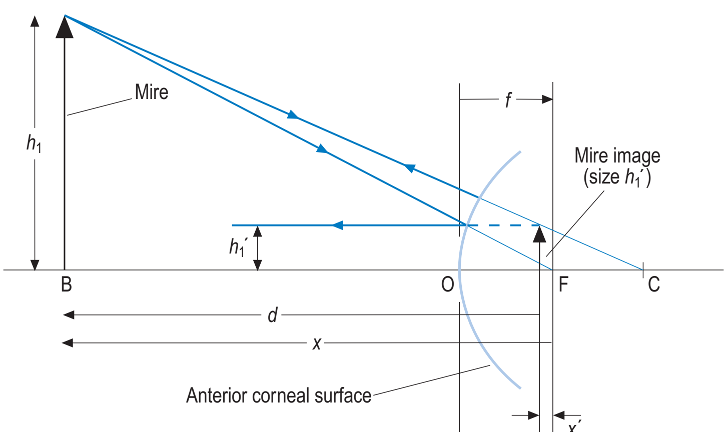

Figure 5.1 The formation of the mire image. The image is formed by reflection of light rays by the anterior corneal surface. This surface has a centre of curvature at C and a focal plane at F

focal plane we could deduce the corneal radius since $f = \frac{r}{2}$, and so we cannot know x unless we already know r. We must therefore use the approximation that x and d are equal, where d is the distance from the mire to the mire image. The equation now becomes

$$r \approx -2\,\frac{h_1'}{h_1}\,d \tag{5.2}$$

If collimated mires are incorporated into the keratometer then the mire is placed at optical infinity. In these circumstances the image formed by reflection in the cornea will be in the focal plane and x will be equal to d. It therefore follows that, as the distance from the mire to the cornea is reduced, the discrepancy between x and d increases owing to the mire image h_1' being displaced from the focal plane of the cornea and this leads to an increasing error in the equation. In the Bausch and Lomb keratometer where d = 72 mm, the error in r is about 0.02 mm. If d is increased to 150 mm, the error in r is reduced to around 0.003 mm. Since d is smaller than x then r will acquire a value which is smaller than it should be. Therefore the error above must be added to the keratometer reading acquired.

Keratometer calibration is best performed by measuring test spheres of known radius and constructing a calibration table comparing the keratometer setting with the radius of the test spheres.

If we wished to design and build a keratometer then equation (5.2) can be used to illustrate how we might decide on the instrument constants. The radius of curvature of the anterior cornea is determined if we know the values h_1', h_1 and d. As already stated the mire size h_1 would be decided upon during the design of a particular instrument. Let us now look at how the values d and h_1' are deduced.

5.2.1 *To deduce* d

In **Figure 5.2** the keratometer telescope position is adjusted until an image h_1'' is formed in the plane of the eyepiece crosswire. This fixes the image distance l' and must also fix the object distance l. If the mires are mounted on the instrument in the same plane as the objective then l equals d, because d in the keratometer equation is the distance between the mire and the corneal image.

In designing a keratometer we would select a value for the power of the objective lens and the distance between the objective and the crosswire, i.e. we would know the values for the lens power and the image distance and so deduction of the object distance is a simple matter of using the equation

$$\frac{1}{f} = \frac{1}{l'} - \frac{1}{l} \tag{5.3}$$

Thus

$$\frac{1}{l} = \frac{1}{l'} - \frac{1}{f}$$

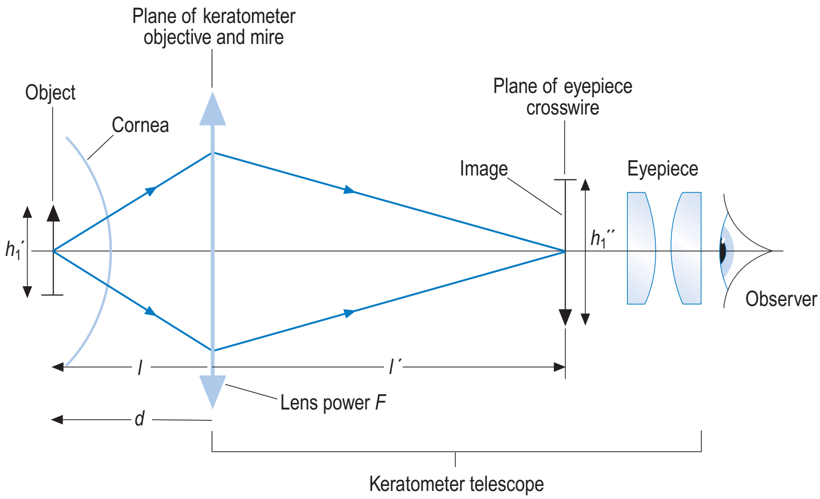

Figure 5.2 The mire image h_1' formed by the cornea becomes the object for the keratometer telescope which is positioned so that an image h_1'' is formed in the plane of the eyepiece crosswire. The distance from the telescope objective to the crosswire is the image distance l' which fixes the value for the object distance l. If the mire is mounted in the same plane as the objective then l equals d (the distance from the mire to the corneal image)

If the mires do not coincide with the objective it is simply a matter of adding or subtracting the distance between mire and objective to or from the value l in order to acquire d.

5.2.2 To deduce h_1'

In Figure 5.2, h_1' and h_1'' represent the size of the object and image respectively for the telescope objective lens. We can therefore use the general relationship

$$\frac{\text{size of image}}{\text{size of object}} = \frac{\text{image distance}}{\text{object distance}}$$

which in Figure 5.2 is

$$\frac{h_1''}{h_1'} = \frac{l'}{l} \tag{5.4}$$

Therefore

$$h_1' = \frac{l}{l'}\,h_1'' \tag{5.5}$$

Thus we can deduce h_1' if the other three values are known.

The values l and l' have already been decided upon during the design of the instrument as described above. We must therefore measure the size of the image in the plane of the eyepiece crosswire (h_1''). If we can measure this image size then we have everything required for substitution of values in the keratometer equation (5.2). This allows us to deduce the radius of curvature of the anterior surface of the cornea r.

5.3 Doubling

Head and eye movements on the part of the patient will cause the image, in the plane of the eyepiece crosswire, to move and consequently any attempt at direct measurement of the image size h_1'' will prove difficult and imprecise. The solution to this problem, in keratometers, is to use some type of doubling device and adjust the doubling until the base of one image coincides with the apex of the other. In these circumstances the doubling is equal to the image size.

In **Figure 5.3** the doubling prism is moved along the optical axis of the telescope (from left to right in the diagram) until the base of the lower image just touches the apex of the upper image. If the prism were moved further to the right the images would overlap. With the prism positioned as shown in **Figure 5.3** the doubling of the images is equal to the image size h_1''.

Prism power P^Δ is defined as the tangent of the angle of deviation multiplied by 100. Thus in **Figure 5.3**

$$P^\Delta = \frac{h_1''}{i} 100$$

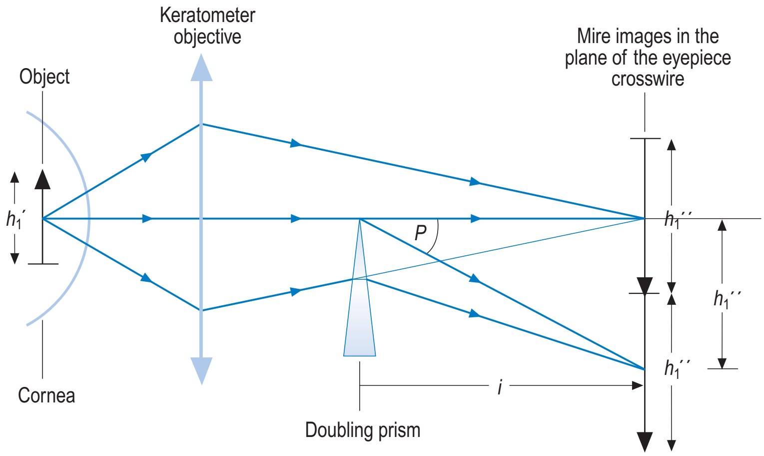

Figure 5.3 The doubling prism

Therefore

$$h_1'' = \frac{Pi}{100} \tag{5.6}$$

Therefore, if we know the prism power and the position of the prism we can deduce the size of the image h_1''. Also from this last equation we can conclude that

$$h_1'' \propto i$$

If we go back to consider the relationship

$$h_1' = \frac{l}{l'}\, h_1'' \tag{5.5}$$

we can conclude that

$$h_1'' \propto h_1'$$

Therefore

$$i \propto h_1'$$

Finally considering the keratometer equation

$$r = -2\,\frac{h_1'}{h_1}\,d \tag{5.2}$$

we can conclude that

$$r \propto h_1'$$

Therefore

$$r \propto i$$

That is, the movement of the doubling prism is directly proportional to the radius of curvature of the cornea. Therefore the radius scale derived from the doubling prism movement will be a linear scale. Thus the end point is reached by moving the prism along the instrument optical axis and this results in the instrument being classified as a variable-position doubler type.

An alternative option would be to fix the position of the prism and vary its power, giving a variable-power doubler. Since

$$h_1'' = \frac{Pi}{100} \tag{5.6}$$

once again

$$r \propto P$$

and therefore the radius scale of the keratometer will be linear.

There is, however, another option which involves a fixed amount of doubling in the keratometer telescope with the mire size (h_1) being increased

or decreased until the crosswire image size is made equal to the doubling. This is described as a variable mire size or fixed doubling instrument, and the keratometer equation

$$r = -2\,\frac{h_1'}{h_1}\,d \qquad (5.2)$$

leads to the conclusion that

$$r \propto \frac{1}{h_1}$$

Therefore the radius scale will not be linear, with crowding occurring for longer radii. However, since

$$\text{power} \quad F = \frac{(n-1)}{r}$$

$$F \propto \frac{1}{r}$$

Thus

$$F \propto h_1$$

This means that the corneal power scale will be linear in this type of instrument.

Repeat measurement studies on various keratometers suggest that the range of radius values will vary on repeat measurement by around ±0.05 mm.

5.4 Corneal region measured

5.4.1 Mire image formation

In **Figure 5.4** the mire is represented as two plus signs which mark its upper and lower extremities, i.e. the separation of the plus signs gives us the mire size h_1. In making an actual keratometer measurement we superimpose the two extremities and it is therefore only the mire extremities that we are concerned with. The light rays drawn in the diagram illustrate the limiting rays entering the telescope. From these we can see that the corneal image is produced by reflection from only a small off-axis region of the cornea.

Obviously the lower extremity of the mire will use an identical corneal region below the optical axis of the system. Thus, remembering that the principal meridians can be at any orientation, we can conclude that the corneal area used to provide the corneal mire image is an annular region. The size of the annulus will vary according to the instrument characteristics and the radius of curvature of the cornea. However, as a generalization, we can say that the internal diameter of the annulus is around 2.5 mm with an approximate annulus width of 0.7 mm. This obviously means that the central

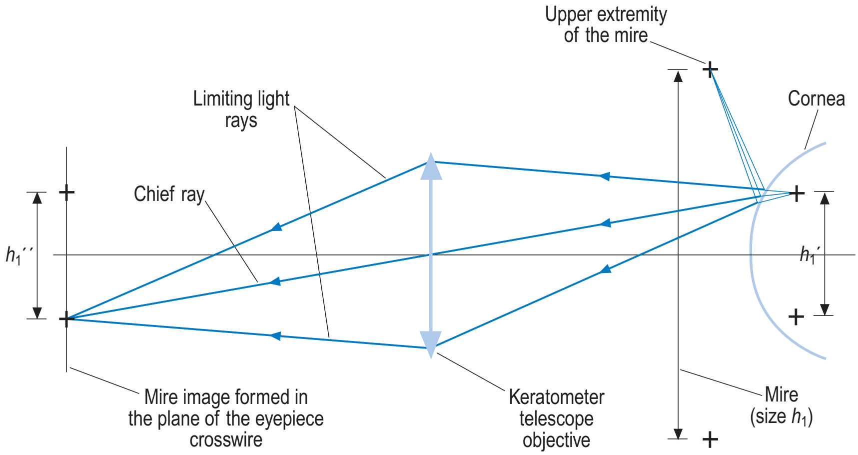

Figure 5.4 The limiting light rays entering the telescope reveal the corneal region involved in producing the mire image

2.5 mm of cornea is not involved in producing the mire image. The diameter of the annulus can be reduced by decreasing the separation of the mire extremities. Reference to the keratometer equation (5.2) indicates that this will lower the sensitivity of the instrument because a small value for the mire size (h_1) means a small change in mire image size (h_1') for a given change in radius.

5.4.2 *Tangential or sagittal radius measurement?*

The corneal section approximates to a conic section (Mandell and St Helen, 1971). The question that therefore arises concerns whether the keratometer measures the sagittal or tangential radius of the annular corneal region. This topic had, surprisingly, received little attention until Bennett and Rabbetts (1991) produced a paper in which ray tracing was used to conclude that the keratometer measures sagittal radius. Douthwaite and Burek (1995) used a modified keratometer to measure a set of convex aspheric buttons of known form. The results were analysed assuming that the keratometer was measuring tangential radius and then re-analysed assuming that the instrument was measuring sagittal radius. The results lead to the conclusion that the standard keratometer measures sagittal radius of curvature of aspheric surfaces that possess conic sections and this supports the conclusions of Bennett and Rabbetts.

5.5 The telecentric keratometer

Figure 5.5 illustrates the features unique to the telecentric design. The first thing to note is that the mires are collimated so that the corneal image is

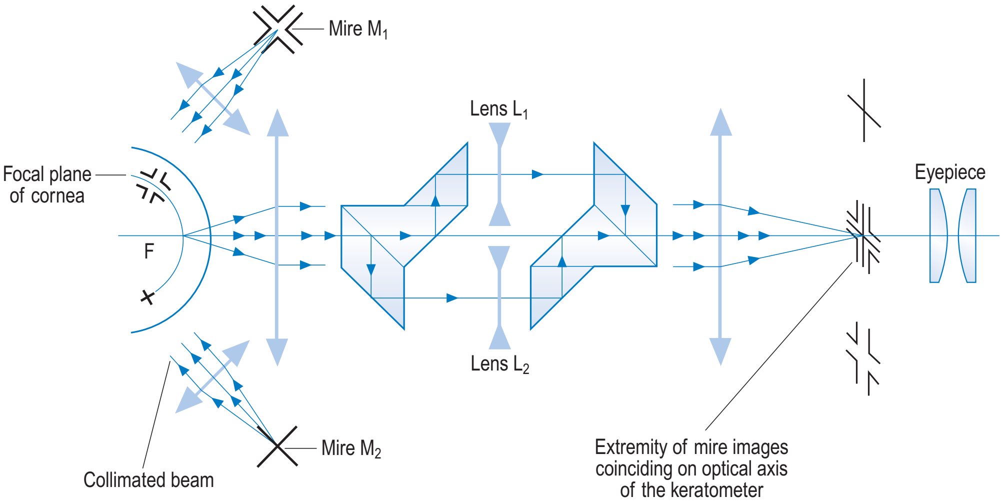

Figure 5.5 The telecentric keratometer. The collimated mire produces an image in the focal 'plane' of the cornea (F). The light from M_1 and M_2 passes through the two optical systems of the instrument to form two images near the eyepiece. These images are placed on the optical axis of the keratometer when the end point setting is achieved. L_1 and L_2 are the prismatic doubling lenses

formed at (rather than near) the focal plane of the cornea. This eliminates the approximation in the keratometer equation (5.2) because d is now equal to x.

In **Figure 5.5** the lenses L_1 and L_2 will induce a variable prism power by decentration and so this system uses a variable-power doubler based on this principle. In order to illustrate a major further advantage of the telecentric keratometer it will be necessary to simplify **Figure 5.5**. We will consider what happens to the light passing through the doubling lens L_1.

Figure 5.6 illustrates what happens in a non-telecentric keratometer. When the keratometer is positioned correctly the corneal mire image A will produce an image A′ in the plane of the eyepiece crosswire. If the keratometer is positioned too far from the corneal mire image as is the case with the image extremity B, then the image B′ formed in the telescope is nearer the objective and **Figure 5.6** illustrates that the image size is reduced. We have established that the keratometer measures the size of this image to deduce the radius of curvature. A reduced image size will give a reduced radius of curvature reading. It is therefore obvious that in a conventional keratometer the examiner must be sure that the eyepiece image is in the plane of the eyepiece crosswire. This is best achieved by establishing that the observed mire image and the eyepiece crosswire image are *simultaneously* clear. If the image is not formed in the plane of the crosswire then the radius reading acquired using a standard keratometer will be inaccurate.

The telecentric system is illustrated in **Figure 5.7** where it can be seen that the variable prism doubler (lens L_1) is placed at F′ (the principal focus of the

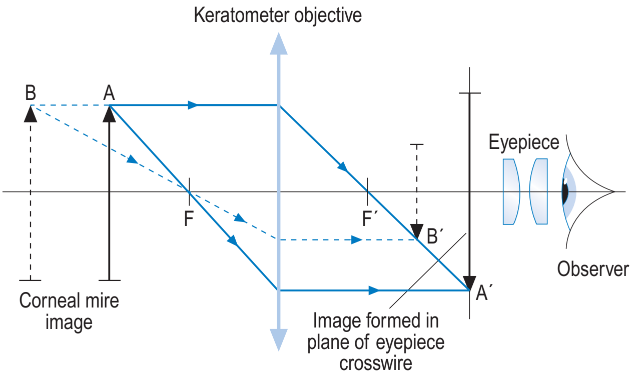

Figure 5.6 Eyepiece image formation in a non-telecentric keratometer

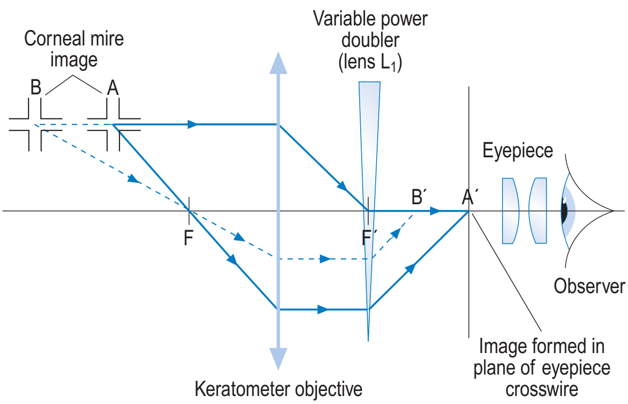

Figure 5.7 Eyepiece image formation in a telecentric keratometer

keratometer telescope objective). **Figure 5.7** illustrates that, if the keratometer is positioned at an inappropriate distance from the corneal mire image B (the centre of the hollow cross), the image point B′ in the telescope will still lie on the optical axis of the instrument, although it will be out of focus due to the cornea being too far from the objective.

The mire images near the eyepiece in **Figure 5.5** are doubled by lenses L_1 and L_2. The degree of doubling is altered until the lower image of the open cross coincides with the upper image of the solid cross and both cross centres lie on the optical axis of the keratometer. If the keratometer is then withdrawn from, or is moved towards the patient's eye, the observed images will defocus but the superimposition will be maintained with the cross centres remaining on the axis as illustrated in **Figure 5.7**. We now have a keratometer that does not rely on the observed images being positioned exactly in the plane of the eyepiece crosswire. The telecentric keratometer therefore does not require an adjustable eyepiece or an eyepiece crosswire and is undoubtedly the easiest keratometer to use.

5.6 One- and two-position instruments

A final option open to the designer of a keratometer is the choice between a one- or two-position instrument. In the two-position instrument, a single doubling device is used and the keratometer orientation is successively set on each of the two principal meridians of the cornea in order to acquire the anterior corneal radii of the flattest and steepest meridians. In the one-position instrument, a pair of doubling devices set mutually at right angles allows a simultaneous measurement of the two principal meridians. However, this is subject to some qualification. The assumption is made that the two principal meridians of the cornea are mutually at right angles and this is not always the case, particularly after prolonged contact lens wear. Also, in the case of a toric cornea, the position of the corneal mire images will differ for the two principal meridians. Therefore if one principal meridian image is in the plane of the eyepiece crosswire then the image determined by the other principal meridian is not. It is necessary to reposition the keratometer for this second image in order to acquire an accurate value for the radius. In **Figure 5.6** the corneal mire images A and B could be considered to be due to the steep and flat meridians respectively (in fact image B would be a little larger than image A due to the flatter corneal curve). The radius value acquired from A will be correct because image A′ is formed in the eyepiece crosswire. However, the flatter meridian produces an image at B which results in a keratometer image at B′ which, as illustrated in **Figure 5.6**, is smaller than it should be. A small image is associated with a short radius of curvature; therefore we can deduce that the corneal radius value acquired will be shorter than if the keratometer were repositioned correctly for this meridian.

For all keratometers, the principal meridians are determined by setting the mire images *in-step* (see **Figure 5.8**). The observation that these images are *in-step* only when the mires are parallel to either of the principal meridians of the cornea arises from the behaviour of light when passing through a toric

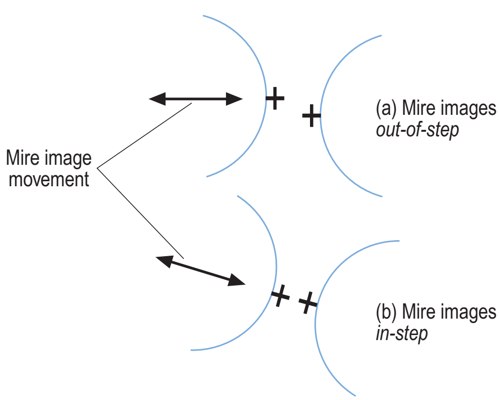

Figure 5.8 (a) The keratometer mire images *out-of-step*. The keratometer must be rotated clockwise to the orientation shown in (b). (b) The mire images are *in-step*, which means that the keratometer mire lies along one of the princial meridians of the cornea. The principal meridian is along 165 in the diagram

system. If we consider the 'scissors movement' observed in a toric spectacle lens which is rotated in front of a cross chart then we can relate this to the keratometer, since a mire image moving *out-of-step* does so for the same reason that the two limbs of the cross in a cross chart fail to intersect at 90° when the principal meridians of a toric lens are not horizontal and vertical. **Figure 5.8(a)** illustrates mire images that are *out-of-step*. In these circumstances it is impossible to superimpose the two cross images. The left-hand cross is higher than the right-hand cross. The keratometer must be rotated clockwise into the position shown in **Figure 5.8(b)** where the mire image movement is parallel to one of the principal meridians of the cornea. The mire image movement will now allow one of the crosses to completely cover the other. These mires are *in-step*. In **Figure 5.8(b)**, the keratometer is measuring the radius of curvature of the principal meridian that runs along 165.

5.7 Extending the range of measurement

There are occasions when the keratometer may be required to measure radii outside the standard range of values, e.g. checking the scleral radii of a scleral contact lens or measuring soft lenses in a wet cell (this gives apparent radii that are steeper than the real radii). The range can be extended, to measure longer radii, by using a negative auxiliary lens fixed to the keratometer objective. A positive auxiliary lens will extend the range to measure steep radii.

In **Figure 5.9(a)** we see a standard keratometer arrangement with the keratometer image h_1'' formed in the plane of the eyepiece crosswire. In **Figure 5.9(b)** the incorporation of a negative auxiliary lens results in a need for the keratometer to be moved further away from the corneal surface in order to ensure that the keratometer image h_2'' is still focused in the plane of the eyepiece crosswire. The increase in the distance between the mire and the cornea will move the corneal image towards the focal plane of the cornea, where it is already assumed to be since we have incorporated the approximation that d equals x in the keratometer equation. What we are really assuming is that the mires represent a distant target. We can conclude that the corneal image size h_1' remains unaltered. In fact, the image size will decrease very, very slightly.

Thus the introduction of the auxiliary lens leads to an increase in the object distance. The object size and the image distance remain unchanged.

If we now consider the relationship

$$\frac{\text{image size}}{\text{object size}} = \frac{\text{image distance}}{\text{object distance}}$$

Then

$$\text{image size} = \frac{\text{object size} \times \text{image distance}}{\text{object distance}}$$

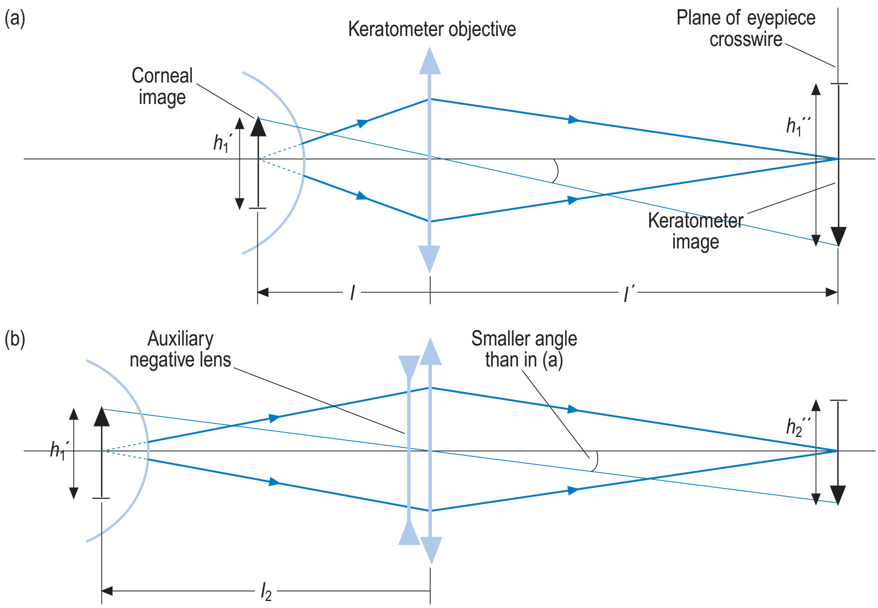

Figure 5.9 (a) The standard keratometer. (b) The same keratometer with a negative auxiliary lens fixed to the keratometer objective which produces a decrease in the size of the image h_1''

The increased object distance means that the image size is decreased. This is confirmed by inspection of **Figure 5.9**. Thus the negative auxiliary lens has reduced the size of the image in the eyepiece crosswire. A reduction in size of this keratometer image is normally due to a reduction in the size of the corneal image and this is produced by a reduction in the radius of curvature of the convex surface being examined. We can therefore conclude that the keratometer with the negative auxiliary lens will read steep. This makes it suitable for measuring radii which are longer than the standard range.

In practice the most convenient method of range extension is to fix an auxiliary lens in place, measure the radius of ball bearings with the modified keratometer and then measure the diameter of the ball bearings with a micrometer. This will allow the practitioner to construct a conversion scale.

In order to clarify the picture, consider the following example.

A keratometer has a working or object distance of 100 mm. The mires are collimated and of fixed separation. The maximum radius reading on the scale is 9 mm.

(a) In order to extend the scale up to 12 mm, what supplementary lens must be placed over the objective?
(b) Describe how the instrument is re-calibrated when using the supplementary lens. What is the radius of the surface when the keratometer radius reading is 6, 7, 8 and 9 mm?

(a)
Figure 5.10 illustrates the size and position of the mire and mire image before the supplementary lens is used. Let us assume that we are measuring a radius of 9 mm, i.e. the instrument is set to read the maximum radius of curvature (maximum doubling). The mire image will be formed in the focal plane of the cornea because the mires are collimated.

In triangle ABF

$$\frac{h_1'}{h_1} = \frac{f}{-d} = \frac{4.5}{100} = \frac{9}{200}$$

When the auxiliary lens is used, the same keratometer setting (the doubling has not been changed) will be used to measure a surface of radius 12 mm.

In these circumstances h_1 and h_1' will remain unchanged. Thus

$$\frac{h_1'}{h_1} = \frac{6}{-d}$$

Therefore

$$\frac{9}{200} = \frac{6}{-d}$$

$$-d = \frac{1200}{9} = 133.33 \text{ mm}$$

$$d = -133.33 \text{ mm}$$

A negative lens will be required to increase the working distance from 100 mm to 133.33 mm.

The dioptric working distance of the unmodified keratometer $= \frac{1000}{100} = 10.00$ D

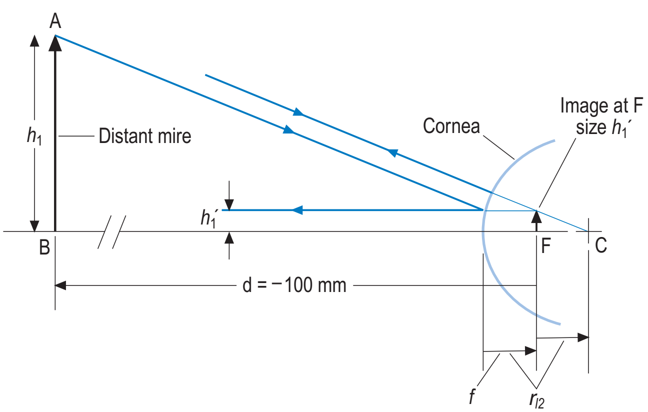

Figure 5.10 The positions and sizes of the mire and mire image before the supplementary lens is used

The dioptric working distance of the modified keratometer with the auxiliary lens $= \dfrac{1000}{133.33} = 7.50$ D

Therefore

the auxiliary lens power $= -2.50$ D

(b)

$\dfrac{h_1'}{h_1} = \dfrac{9}{200} = 0.045$ for the standard keratometer

$\dfrac{h_1'}{h_1} = \dfrac{6}{133.33} = 0.045$ for the modified keratometer

Using the standard keratometer equation

$$r = -2\,\frac{h_1'}{h_1}\,d \tag{5.2}$$

$$r = -2 \times 0.045 \times d$$

for both the modified and unmodified keratometer.

We can see that the ratio of the radius readings for the modified and the unmodified keratometers will be the ratio of their working distances (d).

$$\frac{\text{radius of modified keratometer}}{\text{radius of unmodified keratometer}} = \frac{133.33}{10} = 1.3333$$

Therefore take the instrument reading and multiply by 1.3333.
The scale reading of 6 mm means the surface radius is 8.00 mm.
The scale reading of 7 mm means the surface radius is 9.33 mm.
The scale reading of 8 mm means the surface radius is 10.67 mm.
The scale reading of 9 mm means the surface radius is 12.00 mm.

Note that measuring soft lenses in a wet cell produces apparent radii that are steep. In these circumstances we wish to extend the steep end of the radius scale so the keratometer must read flat. This will be achieved with a positive auxiliary lens.

5.8 The power scale

The keratometer equation allows us to convert the image size into a value for the radius of curvature of the anterior corneal surface. This can be easily converted into dioptric power using the equation

$$F = \frac{n-1}{r}$$

and since the refractive index of the cornea is 1.376 it would seem appropriate to use this as the value for n. However, this indicates the power of the anterior

corneal surface only. The total corneal power must include the influence of the posterior corneal surface. If we take the constants of the exact schematic eye as a basis for illustration, then the anterior surface radius is 7.7 mm. With a posterior surface radius of 6.8 mm and a corneal thickness of 0.5 mm, the total vergence change can be calculated by the step along method.

Power of the anterior cornea

$$F_{ac} = \frac{n_c - 1}{r_{ac}} = \frac{376}{7.7} = +\,48.83\text{ D}$$

where n_c is the refractive index of the cornea and r_{ac} is the anterior corneal surface radius of curvature.

Power of the posterior cornea

$$F_{pc} = \frac{n_a - n_c}{r_{pc}} = \frac{(1336 - 1376)}{6.8} = -\,5.88\text{ D}$$

where n_a is the refractive index of the aqueous

$$\frac{t}{n_c} = \frac{0.5}{1.376} = 0.36\text{ mm}$$

where t is the centre thickness of the cornea

In Figure 5.11

Vergence (D)		Distance (mm)
L_1 = 00.00		
F_{ac} = $\underline{+48.83}$		
L_2 = +48.83 ⟶	$\frac{1000}{48.83}$ ⟶	20.48 = AD
		$\underline{-0.36}$ = t/n_c
L_3 = 49.70 ⟵	$\frac{1000}{20.12}$ ⟵	20.12 = BD
F_{pc} = $\underline{-5.88}$		
L_4 = +43.82		

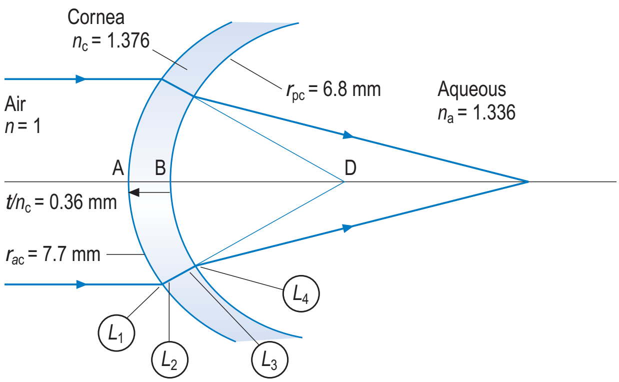

Figure 5.11 The change of vergence of light rays passing through the cornea. Vergence L_4 is the BVP of the cornea

We can conclude that the total vergence change for light from a point source at infinity passing through this cornea is +43.82 D which is the BVP of the cornea.

If we now go back to use the equation $F = \frac{(n-1)}{r}$ but substitute in that equation the total power of the cornea with the radius of the anterior cornea, we obtain a refractive index which can be used to predict total corneal power having measured the radius of curvature of only the anterior corneal surface.

$$n - 1 = F.r = 43.82 \times 0.0077$$
$$n = 1.3375$$

Therefore, if we use a refractive index of 1.3375 to convert anterior corneal surface radius into dioptric power, then the power acquired will be that of the total cornea (anterior and posterior surfaces), provided the cornea under examination has similar characteristics to the exact schematic eye. The dioptric power reported by the keratometer must thus be viewed with caution. If there is a 2.00 D difference between the principal meridians of a toric cornea according to the keratometer, we can conclude that the total corneal astigmatism approximates to 2.00 D.

In the past, some instrument manufacturers have used refractive index values other than 1.3375 and so different instruments may indicate slightly different corneal power values for the same corneal radius. Zeiss, for example, have assumed the refractive index to be 1.332. American Optical have used 1.336 as the refractive index value.

The schematic cornea was derived from a small sample of human eyes and so 1.3375 should not be regarded as the definitive value, it is simply the most common value adopted. The value of 1.336 has some attraction. It is the refractive index of tears and aqueous, it is the refractive index of the single corneal surface in the Bennett and Rabbetts (1989) schematic eye and it has been adopted by some ophthalmologists in calculating intra-ocular implant power.

If the keratometer power scale has been constructed using 1.3375 as the assumed refractive index, then a radius of 7.5 mm gives a power reading of 45.00 D.

One final point to note from the constants of the exact schematic eye is that the ratio of corneal posterior to anterior surface power is

$$\frac{5.88}{48.83} = 0.12$$

This means that the posterior surface power is approximately 12% of the anterior surface power. This assumes that the cornea is regarded as a thin lens. If the corneal thickness is assumed to be 0.5 mm then the ratio changes to 11.8%. Remembering that the exact schematic eye values for thickness and back surface power are derived from a small sample, it may be more appropriate to state that the ratio approximates to 10%. A paper by Dunne

et al. (1991), who examined 60 young subjects, concluded that the posterior corneal surface neutralized around 14% of the anterior corneal surface astigmatism.

5.9 The keratometer used to check concave surfaces

The equations encountered in this chapter are derived assuming paraxial theory. Emsley (1963) and Bennett (1966) have shown this to be an over-simplification. The corneal areas from which the mire images are reflected are too far from the axis of the system to be considered as being in the paraxial region of a surface of reflecting power –260.00 D. Bennett (1966) has shown that the difference in the non-paraxial light rays of convex and concave surfaces can account for the corrections which have to be applied when a keratometer is used to measure the BOZR of a contact lens, whereas the paraxial formulae apply equally to concave and convex surfaces. The error is about 0.02 mm for steep radii (6.50 mm) and around 0.04 mm for flat radii (9.50 mm). For most of the BOZRs encountered in contact lens work it is sufficient to add 0.03 mm to the radius given by the keratometer.

5.10 Precautions

The keratometer is an image size measuring device. It is therefore essential (with the exception of the telecentric keratometer) to have this image in the plane of the eyepiece crosswire.

Thus, when using a keratometer:

(a) Turn the eyepiece fully anticlockwise. Look through the telescope at a distant object and turn the eyepiece clockwise until the crosswire first comes into focus. The crosswire is now at the far point of your eye.
(b) Start the measurement with the instrument distant to the cornea and approach the cornea with the instrument until the mire images first come into focus. The mire images are now at the far point of your eye.
(c) During the time that the mire images are being made to coincide, the practitioner should see the crosswire and mire images clearly *simultaneously*. If they are not simultaneously clear, the mire images are not in the same plane as the eyepiece crosswire. The instrument position must be constantly adjusted to maintain the necessary picture. Therefore, when superimposing the mire images, one hand must be on the joystick constantly adjusting the instrument position while the other hand is used to superimpose the mire images.
(d) The power scale uses an assumed refractive index which allows an approximate prediction of the total corneal power from a measurement of anterior surface radius of curvature. The power scale must therefore be used with this in mind.

(e) The keratometer error arising from the mires being near the cornea increases if the instrument is designed to operate near the eye, i.e. where d and x in the keratometer equation have a small value.
(f) The mire images are formed by a corneal region some 1.5 mm from the keratometric pole. The keratometer is, therefore, indicating the radius of curvature of this area and this area only.

5.11 The topographic keratometer

The human cornea is typically relatively steep at the corneal apex with the corneal radius of curvature increasing as the limbus is approached. A section through one of the principal meridians approximates to a conic section. It would therefore be useful to be able to make curvature measurements across the cornea. This would allow a mathematical description of the corneal topography to be made.

If we return to Section 5.4.1 we can see in **Figure 5.4** that, when taking a keratometric measurement of the central cornea, the image is produced by two corneal regions. These are typically approximately 0.7 mm in diameter and around 1.5 mm from the corneal vertex. If the visual axis of the eye in **Figure 5.4** is directed inferiorly then the light rays from the lower extremity of the mire may well strike the cornea at or near its apex. At the same time the light rays from the upper extremity of the mire would be reflected by a region of the anterior cornea near the upper limbus. The lower mire extremity image will be formed by the central cornea with the upper extremity formed by the flatter periphery as shown in **Figure 5.12**. Any reading taken in these circumstances is of questionable value.

Essentially the problem described above arises from the mire size h_1 being too large. We could measure the radius of curvature of the peripheral corneal regions if we reduced the size of the mire, and this was the approach adopted in the topographic keratometer designed by Bonnet and Cochet (1962). If, in the circumstances illustrated in **Figure 5.12**, we switch off the lower mire extremity and regard the upper extremity as the entire mire, then the corneal image is formed by the peripheral corneal region only. We then simply need to double the extremity to measure its size. This inevitably requires a considerably weaker doubling device.

The disadvantage of a small mire can be seen when the keratometer equation (5.2) is examined

$$r = -2\frac{h_1'}{h_1}d \tag{5.2}$$

A small value for h_1 means that a smaller change in image size h_1' will occur for any given change in radius. Thus the precision of the keratometric measurement is reduced due to reduced sensitivity of the instrument arising from a small mire size. Therefore, when using the topographical keratometer,

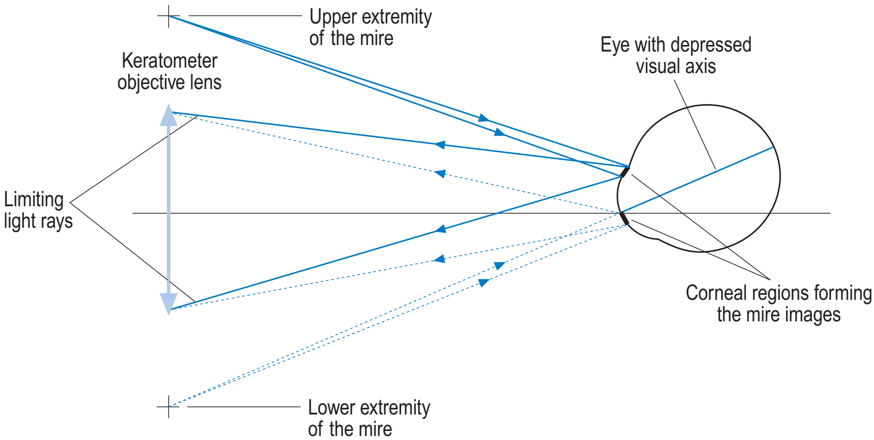

Figure 5.12 An attempt to use a standard keratometer to measure the radius of curvature of the corneal periphery

the central keratometer reading is taken by *classical keratometry*, with the small mire size being used for peripheral radius measurement only. The peripheral radius measurements are made by displacing the fixation spot from the optical axis of the instrument. It must be noted that the eye rotates about the centre of rotation of the sclera and this results in the cornea being tilted as the eye moves from the primary position. Thus the examiner must laterally shift the position of the keratometer when making peripheral radius measurements in order to keep the mire image in the middle of the field of view.

The topographic keratometer should not be confused with the topogometer which is simply a movable fixation device attached to a standard keratometer. The fixation device is displaced from the optical axis of the instrument until the observed image starts to change size. It is thus simply a means of assessing the diameter of the corneal cap, i.e. the central area of the cornea over which there is no appreciable change in radius of curvature.

5.12 The photokeratoscope

It will be recalled from Section 5.4.1 that the corneal region used to generate the mire image is approximately 1.5 mm from the corneal apex. If in **Figure 5.4** the mire size h_1 is increased, then a more peripheral region of the cornea will be used to generate the mire image h_1'. This is the underlying principle of the photokeratoscope. The mire is replaced by a set of concentric circles of increasing diameter (a Placido disc target) as illustrated in **Figure 5.13**. The size of the corneal image of the small central circle can be used to calculate the radius of curvature of the central cornea, whereas the corneal image of the largest circle is used to calculate the radius of curvature of the corneal

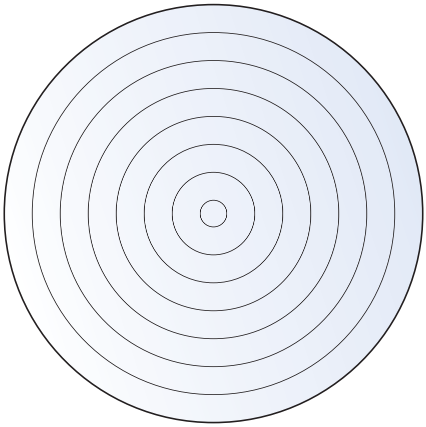

Figure 5.13 A Placido disc target

periphery. The image size is measured from a photograph of the ring images formed by the anterior corneal surface.

Criticisms of this approach are that:

(a) Film shrinkage may occur during development.
(b) The orientation of the principal meridians is not obvious.
(c) The film grain inevitably means that the edges of the image will show some blurring under magnified inspection.

However, photokeratoscopes have been used successfully in practice and possess the significant advantage of assessing the topography of the cornea. The first popular clinical instrument was the Wesley Jessen System 2000 PEK which used a target of seven concentric circles to analyse the cornea in 1 mm increments from an approximate corneal diameter of 3 mm to 9 mm. The image plane for these circles must be flat if all seven circle images are to be focused on the camera film simultaneously. This is essential for accurate radius measurement. The arrangement of the circles will depend on the topography of the surface under examination and so some assumptions about the surface will have to be made. The instrument will be designed to measure a surface that is similar to a typical cornea. The instrument will make accurate radius measurements on this surface. However, the accuracy will deteriorate when measuring steeper or flatter surfaces. It will also deteriorate for surfaces that are more or less aspheric than the design surface.

A toric cornea will produce elliptical rather than circular images and the principal meridians are revealed by the long axis (flat meridian) and short axis (steep meridian) of the ellipse, but of course the image of even a single ring cannot form in a plane in these circumstances. If the section of the ring generated by the flat meridian is in focus, the region generated by the steep meridian will be out of focus.

For the Wesley Jessen photokeratoscope, the circles were arranged so that their loci formed an ellipsoid. The ellipsoidal target surface will theoretically produce a flat image plane when the light is reflected from a spherical surface

of specified radius. Reproducibility studies on the Wesley Jessen instrument claimed a maximum standard deviation from the mean of 0.01 mm; however, this applied to measurements on three test spheres. The target rings on this instrument are set on an ellipsoid which ensures that the photographic image is totally clear only when the light rays are reflected from a spherical surface of radius 7.7 mm. The accuracy will deteriorate as the reflecting surface deviates from this particular form and curvature.

The most recent versions of the photokeratoscope employ the same fundamental principles as the Wesley Jessen PEK but instead of photographing the image with a Polaroid camera, the newer instruments use a computer to download the image from a digital CCD camera. These instruments are called videokeratoscopes.

5.13 Videokeratoscopes

There are now a number of manufacturers producing videokeratoscopes (VKs). These instruments use the Placido disc target with the image captured by a CCD camera that then downloads the image into a computer. The computer will enhance and analyse the incoming data from the corneal image rings with instrument operation performed using mouse driven menus to make the instrument user friendly. All this high tech computer wizardry is very impressive but we must not lose track of the fact that the instruments are still using a Placido disc target consisting of a number of concentric rings with the limitations that have already been described. VKs produce colour coded maps of the corneal topography and this has proved to be popular with ophthalmologists for pre-, followed by post-operative pictures involving all types of corneal surgery. This obviously has some application in the contact lens field where the influence of lens material and contact lens back surface design on the corneal topography can be assessed easily and conveniently in the clinical situation. This is particularly useful in orthokeratology. Contact lens fitting programs are also available. It is, however, worth emphasizing that the target design means that the accuracy of the measurement will deteriorate when atypical corneal surfaces are examined. This may limit the usefulness of these instruments for the prediction of the fit of contact lenses on the unusual cornea.

The fundamental principle of the VK is illustrated in **Figure 5.14** where one of the rings of the Placido disc target provides a chief ray that subtends angle 2α with the optical axis. This ray strikes the anterior cornea at point A where it is reflected in a direction parallel to the VK optical axis. The ray strikes the CCD chip at distance h' from the optical axis. This distance is the semi-diameter of the image of the Placido disc ring formed in the plane of the CCD chip. Thus the VK, like the keratometer, is an image size measuring instrument. The VK is essentially a keratometer with a number of different mire sizes. The geometry of **Figure 5.14** demonstrates that, providing the

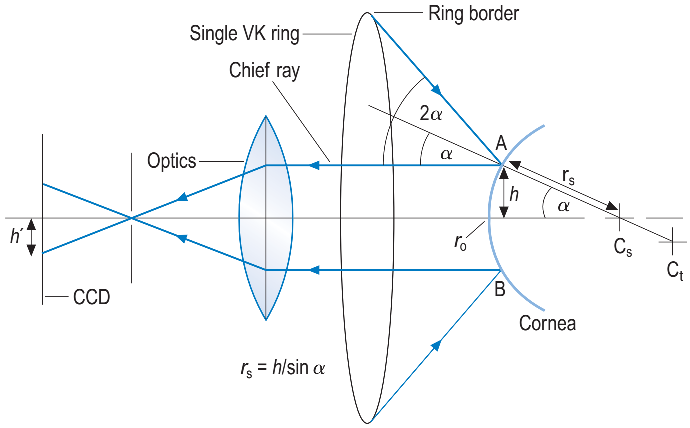

Figure 5.14 The fundamental working principle of the videokeratoscope

constants of the camera are known then measurement of h' allows calculation of distance h by similar triangles. Distance h is the perpendicular distance from the optical axis of the VK to the corneal point being measured (point A). The normal to point A will bisect angle 2α. Thus the normal subtends an angle α with the VK optical axis. The normal to the corneal surface must also pass through the sagittal centre of curvature and the tangential centre of curvature of the corneal section if we assume it to be a conic section.

Figure 5.14 illustrates that

$$\sin\alpha = \frac{h}{r_s}$$

Therefore

$$r_s = \frac{h}{\sin\alpha} \qquad (5.7)$$

where r_s is the sagittal radius of curvature of the cornea at point A. Measurement of the distance h' and knowledge of angle α allows the calculation of perpendicular distance h and sagittal radius of curvature r_s. These two parameters will also be determined for point B. We therefore have a superior semi-meridian distance and radius and an inferior semi-meridian distance and radius for this single ring. A typical VK might have 18 ring borders. This means that there will be 18 pairs of perpendicular distances with 18 pairs of sagittal radii for each semi-meridian. Any meridian can be selected. The instrument default is to select the flattest and steepest corneal meridians. In other words the standard tabular display indicates radii and distances for the two principal corneal meridians. This standard tabular display is a feature of the EyeSys VK. These are the raw data from which the colour coded displays are derived. A simplified version of the tabular display of the EyeSys VK is illustrated in Table 5.1.

Table 5.1 The tabular display of the EyeSys VK (18 ring borders) for an aspheric surface. The perpendicular distances and the sagittal radii are given for the nasal and temporal semi-meridians along 180 with the superior and inferior semi-meridians along 90

	Along 180				Along 90			
Ring no.	**Nas dist**	**Nas rad**	**Tem dist**	**Tem rad**	**Sup dist**	**Sup rad**	**Inf dist**	**Inf rad**
1	0.26	7.44	0.25	7.42	0.24	7.39	0.24	7.36
2	0.53	7.44	0.52	7.41	0.52	7.43	0.50	7.34
3	0.76	7.45	0.75	7.41	0.74	7.43	0.73	7.36
4	1.03	7.44	1.04	7.43	1.01	7.43	0.97	7.35
5	1.27	7.45	1.26	7.43	1.23	7.43	1.23	7.38
6	1.53	7.45	1.54	7.45	1.51	7.45	1.48	7.39
7	1.77	7.46	1.78	7.46	1.73	7.46	1.72	7.41
8	2.02	7.46	2.04	7.46	2.00	7.48	1.97	7.44
9	2.28	7.49	2.28	7.48	2.24	7.50	2.22	7.46
10	2.54	7.5	2.55	7.50	2.50	7.52	2.48	7.48
11	2.73	7.53	2.61	7.53	2.74	7.55	2.73	7.50
12	3.04	7.55	3.08	7.55	3.00	7.57	2.98	7.52
13	3.31	7.59	3.34	7.58	3.25	7.60	3.23	7.55
14	3.56	7.62	3.60	7.61	3.51	7.63	3.50	7.58
15	3.82	7.65	3.87	7.65	3.77	7.65	3.76	7.62
16	4.1	7.68	4.14	7.68	4.02	7.66	4.01	7.65
17	4.34	7.7	4.41	7.71	4.25	7.59	4.28	7.68
18	4.56	7.71	4.63	7.73	4.38	7.54	4.50	7.71

VK displays offer a variety of presentations. The colour coded topographical maps are usually based on radius of curvature with steep areas represented by colours at the red end of the spectrum and blue colours representing flatter curves. Often two radius options are offered. These are either an instantaneous or an axial map. If we assume that the human cornea

possesses a conic section, then the instantaneous radius is synonymous with the tangential radius and the axial radius with the sagittal radius. It will be recalled that in a prolate ellipse the tangential (instantaneous) radius will be longer than the sagittal (axial) and so the instantaneous display will give a more exaggerated topographical variation. It must be remembered, however, that the VK measures sagittal (axial) radius and that the instantaneous radius is calculated from this having made assumptions about the surface characteristics.

Other VK displays include corneal section profiles, simulated keratometer results, changes in the orientation of the principal meridians across the cornea and a facility to subtract one colour coded map from another. This latter option leaves a display that illustrates the regions of the cornea that have changed between the acquisition of the first and second maps. There are also contact lens options that will produce simulated fluorescein pictures.

Simulated fluorescein pictures produce a striking display but is this what the contact lens practitioner requires? In conventional RGP fitting we use fluorescein out of necessity in order to assess the fitting relationship. To do this we need to create a mental image of the fitting relationship from the fluorescein picture. With the VK, we could directly observe a picture of the fitting relationship represented as a tear layer thickness profile. Some of the software available provides this as an option but, because this is a less impressive display and presumably because the software writers think it is unlikely that contact lens practitioners will adapt to a new approach having used fluorescein for decades, this type of display is not well presented. This is a pity. A tear layer profile selected for the flat and the steep meridians directly shows the contact lens practitioner what s/he needs to know.

5.13.1 *Derivation of the apical radius and the p-value*

The tabular data illustrated in **Table 5.1** can be used to determine the apical radius and the p-value of the section of the surface being examined.

Bennett (1988) derived the equation for an elliptical section

$$r_s = \sqrt{[r_o^2 + (1 - p)y^2]} \tag{5.8}$$

where r_s is the sagittal radius for a point perpendicular distance y from the major axis of the elliptical surface, that has an apical radius r_o and a p-value p. This equation can be written as

$$r_s^2 = r_o^2 + (1 - p)y^2 \tag{5.9}$$

and since the apical radius r_0 and the p-value p are constants of the surface section, the equation is the familiar equation for a straight line graph. If we plot a scatter plot graph of r_s^2 (on the y-axis) versus y^2 (on the x-axis) and calculate the best fitting line by the method of least squares, then the equation for that line will be equation (5.9). The intercept on the y-axis gives r_o^2 ($r_s^2 = r_o^2$ where $y = 0$) and the slope of the line is $1 - p$. So the apical radius

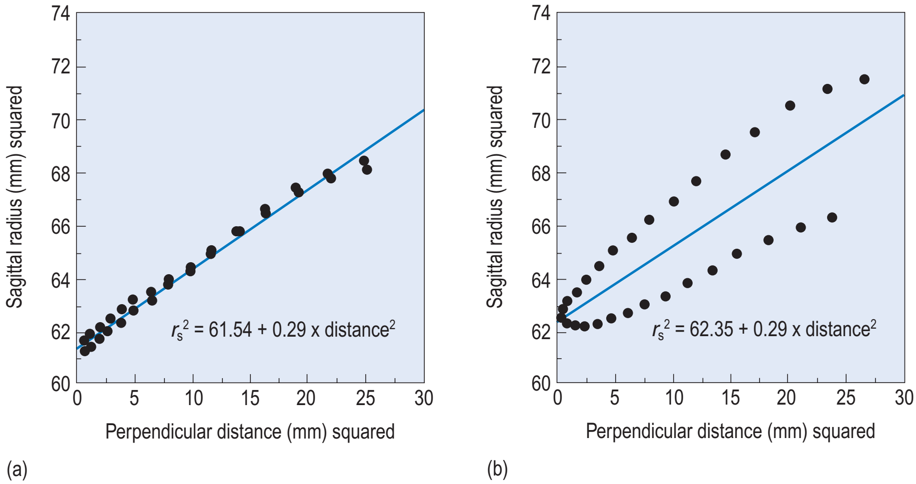

Figure 5.15 Scatterplots of radius squared versus distance squared VK measurements on the horizontal meridian of an ellipsoidal surface. (a) Measured with the surface not tilted. (b) Measured with five degrees of horizontal tilt

of the section is acquired by calculating the square root of the intercept on the *y*-axis and the p-value is calculated by subtracting the slope from unity.

This analysis was used by Douthwaite (1995) using calibrated aspheric surfaces to assess the accuracy of the EyeSys VK. The instrument performed well but measurement error increased for the more aspheric surfaces.

VK displays derived from human corneas often reveal a nasal/temporal asymmetry with little superior/nasal asymmetry. Douthwaite and Pardhan (1998) investigated the nasal/temporal asymmetry and concluded that the human cornea was not notably asymmetrical across its horizontal meridian. The VK asymmetry was a measurement artefact due to corneal tilt in the horizontal meridian. **Figure 5.15** illustrates the r_s^2 versus y^2 scatter plot derived from a VK measurement of the horizontal meridian of an ellipsoidal surface (a) not tilted and (b) tilted 5 degrees.

Figure 5.15(a) gives an apical radius of 7.84 mm ($\sqrt{61.54}$) with a p-value of 0.71 (1 – 0.29) and the points on the graph are all very close to the regression line as expected. **Figure 5.15(b)** shows the effect of surface tilt. The sagittal radius decreases in one semi-meridian and increases in the other and, in consequence, any topographical display will be asymmetrical. Note that the equation for the regression line gives an apical radius of 7.90 mm with a p-value of 0.71 so the regression line is shifted only slightly by the tilt. Douthwaite (2003) continued investigations into the best ways to analyse the results and noted that the scatter plot for a tilted surface could be drawn using points which are the average of the pairs of distances and pairs of radii for each ring. If this is done for **Figure 5.15(b)** then the scatter plot will look like **Figure 5.16** which demonstrates that the points are now placed very close to the regression line. Mathematical modelling showed that all the averaged

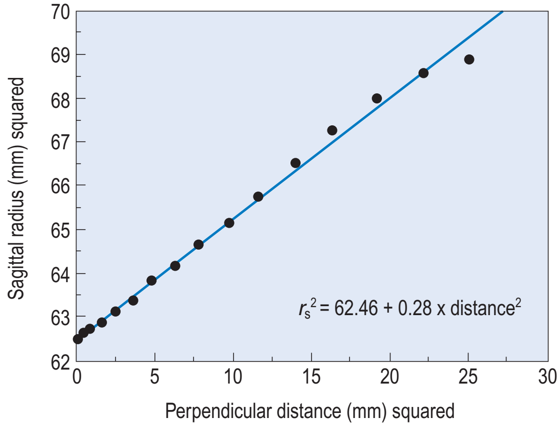

Figure 5.16 Scatterplot of radius squared versus distance squared VK measurements on the horizontal meridian of an ellipsoidal surface with five degrees of horizontal tilt. The nasal and temporal distances of each ring have been averaged to a single distance as have the nasal and temporal radii

points will theoretically lie on the regression line for a tilted ellipsoidal surface and this regression line gives the best estimate of the apical radius and p-value for the section. The big advantage of averaging the points when measuring a human cornea is that the assumption of an elliptical section can be confirmed by a high correlation coefficient (points on or near the regression line). If this does not occur then the cornea being examined does not have an elliptical section and so the apical radius and p-value derived are suspect.

Figure 5.17 shows the scatter plots produced from measurements of the two principal meridians of a human cornea plotted with and without averaging of the points. This shows the apparent asymmetry along the horizontal meridian with no evident asymmetry in the vertical meridian. The figure also indicates that when semi-meridian averaging is performed the points display close proximity to the regression line. This confirms the elliptical form of the two meridian sections.

Figure 5.15 indicates that the points derived from the smaller ring images bunch around the regression line. Images of the small rings in many cases display a spread of points arising from the lack of precision of the results derived from the small rings of the Placido disc. This occurs for exactly the same reason that small mire keratometry lacks sensitivity. The larger rings often fail to generate complete images due to the nose and the brow getting in the way. Douthwaite (2003) therefore derived mean values for a group of 98 subjects using images from rings 5 to 12 inclusive with the EyeSys VK. He also used the asymmetry seen in Figures 5.15(b) and 5.17(a) to determine the approximate tilt of the cornea in relation to the VK optical axis.

5.13.2 Derivation of the approximate tilt

When a rotationally symmetrical surface is not tilted, the perpendicular distances for the two semi-meridians of a VK ring will be equal. This is also

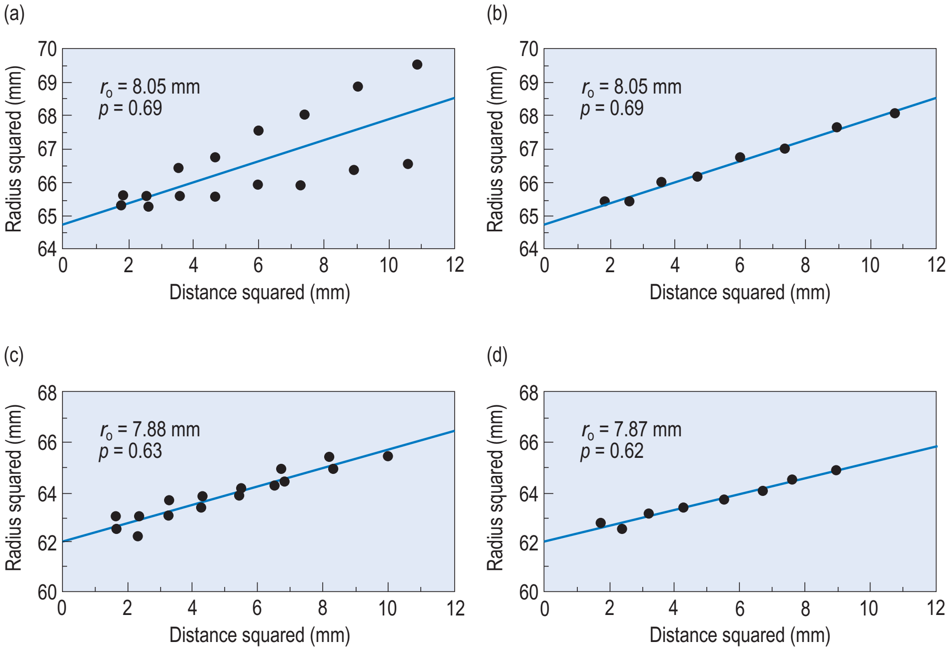

Figure 5.17 VK scatter plots from the measurement of a human cornea. (a) The near horizontal principal meridian. (b) The near horizontal principal meridian after semi-meridian averaging. (c) The near vertical principal meridian. (d) The near vertical principal meridian after semi-meridian averaging. Each scatter plot indicates the apical radius and p-value derived.

true for the two sagittal radii. When the surface is tilted, the sagittal radius decreases in one semi-meridian and increases in the other (see **Figure 5.15**). If a spherical surface is tilted the distances and radii remain unchanged. Thus a given difference in the temporal and sagittal radii could be due to a slightly aspheric surface tilted through a large angle or a very aspheric surface tilted through a small angle. Tilt deduction is therefore dependant on the difference in the pairs of sagittal radii and the asphericity (p-value) of the surface.

The approximate tilt for ring five of the EyeSys VK was determined using

$$\text{tilt (degrees)} = \left(\frac{r_a}{r_b} - 1\right)/[0.0066(1-p)] \tag{5.10}$$

where r_a and r_b are the sagittal radius values from the tabular display for the two semi-meridians of ring number five. The p-value of this meridian is p.

The derivation of equation (5.10) is discussed fully by Douthwaite (2002). The numerical value in the denominator of equation (5.10) is different for each ring. The tilt was derived for each ring and the tilt recorded was the average of the tilts of the eight rings.

5.13.3 *Corneal characteristics in the normal human eye*

The results of the investigations (Douthwaite *et al.*, 1999; Douthwaite, 2003) on 98 normal subjects provided the following information about human corneal topography.

1. The average apical radius was around 7.90 mm in the near horizontal principal meridian and 7.80 mm in the near vertical principal meridian.
2. The average p-value was around 0.77 in the near horizontal principal meridian and 0.82 in the near vertical principal meridian.
3. The average corneal surface tilt was around 2 degrees (abduction) in the horizontal meridian and zero degrees in the vertical meridian.
4. The radius of curvature is likely to be the same when comparing the same principal meridian in the right eye with that of the left eye.
5. The p-value is likely to be the same when comparing the same principal meridian in the right eye with that of the left eye.
6. The radius of curvature is likely to be different when comparing the near horizontal principal meridian with the near vertical principal meridian of the same eye.
7. The p-value is likely to be different when comparing the near horizontal principal meridian with the near vertical principal meridian of the same eye.
8. A steep cornea is likely to be no more or no less aspheric than a flat cornea.
9. There was no association between corneal curvature, asphericity or tilt.
10. Within the age range investigated (20 to 59 years), age showed no association with corneal curvature, asphericity or tilt.
11. Males are likely to have flatter corneas than females but both asphericity and tilt are likely to be the same in the two genders.

5.14 Corneal topography by keratometry

Wilms and Rabbetts (1977) proposed using a keratometer to assess corneal topography. Douthwaite and Evardson (2000) used the Zeiss telecentric keratometer to determine the apical radius and the p-value of the corneal principal meridians. This involves moving the fixation point off the optical axis of the keratometer and making keratometric measurements at various oblique angles as illustrated in **Figure 5.12**. Part of the keratometer mire was used as a fixation target. **Figure 5.18** shows the hollow cross extremity of the Zeiss telecentric keratometer. Each square block was used as a fixation point. The values below the cross indicate the angular subtense of the centre of each block from the telescope optical axis. The hollow cross was alternated to occupy a position in both semi-meridians with the pairs of radius measurements being averaged to give a mean radius for each block. This series of radius measurements produced a sagittal radius derived by conven-

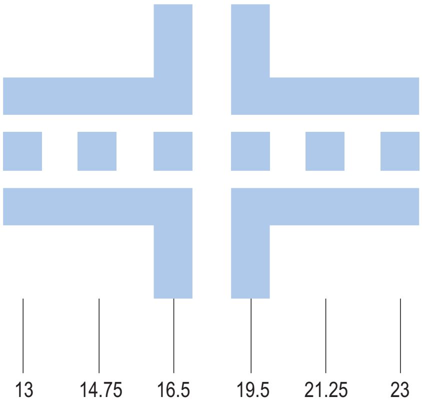

Figure 5.18 Diagram showing the hollow cross mire extremity of the Zeiss telecentric keratometer. The numbers indicate the angular subtense (degrees) of the centre of each block from the telescope optical axis

tional keratometry and six additional sagittal radii along one of the principal corneal meridians.

Douthwaite and Evardson (2000) derived equations to approximate the perpendicular distance from the keratometer axis to the corneal point being measured in each case. They then used calibrated aspheric buttons to derive a correction factor for this approximate distance. The corrected perpendicular distance and the sagittal radius for each fixation position provided the same information as the tabular data of the VK and so a scatter plot and equation (5.9) could be used as before to derive a regression line that gives the apical radius and the p-value. The results of this technique were compared to those of the EyeSys VK. The p-value derived was not significantly different in the two instruments. There was a slight but significantly different result for the apical radius. The keratometer was under-reading by 0.03 mm compared to the VK.

The advantage that the keratometer possesses over the VK is that each sagittal measurement is made with individually focused mire images. The VK must assume that all ring images are simultaneously in focus and this is extremely unlikely. The keratometer measurement however takes much longer than the VK measurement. There is a program in Chapter 11 that will calculate the apical radii and p-values of the two principal corneal meridians using the Zeiss telecentric keratometer. The measurement technique is described in Section 11.11.

5.15 The autokeratometer

This term is used to describe computerized keratometers. These instruments require an initial alignment and the pressing of a single button provides the operator with the corneal data. For purposes of illustration, the Topcon KR-3500 will be described. The fundamental working principle is the same as

any other keratometer, however the instrument makes a measurement with two different size mires. This means that two sagittal radii at two different locations (one more peripheral than the other) will be acquired. The measurement of sagittal radius at two points on the corneal surface allows calculation of the p-value and apical radius of the corneal section. Unfortunately the KR-3500 only prints the eccentricity but it does this for the temporal, nasal, superior and inferior semi-meridians.

5.15.1 The calculation of the apical radius and the p-value.

$$r_s = \sqrt{[r_o^2 + (1 - p)y^2]} \tag{5.8}$$

Therefore the p-value can be calculated from the equation

$$p = 1 - [(r_s^2 - r_o^2)/y^2] \tag{5.11}$$

There will, in fact, be two values for r_s which will be called r_{s1} and r_{s2} and these will produce two values for y which will be called y_1 and y_2.

Thus for the keratometer

$$p_1 = 1 - \frac{(r_{s1}^2 - r_o^2)}{y_1^2}$$

$$p_2 = 1 - \frac{(r_{s2}^2 - r_o^2)}{y_2^2}$$

and

$$p_1 = p_2 \quad \text{for the given surface}$$

Therefore

$$\frac{(r_{s1}^2 - r_o^2)}{y_1^2} = \frac{(r_{s2}^2 - r_o^2)}{y_2^2}$$

$$y_2^2 \,.\, r_{s1}^2 - y_2^2 \,.\, r_o^2 = y_1^2 \,.\, r_{s2}^2 - y_1^2 \,.\, r_o^2$$

$$r_o^2\,(y_1^2 - y_2^2) = y_1^2 \,.\, r_{s2}^2 - y_2^2 \,.\, r_{s1}^2$$

Therefore

$$r_o^2 = \frac{(y_1^2 \,.\, r_{s2}^2 - y_2^2 \,.\, r_{s1}^2)}{(y_1^2 - y_2^2)} \tag{5.12}$$

Therefore we can calculate the apical radius (r_o) if we know the more central and more peripheral keratometer readings (r_{s1} and r_{s2}) and the values for y_1 and y_2. Once we have calculated r_o then we can substitute this into equation (5.11) in order to calculate the p-value. The p-value and the apical radius are all that we require to describe the corneal ellipse. The apical radius indicates how steep or flat is the cornea under investigation and the p-value indicates the rate of flattening or steepening of the curve from the apex to the limbus.

5.16 The Orbscan

The original Orbscan instrument was designed using *slit scan technology* to determine:

(a) the anterior corneal topography by assessment of elevation.
(b) the corneal thickness across the cornea.
(c) the posterior corneal topography by assessment of elevation.

The basic principle of slit scan technology is to use a slit beam and to observe this beam in an optical section arrangement. In these circumstances, specular reflection takes place at the anterior tear film and diffuse reflections occur at tissue surfaces (cornea, iris, lens) encouraged by the relatively rough nature of these surfaces. Because diffuse reflections are omni-directional then a standard fixed instrument set-up can be used to generate the image. It is therefore possible to use triangulation on isolated surface points as illustrated in **Figure 5.19** for the anterior cornea.

Triangulation is a principle that has been used in navigation for centuries. Suppose you wished to know the distance to the moon. One person could measure the angle of the moon in London and, at the same time, another could measure the angle in Moscow. We now have a triangle with the three points being London, Moscow and the moon. We have measured the angles of the triangle at London and Moscow. The internal angles of a triangle add up to 180°. A little mental arithmetic will give us the third internal angle of the triangle. We can also measure the distance between London and Moscow so we have a triangle where we know all three internal angles and the length of one side. We can now obviously calculate the lengths of the other two sides. So triangulation allows us to locate the position of the moon. In **Figure 5.19**, if we know the constants of the instrument like the angle between the projector and camera axes and the separation distance between the projector and camera then we can locate the isolated surface point on the anterior

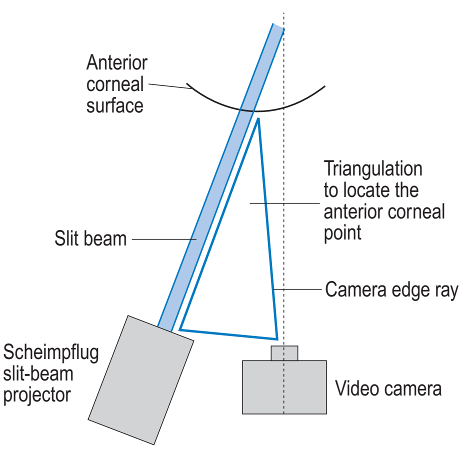

Figure 5.19 The slit scanning system. Triangulation of the camera edge ray allows for the calculation of the position of the surface point

cornea. The Scheimpflug projection system is one that produces a slit image of the anterior cornea, posterior cornea, iris and anterior lens surfaces where all these surfaces are simultaneously in focus. Obviously this is not the case in a conventional optical section. The triangulation of an edge point in the camera object space (x, y, z) is made by mathematically intersecting the diffuse reflected camera edge ray with the calibrated slit beam and this can be done for all points on the beam. The Orbscan moves across the cornea and projects 20 slit images projected from the right-hand side and 20 slit images projected from the left as illustrated in **Figure 5.20**. A total of 8000 points are processed. Each slit triangulates one slice of the ocular surface. The 40 slit images are processed in two 0.7 sec periods. Eye movement is measured during this time and the movement artefacts are subtracted from the results.

The distance between slices averages 250 μm (for a coarse scan limbus to limbus). Smoothly connected low- order polynomials (splines) are used locally to interpolate between the data slices. The instrument was improved and released to the market as Orbscan II. Orbscan II retains the triangulation principle but also incorporates VK specular reflection as a complementary method of measurement. The reason for the addition of the VK principle, using Placido rings, is that both approaches have their advantages and disadvantages and generally where the one system is weak, the other is strong. A specular reflection system is best for deriving curvature information in a typical normal cornea. A triangulation system is best for deriving elevation particularly where the surface is irregular.

In the Orbscan II:

1. triangulation faithfully follows complex surfaces.
2. directly triangulated elevation extracts the ultimate accuracy inherent in perspective reflection.
3. surface normal (specular reflection) data enhance curvature accuracy and elevation accuracy.
4. curvature is unambiguous and known in all directions thanks to triangulation.
5. double the data increases spatial resolution.

The Orbscan literature suggests that tangential (instantaneous) curvature be called meridional curvature and sagittal (axial) curvature be called axial curvature.

One other interesting feature of the Orbscan II instrument is that the software, combined with axial length measurement, can be used to predict the size and shape (point spread function) of the retinal image of a distant point source.

5.17 The pachometer

The pachometer is an instrument which, in conjunction with a slit lamp, measures the corneal thickness. This information is useful in contact lens after

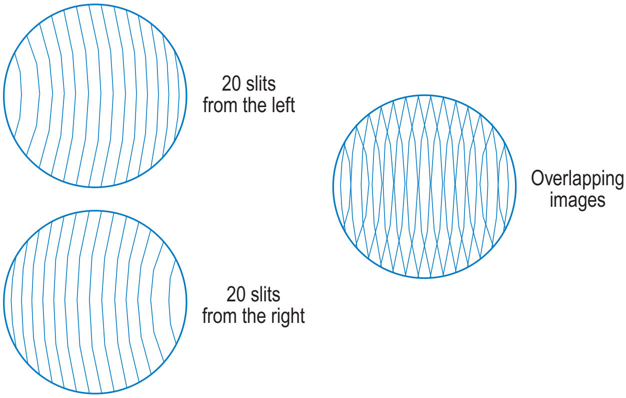

Figure 5.20 The Orbscan projects onto the cornea, 20 slits from the left and 20 slits from the right in two 0.7 sec periods. This produces the overlapping data illustrated on the right-hand cornea

care examinations because it allows the quantification of corneal oedema. If the corneal thickness was measured as 0.50 mm in the initial patient examination but was found to be 0.55 mm at the first after care examination then the thickness has increased by 10%. We can deduce that the contact lens has induced stromal oedema. When the cornea swells, it can only expand in an anterior–posterior direction. Thus the thickness increase and volumetric increase are proportional. A 10% increase in corneal thickness is thus assumed to indicate a 10% increase in corneal oedema. The cornea will swell around 3% overnight and this oedema will decrease during the waking hours.

Oedema around 5 to 6% would not be acceptable in a daily wear contact lens patient. It might be just about acceptable for an extended wear patient if observed shortly after wakening providing the oedema decreased during waking hours. You would expect to observe signs of striae beginning to appear with this level of oedema. Stromal folds develop with oedema around 10 to 12% and this is obviously not acceptable. Corneal oedema in excess of 15% must be considered to be pathological oedema (Efron, 1999).

To use a pachometer, the slit lamp is set up to produce an optical section. The working principles are illustrated in **Figure 5.21**, where the eye under examination fixates the slit beam which therefore passes through the central cornea. The observer views the optical section by using the biomicroscope set at the specified angle θ. The observed thickness of the cornea in these circumstances is BE. From **Figure 5.21** we can deduce in triangle BQ′E

$$\angle BQ'E = \theta$$

Apparent thickness $$BQ' = \frac{BE}{\sin\theta} \tag{5.13}$$

For the observation system, Q is a point object and Q′ is the point image in the cornea of refractive index n and anterior surface radius r. We can calculate the position of the object (Q) if we know the image position (Q′). Distances BQ, BQ′ and r are all measured in the same direction, which is negative (against the direction of the incident light). We can eliminate the negative signs by reversing the directions of the light rays, so that we consider a light ray incident on the anterior corneal surface directed to Q′ which is refracted to the point Q on the posterior corneal surface. Thus the object distance is BQ′ and the image distance is BQ.

$$L' = L + F$$

substituting the distances in **Figure 5.21**

$$\frac{n}{BQ} = \frac{1}{BQ'} + \frac{(n-1)}{r}$$

$$\frac{n}{BQ} = \frac{r + BQ'(n-1)}{r.BQ'}$$

$$\frac{BQ}{n} = \frac{r.BQ'}{r + BQ'(n-1)}$$

$$BQ = \frac{n.r.BQ'}{r + BQ'(n-1)} \qquad (5.14)$$

Thus the real thickness can be calculated by taking the refractive index and the anterior surface radius of the cornea into account. The instrument scale is calibrated to give a good approximation of real thickness, with tables supplied to compensate for k-reading differences when a more accurate result is required.

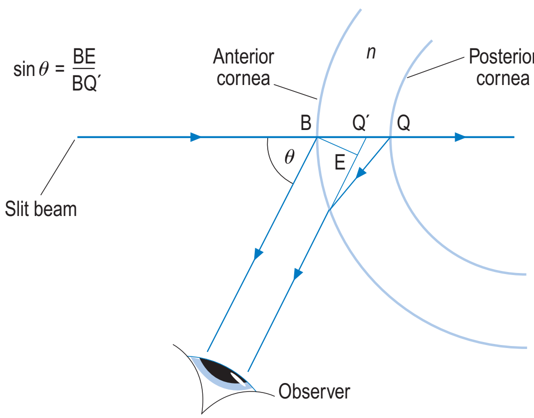

Figure 5.21 The optical section observed during a pachometric measurement. The biomicroscope has been omitted for simplicity

As with keratometry we must devise some means of measuring the size of BE and once again a doubling system is incorporated into the instrument. This consists of a thick glass plate placed into the observation system. Half of the aperture is covered by a fixed plate, the other half by a plate which can be tilted as shown in **Figure 5.22**.

This provides an ability to displace half the image of the optical section as shown in **Figure 5.23**. The fixed plate is included in the system so that the length of the optical path in the two halves of the system does not differ significantly. The end point is reached when the posterior corneal surface of one half of the aperture coincides with the anterior corneal surface of the other.

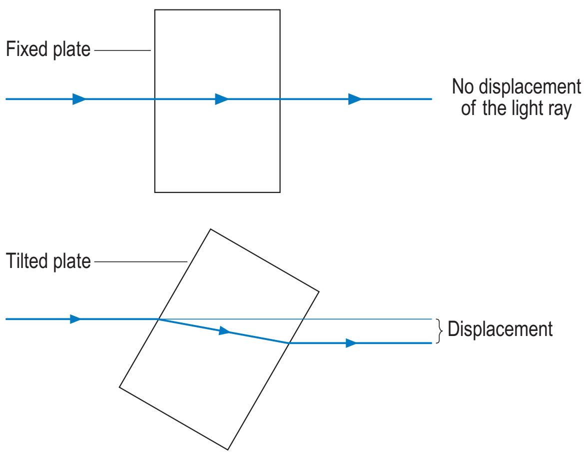

Figure 5.22 Light ray displacement by tilting a thick glass plate

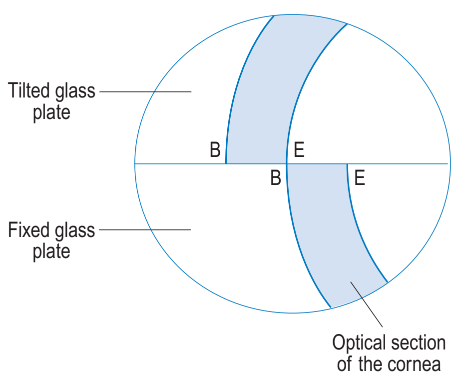

Figure 5.23 The optical section appearance when the pachometer is set to measure the corneal thickness. The image displacement is equal to the distance BE

5.17.1 Precautions

1. The illuminating beam must be normal to the corneal surface. This is best confirmed by asking the patient to look at the light source and checking that the reflected light returns to the slit aperture of the pachometer. This requires a small screen mounted just above the slit aperture.
2. Vertical alignment is best achieved by bisecting the first Purkinje image with the dividing line between the upper and lower glass plates.

   Points 1. and 2. will help to ensure that the corneal thickness measurement is always made at the same point on the cornea. This is important when repeat measurements are made for monitoring changes in corneal oedema.
3. As with any optical section the slit beam should be as narrow as possible, with the light source as bright as possible to help compensate for the light loss.

With some instruments the observation is always made to the right of the slit beam. This means an observation on the nasal side of the corneal apex for a right eye and the temporal side for a left eye. This has been suggested as the cause for the significant differences observed between the right and left eyes in some investigations.

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CHAPTER 6

Contact lens design

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Although soft lenses are popular because of their comfort and lack of adaptation difficulties, they have a relatively short life, produce less stable vision, suffer from surface deposition, are more difficult to replicate and check, involve the patient in higher running costs for lens replacement, cleaning and disinfection and are more likely to induce long term complications. Spherical soft lenses transmit all the ocular astigmatism and it is therefore likely that the practitioner will have to resort to toric soft lens fitting on a regular basis. There is most definitely, therefore, still a place for rigid corneal lens designs and many practitioners consider the rigid gas permeable (RGP) lens as the lens of choice when fitting young patients who intend to wear lenses all day every day for the foreseeable future. This chapter is concerned with the back surface design of these corneal lenses in order to arrive at the optimum fitting relationship between the lens and the cornea.

6.1 Basic requirements

The basic requirements are simple and straightforward.

6.1.1 The central region

The lens must be a near alignment fit in the central region which is considered to be the region inside the BOZD. Some practitioners fit on the steep side of alignment and claim a high success rate, others fit on the flat side of alignment and yet others claim that the only way to fit is to align the lens as precisely as possible with the cornea. It therefore appears that a fit somewhere near alignment is going to be acceptable. If the fit is obviously steep then the fluid trapped in the resulting central pool will stagnate and the corneal metabolism will be compromised to the point that the lens will be unwearable. This steep fit also results in a limited contact area between the lens and cornea in the region of the first transition (which marks the BOZD) and this may result in arcuate staining. If the lens fit is very flat, then the contact zone between the lens and the cornea will be a small area at the corneal apex. This will result in considerable pressure exerted on this region by the contact lens because

$$\text{pressure} = \frac{\text{force}}{\text{area}}$$

This will result in local trauma to the anterior layers of the central cornea. The lens will be uncomfortable and unstable on the eye. It will move around excessively and will be easily displaced off the cornea or may even fall out of the eye.

6.1.2 The lens periphery

Corneal lenses are manufactured with flattening peripheral zones which are there to assure that there is an edge clearance from the cornea. This edge clearance will provide:

- for the removal of the lens by the eyelids.
- a reservoir of tears that will encourage tear flow beneath the lens when the eye moves and also during blinking. This allows maintenance of corneal function even when using low oxygen transmission lenses.
- the tears meniscus which will generate adequate capillary attraction in order to maintain the lens in a stable and reasonably well-centred position on the cornea.
- a lens edge that does not dig into the corneal epithelium near the limbus when the lens moves.

It must be obvious from the above list that deciding on an appropriate peripheral design and specification is going to have a major influence on the

success or failure of the fitting. However, in current contact lens practice, optometrists are accepting the standard lens designs provided by the contact lens laboratories. The question arises as to who is in the best position to decide on the peripheral specification of the contact lens. It may be fine for an inexperienced contact lens fitter to rely on proprietary lens designs in order to build some confidence but once this stage is passed the practitioner should accept that s/he is going to be in complete control of the fitting exercise and will dictate the lens design and specification to the laboratory rather than have the laboratory dictate the design to the optometrist. This chapter is concerned with indicating how easy this is and to suggest how designs and full specifications can be devised.

6.2 The edge clearance

6.2.1 Results of peripheral edge clearance being excessive

If a lens produces too much edge clearance then some of the following problems may arise:

- The lens will be uncomfortable and may be displaced onto the sclera during wear.
- The lens may fall out of the eye.
- Bubbles may form under the lens edge and then break down into froth.
- The lens may not centre well.
- The excessive edge clearance may induce 3 and 9 o'clock staining.

6.2.2 Results of insufficient peripheral edge clearance

An inadequate clearance may induce any of the following:

- The lens may be difficult to remove.
- The lens edge may dig into the corneal epithelium which will cause epithelial trauma along the flat corneal meridian in the limbal region.
- The lens will be uncomfortable in wear.
- There will be a poor tears exchange under the lens and this may result in inadequate oxygenation of the central cornea, leading to lens discomfort and oedema.

6.3 The history of back surface design

The first corneal PMMA lens design was a monocurve back surface design by Tuohy in 1948 but its success was limited. The first successful corneal lens design in the UK was the Bier Contour lens which was a bicurve design with the back peripheral radius (BPR) around 0.5 mm flatter than the BOZR throughout the fitting set. The BOZD was around 6.50 mm with the total

diameter (TD) around 9.50 mm. The design was subsequently altered to the Modified Contour Lens which was a tricurve design. This was followed by other tricurve, tetracurve and pentacurve designs. These multicurve designs were also made with the peripheral curves flatter than the BOZR by a fixed amount. Stone (1975) noted that this results in a reduced axial edge lift (AEL) in the flatter lenses in the fitting set. She, and others, proposed a number of alternative designs that were described as constant axial edge lift (CAEL) lenses. A typical BOZD was now around 7.50 mm and a typical TD around 9.00 mm. She believed that CAEL lenses represented an improvement based on her extensive experience as a contact lens practitioner. She considered that the fixed difference designs caused problems with flatter corneas having too little axial edge clearance (AEC) from the cornea and steeper corneas having excessive AEC. It is possible to confuse AEL on the *lens* with AEC from the *cornea*, however there was no confusion here because she states that it is important to remember that the edge lift is greater than the edge clearance due to corneal flattening. The point that must be emphasized is that a CAEL design does *not* produce a constant axial edge clearance from the cornea. It is regrettable that, subsequently, other authors have not made this distinction.

The introduction of gas permeable materials provided an obvious step in the right direction. PMMA lenses were troubled by the absence of oxygen permeability in the material. This meant that large diameter lenses often produced central corneal hypoxia problems. It was sometimes not possible to use a small BOZD without troublesome visual instability, arising from a large diameter pupil. On the other hand, a large BOZD would often produce central corneal oedema due to stagnation of the tears trapped behind the lens. The advent of the RGP contact lens has been accompanied by the, not totally desirable, trend towards larger BOZDs, TDs and reduced AEC.

The manufacturers by this time were offering aspheric and offset lenses which have the distinct advantage of being transition free, due to the central and peripheral curves running into each other in a tangential manner. The manufacturers of aspheric designs claimed that this was the ideal back surface because the ellipsoidal cornea could be matched exactly with an ellipsoidal back surface contact lens. However, if the lens really did achieve a perfect matching fit then there would be no edge clearance resulting in high capillary attraction forces, which would make the lens unwearable and so the manufacturers recommended that the lenses were fitted around 0.1 mm flatter than the keratometer reading and some were forced to incorporate a peripheral, flat, spherical curve to produce a reasonable AEC. This re-introduced a transition on the lens back surface Also the matching fit can only be achieved if the cornea under investigation has the same characteristics as the one which was taken by the manufacturer as their model of a typical cornea.

In the past there was no convenient clinical equipment for measuring the corneal topography and the practitioner was limited to keratometric measurements of the radius of curvature of the cornea, which assess an

annular area approximately 1.5 mm from the corneal apex. This, of course, gives no indication of the rate of flattening from the apex to the limbus. Help was on the way in the form of the foundation work of Townsley (1970) who suggested that the fit of a corneal lens could be expressed in terms of the apical clearance, which became known as the tear layer thickness (TLT) and the axial edge clearance (AEC). When these two values are optimum, Townsley believed that the resulting lens fit will be a good one. This approach, however, most definitely requires measurement of the corneal asphericity so that we know both the curvature and the rate of flattening from the corneal apex to the limbus. Townsley's foundation work led to the development and production of the Wesley Jessen System 2000 PEK which was the first commercially available clinical instrument that could measure and record the eccentricity of the corneal ellipse. This instrument was used to determine the characteristics of the average human cornea by a number of workers (Bibby, 1976; Guillon *et al.*, 1983, 1986). These investigations appear to suggest that the cornea is ellipsoidal in form with a p-value around 0.8 in an average eye. It may come as a surprise that Helmholtz, in his book *Physiological Optics*, described the cornea as an ellipsoid. Helmholtz examined the corneas of three subjects and his results can be used to calculate the p-value, which comes out as 0.56, 0.76 and 0.7 for the three individuals. More recent investigations have revealed that the p-value can vary considerably in the normal population.

There are now an increasing number of clinical instruments that are capable of measuring the corneal curvature and asphericity. This means that it is now possible to use Townsley's approach to the problem of matching the curves of the cornea and contact lens, for any of the back surface designs currently available. The introduction of the videokeratoscope (VK) has made the acquisition of corneal topography data easy and instrument software is capable of calculating the optimum contact lens back surface design for each cornea examined.

6.4 Tear lens thickness and edge clearance

6.4.1 Optimum values for the TLT and AEC

If we are to use Townsley's approach to the problem of matching the curves of the contact lens with those of the cornea, we need to know what the optimum values are for the TLT and the AEC. He suggested that the TLT should be 0.025 mm. According to Guillon *et al.* (1983) the TLT should be between 0.015 and 0.025 mm. Tomlinson and Bibby (1977) concluded that the optimum value is 0.015 mm.

The optimum AEC from the cornea was considered, by Townsley, to be around 0.08 mm. Tomlinson and Bibby (1977) suggested that 0.08 mm is an appropriate radial edge clearance (REC) and the Wesley Jessen PEK was used to allow the production of a contact lens which gives a TLT of 0.015 mm and

an AEC of 0.05 mm. Guillon *et al.* (1983) proposed a value of 0.08 mm as an AEC. Atkinson (1984) considered the optimum AEC to be 0.08 mm for gas permeable lenses and stated that an REC of 0.08 mm is probably a little excessive.

Thus the consensus figure for the optimum TLT appears to be around 0.02 mm with that for the AEC along the flat corneal meridian at 0.08 mm.

If a central alignment fit is required then the TLT will be zero. An AEC of 0.08 mm approximates to an REC of 0.065 mm at a diameter of 9.00 mm on an average cornea.

6.4.2 *Fitting sets*

Any fitting set designed to incorporate the concept of an optimum TLT and AEC must be based on a particular corneal form. This inevitably means that the atypical cornea, when fitted with this type of lens, will produce values for the TLT and AEC which are not optimum. It is, therefore, unlikely that this approach can be used to design standard lenses for fitting sets. Guillon *et al.* (1983) used the results of the Wesley Jessen PEK to devise an aspheric back surface fitting set that was claimed to give optimal fitting characteristics in 95% of the prospective contact lens wearing population. The problem with this solution is that the fitting set incorporated three lens TDs with three BOZDs. This may add to the optometrists workload.

A fitting set may be better used to verify the value of the BOZR determined by the TLT calculation and to assess all the other factors that must be considered when fitting contact lenses. Factors like lens diameters, lens movement, lens position, patient reaction, the effect of the lids and the palpebral aperture shape will all need consideration and this is best achieved by using lenses from a fitting set. We are currently happy to use fitting set lenses which have the wrong BVP. We perform an over-refraction to determine the correct BVP and then order this. Why not extend this philosophy to include the peripheral curves of the lens and determine their radii by quantifying the corneal topography. The subsequent use of suitable equations allows the calculation of the optimum central and peripheral curves. It is likely that the peripheral curves of fitting set lenses will be reasonably near to the optimum values ultimately ordered. The ordered values for the peripheral curves would be those calculated from the characteristics of the corneal topography.

6.5 Calculation of the TLT and AEC

The corneal principal meridians can be considered to approximate to conic sections (Douthwaite and Pardhan, 1998; Douthwaite *et al.*, 1999; Douthwaite, 2002, 2003).

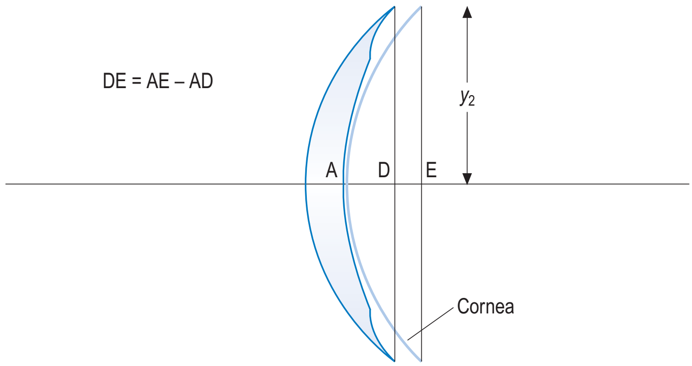

Figure 6.1 A flat fitting bicurve contact lens. DE is the axial edge clearance (AEC). AD is the overall sag of the contact lens, and AE is the sag of the cornea over semi-diameter y_2

The relationship between the cornea and the contact lens is illustrated in **Figures 6.1** and **6.2** which both show that we must concern ourselves with the sags of the anterior corneal surface and the sags of the contact lens.

In **Figure 6.1** we see a flat fitting bicurve RGP lens centred on the cornea. In this case and in the case of an alignment fit, the AEC is the sag of the cornea minus the overall sag of the contact lens for semi-diameter y_2. The TLT will of course be zero.

In **Figure 6.2** we see an apical clearance fit. The TLT is the contact lens sag minus the corneal sag for semi-diameter y_1 (half the BOZD). The AEC is the TLT plus the corneal sag minus the contact lens overall sag for semi-diameter y_2 (half the lens TD).

The sags in these diagrams can be calculated by using for the cornea (conic section)

$$x = \frac{r_o - \sqrt{(r_o^2 - py^2)}}{p} \tag{4.9}$$

and for the contact lens (circular section)

$$s = s - \sqrt{(r^2 - y^2)} \tag{3.1}$$

In **Figure 6.2** the TLT can be deduced from the relationship

$$\text{TLT} = \text{AB} = \text{AC} - \text{BC} \tag{6.1}$$

where AC is the sag of the contact lens and BC is the sag of the cornea over the semi-chord y_1.

The AEC is calculated from

$$\text{AEC} = \text{DE} = \text{AB} + \text{BE} - \text{AD} \tag{6.2}$$

where AB is the TLT, AD is the overall sag of the contact lens ($s_1 + s_2 - s_3$) and BE is the sag of the cornea over semi-chord y_2.

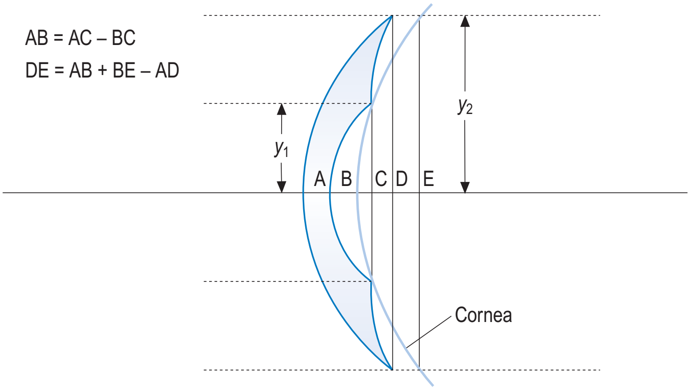

Figure 6.2 An apical clearance fitting bicurve contact lens. AB is the central tear layer thickness (TLT). DE is the axial edge clearance (AEC). AC and BC are the respective sags of the contact lens and cornea over semi-diameter y_1. AD is the overall sag of the contact lens and BE is the sag of the cornea over semi-diameter y_2

Although **Figure 6.2** illustrates a bicurve contact lens, the above treatment can be applied to multicurve, offset or aspheric lenses provided that the overall sag of the contact lens over semi-chord y_2 and the point of contact between the lens and the cornea (which determines y_1) are known. This latter point will always be the first transition in a typical multicurve fitted on the steep side of alignment, thus $2y_1$ = BOZD. In a flat fit, the point of contact will be the corneal apex for any lens design and the contact diameter will be zero, the TLT will be zero and only the AEC will need to be calculated using equation (6.2). The point of contact in offset and aspheric lenses fitted on the steep side of alignment is more difficult to determine. A computer program is presented in Chapter 11 which calculates the lens back surface specification and contact diameter for any requested TLT and AEC. The program will design multicurve, offset and aspheric lenses. The use of this program means that the tedium of lengthy calculation is avoided and the possibility of personal and individual design of the contact lens back surface is an easy and convenient possibility.

The program assumes that the lens position is concentric with the corneal apex. This is no different from conventional fitting because, when assessing the relationship between the curves of the contact lens and those of the cornea using fluorescein, it is essential that the lens is manipulated to a central position. Then, *and only then*, can the optometrist decide whether the lens is steep, flat or aligned. If, for example, a lens which displays an alignment fit is allowed to decentre on the cornea then this will encourage the appearance of a central pool of fluorescein with the possibility of the practitioner interpreting this as a steep fit when in fact the misleading appearance is due to the lens decentration.

The other assumption is that the lens will be in contact with the cornea at one point. This, as already mentioned, will be the corneal/lens apex in a flat

fit and will be the first transition in a steeply fitting multicurve. Obviously the real lens will bed into the cornea. The manufacturer may well have blended the lens transitions and this will affect the TLT and AEC in the real lens. It is, therefore, unlikely that the actual TLT and AEC will be exactly the same as the values calculated by the computer. However, any differences are likely to be small and the precision of this approach is more than adequate for contact lens fitting purposes. According to our experience of fitting with this approach, the precision is better than that achieved by the average practitioner using a fluorescein assessment. It must, never the less, be pointed out that the programs which give the TLT and AEC to three decimal places of a millimetre, i.e. to the nearest micron, give the impression of a level of precision which is optimistic.

This treatment assumes that the lens/cornea system is axially symmetrical and the fitting of the curves of the contact lens and cornea are considered in terms of their cross-section. Any corneal asymmetry (between the nasal and temporal sides for example) must be averaged out to allow a match between the cornea and the lens because contact lenses are not manufactured in an asymmetrical form.

In the case of the toric cornea, the section of the flat principal meridian is considered for the provision of assessment of the TLT and edge clearance. The AEC for the steep meridian is calculated using equation (6.2) and using the TLT calculated for the flat meridian. This aspect is covered in more detail in Chapter 7.

The final point that must be made is that the AEC is assumed to be an axial clearance at the edge of the lens. Clearance can be measured at any specified diameter, not only the total diameter. However, in normal usage, unless qualified otherwise, total diameter clearance is assumed. It must be noted that the effect of edge shaping and polishing on the estimate of edge clearance is disregarded.

Let us take an example.

A cornea has an apical radius of 7.80 mm with a p-value of 0.7. This cornea is fitted with an RGP lens having the following specification:

C3 7.80:7.00/8.30:8.00/8.80:9.00

Calculate the TLT and the AEC at diameter 9.00 mm. You may ignore the effect of edge taper.

The cornea has a p-value of 0.8. This means that the corneal radius of curvature will increase from the apex to the limbus. Thus this lens will fit with apical clearance. **Figure 6.2** illustrates the fitting relationship.

Calculate the TLT

TLT = AB = AC – BC

AC is the sag of the central curve over the BOZD which we have called s_1. BC is the sag of the cornea over the same diameter. Thus

$$AC = 7.8 - \sqrt{(7.8^2 - 3.5^2)} = 0.8293 \text{ mm}$$

$$BC = \frac{7.8 - \sqrt{(7.8^2 - 0.7 \times 3.5^2)}}{0.7}$$

$$= 0.8151 \text{ mm}$$

$$TLT = AB = 0.8293 - 0.8151 = 0.0142 \text{ mm}$$

Calculate the AEC

$$AEC = DE = AB + BE - AD$$

$$AB = 0.0142 \text{ mm}$$

$$BE = \frac{7.8 - \sqrt{(7.8^2 - 0.7 \times 4.5^2)}}{0.7}$$

$$= 1.3840 \text{ mm}$$

AD is the overall sag of the contact lens. In the case of a tricurve lens

$$\text{Overall sag} = s_1 + s_2 - s_3 + s_4 - s_5$$

$$s_1 = 7.80 - \sqrt{(7.80^2 - 3.5^2)} = 0.8293 \text{ mm}$$

$$s_2 = 8.30 - \sqrt{(8.30^2 - 4^2)} = 1.0274 \text{ mm}$$

$$s_3 = 8.30 - \sqrt{(8.30^2 - 3.5^2)} = 0.7740 \text{ mm}$$

$$s_4 = 8.80 - \sqrt{(8.80^2 - 4.5^2)} = 1.2376 \text{ mm}$$

$$s_5 = 8.80 - \sqrt{(8.80^2 - 4^2)} = 0.9616 \text{ mm}$$

$$AD = s_1 + s_2 - s_3 + s_4 - s_5 = 1.3587 \text{ mm}$$

$$AEC = DE = AB + BE - AD$$

$$= 0.0142 + 1.3840 - 1.3587 = 0.0395 \text{ mm}$$

Therefore

$$TLT = 0.014 \text{ mm}$$

$$AEC = 0.040 \text{ mm at diameter } 9.00 \text{ mm}$$

The above example illustrates how to calculate the TLT and AEC for a given back surface design. A more likely requirement is the need to be able to calculate a back surface design that gives a specified TLT and AEC. The practitioner must decide on appropriate lens diameters. The radii of curvature of the curves can then be calculated.

For illustration purposes we will consider a tricurve back surface design where the mid peripheral curve is responsible for 67% of the AEL and the final peripheral curve is responsible for 33% of the AEL.

A cornea has an apical radius of 7.70 mm and a p-value of 0.8 along the flat principal meridian. This cornea is to be fitted with a tricurve RGP contact lens with the following diameters

$$BOZD = 7.80 \text{ mm}, \quad BPD_1 = 8.60 \text{ mm}$$

and

$$\text{diameter for edge clearance} = 9.00 \text{ mm}$$

The first peripheral curve must produce 67% of the total AEL at diameter 9.00 mm.

The second peripheral curve must produce 33% of the total AEL at diameter 9.00 mm.

The fitting relationship must give a TLT of 0.02 mm and an AEC of 0.08 mm at diameter 9.00 mm. What radii of curvature are required for this lens?

BOZR

From **Figure 6.2** we can see that

AC = AB + BC

Thus the sag of the central curve of the contact lens is equal to the sag of the cornea plus the TLT.

$$\text{The sag of the central cornea} = \frac{7.7 - \sqrt{(7.7^2 - 0.8 \times 3.9^2)}}{0.8}$$

$$= 1.0443 \text{ mm}$$

$$\text{Sag of BOZR over BOZD } s_1 = 1.0443 + 0.02 = 1.0643 \text{ mm}$$

From **Figure 3.1** we can derive the relationship

$$r = \frac{y^2 + s^2}{2s} \tag{6.3}$$

Thus, for the BOZR

$$r_o = \frac{3.9^2 + 1.0643^2}{2 \times 1.0643}$$

$$\text{BOZR} = 7.6777 = 7.68 \text{ mm}$$

BPRs

From **Figure 6.2** the contact lens overall sag

AD = AB + BE – DE

where

AB = TLT
BE = corneal sag for diameter 9.00 mm

and

DE = AEC for diameter 9.00 mm

$$\text{The corneal sag} = \frac{7.7 - \sqrt{(7.7^2 - 0.8 \times 4.5^2)}}{0.8}$$

$$= 1.4196 \text{ mm}$$

Therefore

$$\text{Contact lens overall sag} = 0.02 + 1.4196 - 0.08$$

$$= 1.3596 \text{ mm}$$

From Figures 3.3 and 3.4

AEL of the contact lens = sag of central curve – overall sag

$$\text{sag of central curve over diameter 9.00 mm} = 7.68 - \sqrt{(7.68^2 - 4.5^2)} = 1.4565$$

$$\text{AEL} = 1.4565 - 1.3596 = 0.0969$$
$$\text{AEL} = 0.097 \text{ mm}$$

We now have a problem that requests the contact lens peripheral radii given the lens diameters and the lens AEL. This can be solved as illustrated in Section 3.2.3. The only extra complication is that we must share the AEL as requested.

BPR_1 will produce an AEL of

$0.097 \times 0.67 = 0.065$ mm at diameter 9.00 mm

BPR_2 will produce an AEL of

$0.097 \times 0.33 = 0.032$ mm at diameter 9.00 mm

For BPR_1 the lens must be considered to be

$7.68{:}7.80/BPR_1{:}9.00$ AEL = 0.065 mm

For BPR_2 the lens must be considered to be

$BPR_1{:}8.60/BPR_2{:}9.00$ AEL = 0.032 mm

BPR1

From Figure 3.7

$$s_1 = 7.68 - \sqrt{(7.68^2 - 3.9^2)} = 1.0639 \text{ mm}$$
$$s_0 = 7.68 - \sqrt{(7.68^2 - 4.5^2)} = 1.4565 \text{ mm}$$
$$x = s_0 - z = 1.4565 - 0.065 = 1.3915 \text{ mm}$$

In triangle BDE

$$\text{BE} = 1.3915 - 1.0639 = 0.3276 \text{ mm}$$
$$\text{DE} = 4.5 - 3.9 = 0.6 \text{ mm}$$
$$\angle\text{BDE} = \theta$$
$$\tan\theta = \frac{\text{BE}}{\text{DE}} = \frac{0.3276}{0.6} = 0.546$$
$$\theta = 28.6345°$$

In triangle BDE, if M bisects BD then P bisects ED. Therefore

$$\text{DP} = \text{PE} = \frac{0.6}{2} = 0.3 \text{ mm}$$

$$\text{MC}_1 = \frac{3.9 + 0.3}{\sin 28.6345} = 8.7642 \text{ mm}$$

$$\cos\,\theta = \frac{DP}{DM}$$

$$DM = \frac{0.3}{\cos 28.6345} = 0.3418 \text{ mm}$$

$$DM = BM$$

In triangle BC_1M

$$BC_1^{\,2} = BM^2 + MC_1^{\,2} = 0.3418^2 + 8.7642^2$$
$$BC_1 = 8.7709 \text{ mm} = BPR_1$$

BPR2

For BPR2 the lens is

$8.77{:}8.60/BPR_2{:}9.00$ AEL = 0.032 mm at diameter 9.00 mm

From Figure 3.7

$$s_1 = 8.77 - \sqrt{(8.77^2 - 4.3^2)} = 1.1265 \text{ mm}$$
$$s_0 = 8.77 - \sqrt{(8.77^2 - 4.5^2)} = 1.2425 \text{ mm}$$
$$x = s_0 - z = 1.2425 - 0.032 = 1.2105 \text{ mm}$$

In triangle BDE

$$BE = 1.2105 - 1.1265 = 0.084 \text{ mm}$$
$$DE = 4.5 - 4.3 = 0.2 \text{ mm}$$
$$\angle BDE = \theta$$
$$\tan\,\theta = \frac{BE}{DE} = \frac{0.084}{0.2} = 0.42$$
$$\theta = 22.7824°$$

In triangle BDE, if M bisects BD then P bisects ED. Therefore

$$DP = PE = \frac{0.2}{2} = 0.1 \text{ mm}$$

$$MC_1 = \frac{4.3 + 0.1}{\sin 22.7824} = 11.3627 \text{ mm}$$

$$\cos\,\theta = \frac{DP}{DM}$$

$$DM = \frac{0.1}{\cos 22.7824} = 0.1085 \text{ mm}$$

$$DM = BM$$

In triangle BC_1M

$$BC_1^{\,2} = BM^2 + MC_1^{\,2} = 0.1085^2 + 11.3627^2$$
$$BC_1 = 11.3632 \text{ mm} = BPR_2$$

The back surface specification required to give a TLT of 0.02 mm and AEC of 0.08 mm on this cornea is

C3 7.68:7.80/8.77:8.60/11.36:9.00

This lens gives an AEL of 0.097 mm at diameter 9.00 mm.

6.6 Radial edge clearance

Up to now, we have considered the edge clearance in an axial direction because the fluorescein picture observations are made axially and it must be admitted that the calculation of radial edge clearance (REC) is more difficult. It may be more appropriate to consider the REC because this represents a minimum distance between the lens and the cornea. The REC may, therefore, have more bearing on the comfort and the tear exchange of any lens under consideration. The conversion from axial to radial or radial to axial edge clearance is not straightforward. Burek and Douthwaite (1993) discussed the problem and presented a solution to the conversion which required an iterative approach. They included computer programs which perform the conversion axial to radial and radial to axial. They also described an approximate conversion and derived the following equations:

Axial to radial conversion

$$b = a\left\{\frac{r_o^2 - py^2}{r_o^2 - (p-1)\,y^2}\right\}^{0.5} \tag{6.4}$$

where a is the axial edge clearance, b is the radial edge clearance at semi-diameter y, r_o is the apical radius of the cornea and p is the corneal p-value.

Radial to axial conversion

$$a = b\left\{\frac{r_o^2 - (p-1)\,y^2}{r_o^2 - py^2}\right\}^{0.5} \tag{6.5}$$

The problem with any radial measurement is that we must define the surface to which we refer. Is the direction radial to the front or back surface of the contact lens or is it radial in relation to the corneal surface? Burek and Douthwaite (1993) decided that the measurement must be made normal to the anterior corneal surface. It is then independent of the back surface design of the contact lens.

The second problem is to decide where the diameter is to be measured. In the case of an axial measurement the diameter does not change but when the measurement is made on a radial line the diameter will increase as we move from the cornea to the contact lens. Burek and Douthwaite (1993) decided that the diameter should be measured at the back surface of the contact lens. This ensures that for all radial measurements (edge lift, edge thickness, edge clearance) the diameter for the edge measurement is made on the back surface of the contact lens. All of this is illustrated in **Figure 6.3**.

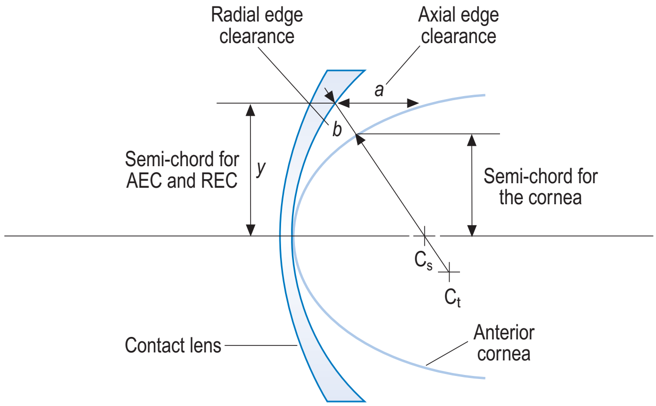

Figure 6.3 The relationship between the AEC and REC. C_t is the tangential centre of curvature of the cornea. C_s is the sagittal centre of curvature

A program that calculates AEC can be converted to REC by entering an axial to radial program as a subroutine. This is incorporated into the programs that calculate REC in Chapter 11.

6.7 The TLT and AEC of previous and present proprietary designs

Atkinson (1987) wrote an excellent paper dealing with the TLT and AEC values using a computer program written by Tony Hough. This paper illustrated the TLT, AEC and tear layer profile produced by a number of popular corneal lens designs when matched to an average cornea. A similar approach to assessing past designs is adopted in the following sections of this chapter. It is interesting to note that Atkinson concluded that a modern aspheric design (the Hydron GP 20/50) produced a tear layer profile very similar to that of the Bier Contour lens which was the first successful PMMA lens in the UK.

The results described below were derived using the programs described in Chapter 11. There are two programs that involve the TLT and the AEC. The most useful is the one is called *Design*. This asks for the TLT and AEC required along with the contact lens diameters. The program calculates the back surface radii required for any given cornea. It will calculate the back surface radii for monocurve to pentacurve, offset bicurve, conicoidal and polynomial back surface designs. The second program is called *TLT AEC*. This requires the contact lens back surface radii and diameters to be entered in order to calculate the TLT and the AEC on a given cornea. This also deals with a similar range of designs and is useful when examining the effects of changes in the back surface specification. Both programs calculate the lens AEL and the REC.

The designs below are compared by assuming, unless otherwise stated, that the cornea being fitted has an apical radius of 7.70 mm (exact schematic eye) along the flat principal meridian and a p-value of 0.8. This cornea would give a keratometry reading of 7.75 mm (to the nearest 0.05 mm) using a Bausch and Lomb keratometer.

The recommended **Bier Contour lens** for this cornea would probably have been

C2 7.75:6.50/8.25:9.50 AEL = 0.074 mm

This gives

TLT = 0.002 mm AEC = 0.048 mm REC = 0.039 mm

This represents a near alignment fit in the optic zone with an inadequate edge clearance.

The **Modified Contour lens** would be

C3 7.75:6.50/8.25:9.10/12.25:9.50 AEL = 0.129

This gives

TLT = 0.002 mm AEC = 0.102 mm REC = 0.082 mm

The addition of the third curve has produced a generous edge clearance.

A **Stone CAEL lens** is recommended to be fitted around 0.1 mm steeper than the k-reading.

C4 7.60:7.00/8.40:7.80/9.70:8.60/10.80:9.00 AEL = 0.149 mm

This gives

TLT = 0.023 mm AEC = 0.116 mm REC = 0.096 mm

This design is giving a near optimum TLT but the AEC appears excessive on this average cornea.

The typical ellipsoidal cornea will display a corneal edge lift (due to peripheral flattening) from the circular arc that describes the corneal apical radius. The investigation of a range of surface radii reveals that, if we assume that the cornea is ellipsoidal with a constant p-value, then flatter corneas display less corneal axial edge lift. The investigations into corneal topography (Douthwaite *et al.*, 1999; Douthwaite, 2003) support the notion that the average p-value of around 0.8 applies across the curvature range from steep to flat corneas. If we wish to maintain the AEL in a contact lens fitting set then the peripheral curves of flatter lenses must possess a greater degree of flattening than that which occurs in the steeper lenses. Stone (1975) and Rabbetts (1976) recommended the concept of constant axial edge lift for corneal lens fitting sets. However, they point out that edge lift refers to a measurement from the central spherical curve of the contact lens and that this is therefore not a clearance from the cornea. It may seem more rational to recommend a constant edge clearance from the cornea but this requires an

accurate knowledge of the individual corneal topography. Accepting the generalization that flatter corneas are likely to display less corneal axial edge lift than steep corneas, perhaps the recommendation of constant edge lift fitting sets requires some reappraisal. The ideal fitting set should, perhaps, contain lenses that also show a decreasing edge lift in the flatter radii. It must be noted that this occurs in fitting sets where the peripheral radii are flatter than the BOZR by a fixed amount. Some contact lens laboratories market designs that they call constant edge clearance designs. Although this is a step in the right direction, these lenses will only produce a constant edge clearance for the corneal model that has been assumed for the calculation. Any individual cornea that does not conform to the model will induce an edge clearance different to the expected value. The only way to guarantee a requested TLT and AEC is to design the contact lens for each individual cornea.

In order to illustrate the relationship between AEL and AEC, we can consider three corneas with apical radii of 7, 7.7 and 8.4 mm. Experimental evidence suggests (Sheridan and Douthwaite, 1989; Douthwaite *et al.*, 1999; Douthwaite, 2003) that there is no correlation between apical radius and the asphericity of the corneal ellipse and so we will assume that the p-value for all three corneas is 0.8. We can then compare a CAEL design with the more traditional design where the peripheral curves flatten by a constant amount when compared with the BOZR.

Table 6.1 lists three tricurve corneal lenses of CAEL design (the AEL is 0.12 mm to two decimal places). The AEL, AEC and REC are given for these lenses. It can be seen that the edge clearances increase in the first three lenses as lenses are examined on flatter corneas, due to the decrease in the corneal

Table 6.1 A comparison of the AEL, AEC and REC in the two basic multicurve designs. Lenses 1 to 3 represent a CAEL design. Lenses 4 to 6 represent the older design where there is a fixed difference between the BOZR and the peripheral radii. All values are expressed in mm. All lenses are matched to a cornea (p-value 0.8) with an apical radius equal to the BOZR of the contact lens. This results in an apical clearance fit in the central back optic zone, i.e. all the lenses are resting on the first transition

The contact lens specification	AEL	AEC	REC
Constant axial edge lift design			
1. 7.00:7.00/7.85:7.80/9.00:8.60	0.120	0.097	0.078
2. 7.70:7.00/8.80:7.80/10.65:8.60	0.119	0.103	0.087
3. 8.40:7.00/9.90:7.80/12.75:8.60	0.123	0.112	0.097
Fixed radius differences			
4. 7.00:7.00/7.85:7.80/9.00:8.60	0.120	0.097	0.078
5. 7.70:7.00/8.55:7.80/9.70:8.60	0.092	0.077	0.064
6. 8.40:7.00/9.25:7.80/10.40:8.60	0.074	0.063	0.054

edge lift. The fourth lens in the table is identical to the first lens. The fifth and sixth lenses are designs where the peripheral curves are flattened by a fixed amount. So, for lenses numbered from four to six the first peripheral curve is 0.85 mm flatter than the BOZR and the second peripheral curve is 2 mm flatter than the BOZR. This results in a decreasing AEL and edge clearance as the cornea flattens.

The REC is the most useful dimension in the table, because it represents the minimum distance between the contact lens back surface and the cornea. This dimension is measured along a line normal to the corneal surface. **Table 6.1** illustrates that the REC variation is marginally less in the CAEL design. In moving from the steep to the flat lens, the REC *increases* by 0.019 mm in the CAEL design and *decreases* by 0.024 mm in the constant radius difference design. The two approaches to lens design produce differences in REC variation and it is not possible to demonstrate the superiority of one approach over the other.

If we take the lenses in **Table 6.1** but consider a cornea with a p-value of 0.4, the REC variation shows an increase of 0.033 mm in the CAEL design with the constant radius difference design showing a decrease of 0.011 mm. Thus the constant radius difference design is showing some superiority here in that its REC is less sensitive to changes in the radius of curvature and a fitting set of this design would encourage smaller variations in REC where the corneal p-value is below average.

Returning to our average cornea (apical radius 7.70 mm, p-value 0.8) and considering a current tricurve design fitted with a BOZR equal to the keratometry reading of the flat corneal meridian, the specification will be something like

C3 7.75:7.80/8.85:8.60/11.55:9.00 AEL = 0.096 mm

This gives

TLT = 0.008 mm AEC = 0.084 mm REC = 0.069 mm

along the flat corneal principal meridian.

This more recent design is giving a TLT and AEC closer to the optimum. The values would be even nearer if the lens radii were all decreased by 0.05 mm, where the TLT would be 0.016 mm, the AEC 0.082 mm and the REC 0.068 mm.

A modern pentacurve design fitted 0.05 mm steeper than the k-reading of the flat corneal meridian might be

C5 7.70:7.50/8.20:7.90/8.70:8.30/9.20:8.60/12.25:9.00 AEL = 0.111 mm

This gives

TLT = 0.014 mm AEC = 0.092 mm REC = 0.076 mm

Again this design is producing values for the TLT and AEC that are close to the optimum values quoted earlier.

The problem is, however, that the patients we see will not all have corneas with an apical radius of 7.70 mm and a p-value of 0.8. The proprietary designs work well in a good number of cases but we have all fitted patients where the fluorescein picture of the first lens, selected on the basis of the keratometry readings, was an unexpected one. Fortunately patients with an unusual p-value are in a minority. However, a contact lens back surface design that has been derived to fit the individual cornea will cope with all patients. This is where the computerized approach to fitting lenses really comes into its own and is demonstrably superior to the use of standard proprietary designs. It is also our experience at the University of Bradford and the Hong Kong Polytechnic University Contact Lens Clinics that the computer calculation approach saves a significant amount of chair time (Lam and Douthwaite, 1994).

6.8 Alignment fitting and the equivalent sphere

When we talk about an alignment fit we are suggesting some sort of matching fit between the contact lens and the cornea. If the contact lens back surface is spherical and the cornea is ellipsoidal then it is obvious that the match cannot be perfect. A flat fit produces contact at the corneal apex and a steep fit results in the lens touching the cornea on the first transition so that the contact diameter is equal to the BOZD. If the sags of the cornea and the contact lens are equal over a chord length equal to the BOZD then we achieve three point touch. The lens and cornea are in contact at the apex and the first transition. The TLT is zero with a contact diameter equal to the BOZD and this is the best match that can be achieved. The fitting relationship is illustrated in Figure 6.4. The BOZR in these circumstances is the radius of the equivalent sphere.

In the computer calculation approach to contact lens fitting, an alignment fitting is one where we request a TLT of 0 mm without the lens being a flat fit so that the contact diameter is still equal to the BOZD.

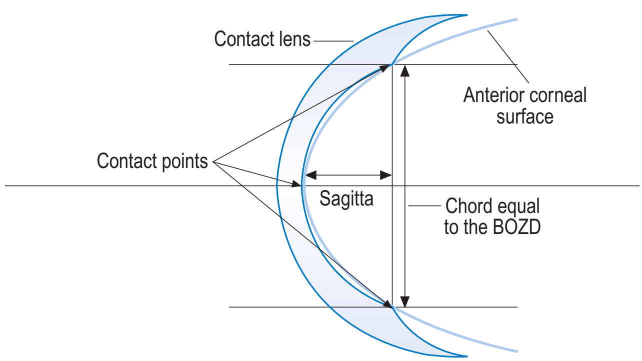

Figure 6.4 An alignment fit. The curves of the contact lens and cornea inside the BOZD produce an identical sag. This is the best match for an ellipsoidal cornea with a spherical contact lens. The lens is fitting with three point touch and the BOZR is equal to the radius of the equivalent sphere

This recommendation will result in a lens supported by the cornea over a wide area and thus we will avoid any local pressure causing local epithelial trauma. Therefore, an alignment fit is achieved by selecting the radius of the equivalent sphere as the contact lens BOZR.

6.9 The fitting of a multicurve lens

The programs *Lens Design* and *TLT AEC* in Chapter 11 can now be used to design the lens back surfaces illustrated in the rest of this chapter. It may well be worth considering reading Section 11.8 which gives instruction on the use of the program *TLT AEC* before reading what follows in this chapter.

Figure 6.5 illustrates how the tear layer profile is obtained for the computer graphic display. The profile can be seen to be an axial thickness profile which should correspond better to fluorescein pictures, because these are observed in an axial direction. The program also calculates the REC. The short red vertical lines below the fluid lens profile indicate the positions of the transitions, the lens edge and the point of contact between the lens and the cornea.

The computer program *Lens Design* is used as follows.

Let us suppose that we intend to order a tricurve RGP lens.

- Left double click the mouse on the *C3* option on the first screen and the tricurve option will then be displayed.

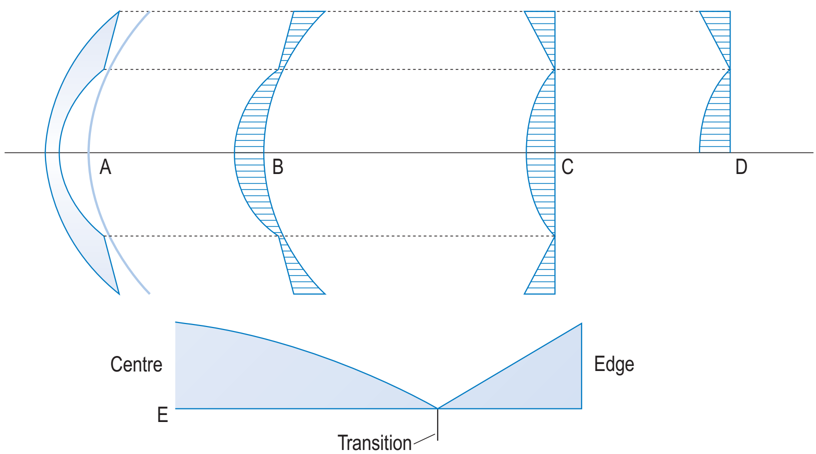

Figure 6.5 Tear layer profile diagrams. A, lens/cornea apposition. B, tear film between the lens and the cornea. C, the same thickness of the tear film referred to a straight line. D, assuming axial symmetry, only one half of the tear film need be considered. E, for convenience the diagram is displayed horizontally with the vertical display (thickness) magnified 350× more than the horizontal

- Left click the mouse on each of the windows and type the following in each window:

Corneal Apical Radius Flat Meridian	type **7.7**
Corneal p-value Flat Meridian	type **.8**
Corneal Apical Radius Steep Meridian	type **7.5**
Corneal p-value Steep Meridian	type **.9**
TLT remains at default value	**0.02**
AEC remains at default value	**0.08**
BOZD	type **7.8**
BPD1	type **8.6**
Diameter for AEC	type **9**

  Contribution of each peripheral curve
  remain at the default values **0.67** (first peripheral curve)
  and **0.33** (second peripheral curve)
- Left click the mouse on the *Calculate flat meridian* command button.

The screen will display the fluid lens profile for the flat meridian from the lens centre to the lens periphery. In this example the following values are displayed on screen:

Flat Meridian

TLT = 0.02 mm *AEC = 0.08 mm* *REC = 0.066 mm*
CD = 7.8 mm (contact diameter)

The lens back surface specification is also stated

C3 7.68:7.80/8.77:8.60/11.36:9.00 *AEL = 0.097 mm* *REL = 0.079 mm*

- Left click the *Calculate steep meridian* command button.

The screen will now display the fluid lens profile for both meridians and the following values are displayed on screen for the steep meridian.

Steep Meridian

TLT = 0.02 mm *AEC = 0.142 mm* *REC = 0.114 mm*

Note that the short red lines mark the position of the transitions and the lens edge. The display is illustrated in **Figure 6.6**.

Note the default setting for the *Contribution of each peripheral curve*. This is 67% for the first peripheral curve and 33% for the second peripheral curve. These can be changed if you wish as can the values for TLT and AEC.

If the radii were rounded to the nearest 0.05 mm the lens specification becomes

C3 7.70:7.80/8.75:8.60/11.35 :9.00

The program *TLT AEC* indicates that this gives

AEL = 0.095 mm
Flat meridian *TLT = 0.016 mm, AEC = 0.079 mm, REC = 0.065 mm*
Steep meridian *TLT = 0.016 mm, AEC = 0.141 mm, REC = 0.114 mm*

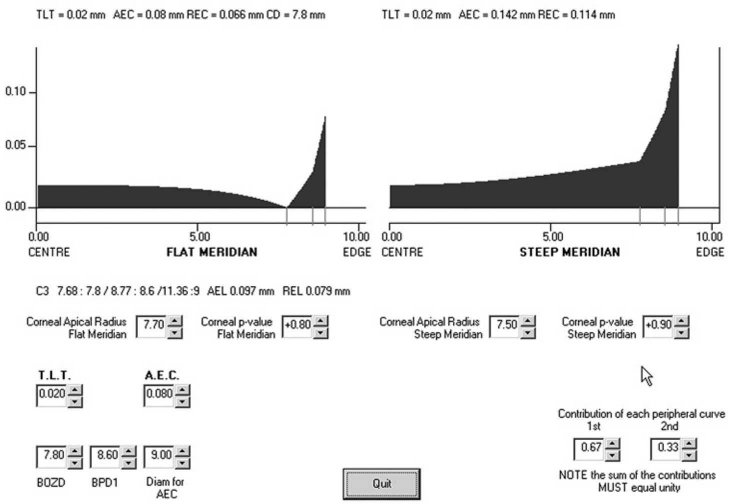

Figure 6.6 Computer display of the fluid lens profile for a tricurve contact lens giving an apical clearance fit

The TLT value is still acceptable and there is no significant change in the edge clearance.

Figure 6.7 illustrates the fit of a pentacurve RGP lens on a cornea with an apical radius of 7.70 mm and a p-value of 0.8 on the flat meridian and an apical radius of 7.50 mm and p-value of 0.9 on the steep meridian. **Figure 6.8** illustrates the fit of the pentacurve lens on a more aspheric cornea with apical radii of 7.70 and 7.50 mm but with a p-value of 0.4 for both meridians. The curves for both corneas were computed to give a BOZR equal to the equivalent sphere in order to achieve an alignment fit with a TLT of zero. The peripheral curves were calculated to give an AEC of 0.08 mm along the flat meridian. Note that the stepped appearance of the edge lift is less pronounced than that of the tricurve.

Two points are of interest here.

1. If we consider the flat meridian of these two corneas and remember that we have requested an alignment fit then it will be interesting to compare the results with the keratometer readings.

   For an apical radius of 7.70 mm and a p-value of 0.8, the Bausch and Lomb keratometer will give a k-reading approximating to 7.75 mm. Note that the contact lens BOZR is 7.80 mm which is 0.05 mm flatter than the k-reading. If we had selected a BOZR equal to the k-reading of the flat meridian we would have achieved a satisfactory fit over the back optic zone. Thus the keratometer reading has provided a good guide to selection of an appropriate BOZR.

   For an apical radius of 7.70 mm and a p-value of 0.4, the Bausch and Lomb keratometer will give a k-reading approximating to 7.80 mm. Note

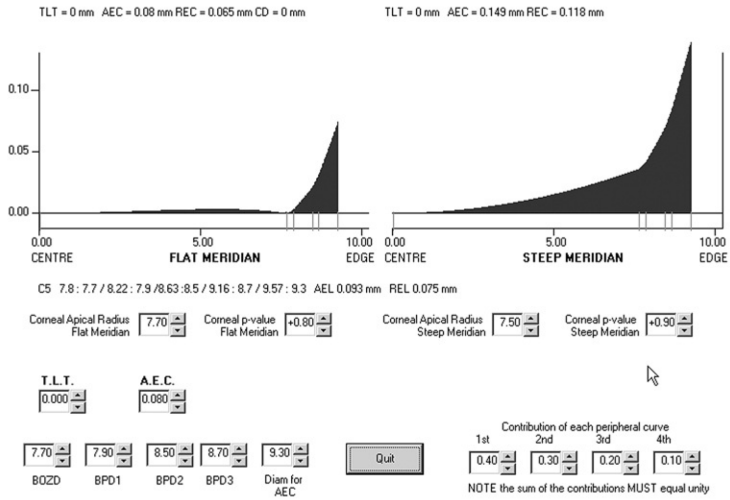

Figure 6.7 A pentacurve contact lens giving an alignment fit on a typical cornea

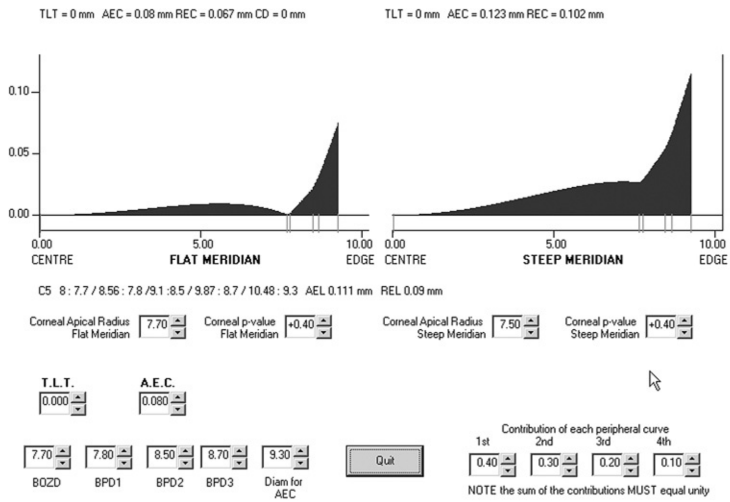

Figure 6.8 A pentacurve contact lens giving an alignment fit on an unusually aspheric cornea

that the contact lens BOZR is 8.00 mm which is 0.20 mm flatter than the k-reading. If we had selected a BOZR equal to the k-reading of the flat meridian we would have achieved a fit that was 0.20 mm steep over the back optic zone. Thus the keratometer reading has provided a poor guide to selection of an appropriate BOZR.

2. Although the TLT is zero there is some fluid collecting in the middle region of the back optic zone. Thus the theoretical fluorescein picture would reveal an annular ring of green just inside the first transition. This annular pool arises as a direct consequence of attempting to align a spherical

contact lens surface to an ellipsoidal cornea, as illustrated in **Figure 6.4**. The pool is more pronounced in the cornea with the lower p-value (more aspheric surface) and some practitioners may recall seeing this type of fluorescein picture in patients' eyes from time to time. It is likely that these individuals have aspheric corneas with a low p-value.

The contact diameter in **Figures 6.7 and 6.8** is zero. This has occurred because the BOZR was rounded to the nearest 0.01 mm and this has made the fitting very, very slightly flat.

6.10 The fit of the offset lens

Figure 6.9 illustrates the fit of a contralateral continuous offset bicurve lens on a cornea with an apical radius of 7.70 mm and a p-value of 0.8 on the flat meridian and apical radius of 7.60 mm and p-value of 0.85 on the steep meridian. A request was made for a TLT of 0.02 mm and an AEC of 0.08 mm on the flat meridian for a lens with a BOZD of 6.00 mm and a TD of 9.00 mm. The specification is

Offset *7.53:6.00/17.98:9.00*
AEL = 0.133 mm *REL = 0.107 mm*

This gives

Flat meridian *TLT = 0.022 mm* *AEC = 0.082 mm* *REC = 0.068 mm*
CD *= 6.42 mm*
Steep meridian *TLT = 0.022 mm* *AEC = 0.112 mm* *REC = 0.092 mm*

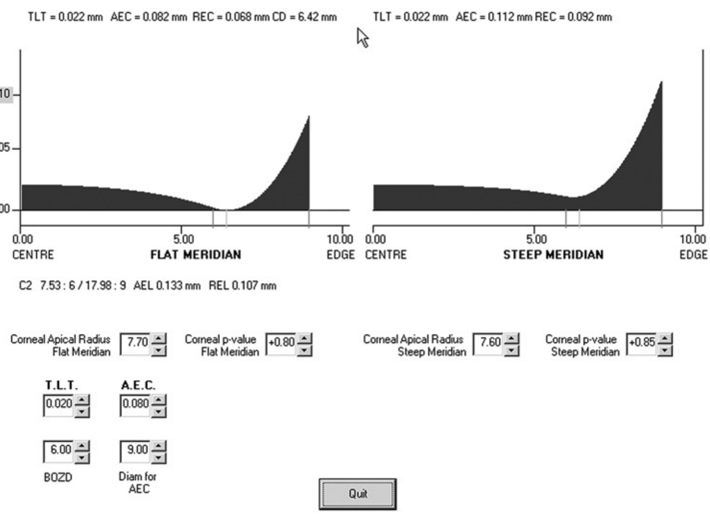

Figure 6.9 A contralateral continuous offset bicurve lens giving an apical clearance fit on a typical cornea (Plate 1)

The lens was requested to give a TLT of 0.02 mm with an AEC of 0.08 mm along the flat meridian. Note that the BOZR is considerably steeper than the keratometer reading of the flat meridian (7.75 mm) and the BPR is very large compared to a conventional bicurve. Also note that this apical clearance fit results in a contact diameter which is not equal to the BOZD. The most central short vertical line in **Figure 6.9** marks the BOZD (6.00 mm) and the short vertical green line to its right indicates the contact point (diameter 6.42 mm). The increase of edge clearance with increasing diameter is much smoother than the stepped appearance produced by the multicurve lenses. The tear profile is very similar to that produced by a conicoidal back surface contact lens. The calculated TLT and AEC for this lens are 2 μm larger than requested. This arises from the fact that the program assumed a contact diameter of 6.00 mm to calculate the lens specification. When this lens was subsequently matched to the cornea, the contact diameter was found to be 6.42 mm. This increase in diameter is responsible for the extra 2 μm present in the TLT and the AEC.

6.11 The aspheric back surface

6.11.1 The limitations of the conicoidal surface

If we start out with the aspheric design favoured by a number of lens manufacturers, namely an ellipsoidal surface with a p-value around 0.8, then we can consider some of the shortcomings of the design. The corneal asphericity varies between individuals. The asphericity of the contact lens back surface would need to be variable in order to acquire a desired fitting relationship between individual corneas and contact lenses. Manufacturers of aspheric lenses offer only a limited number of asphericities. A number of manufacturers offer only one asphericity for a given lens design. Lenses like this are designed for the cornea with a typical asphericity (p-value = 0.8) and thus are not likely to work as well where the corneal asphericity is not typical.

If a cornea with an apical radius of 7.7 mm and a p-value of 0.8 is fitted with an ellipsoidal back surface lens with a vertex radius of 7.7 mm and a p-value of 0.8, then the resulting fit is a perfect match with a TLT of zero and an AEC of zero. This is obviously an unsatisfactory fit with high capillary attraction forces, a very thin tear layer trapped between the cornea and contact lens and in consequence, a very limited tear exchange. This lens is likely to centre well but will not move.

The computer program *Design* has been used to design an aspheric back surface for a typical cornea with apical radius of 7.70 mm and a p-value of 0.7 along the flat meridian. The apical radius for the steep meridian is 7.50 mm with a p-value of 0.8. A design is requested to give a TLT of zero and an AEC of 0.08 mm along the flat meridian.

The program selects a contact lens apical radius equal to the apical radius of the flat corneal meridian. A flat fitting will be required to produce the

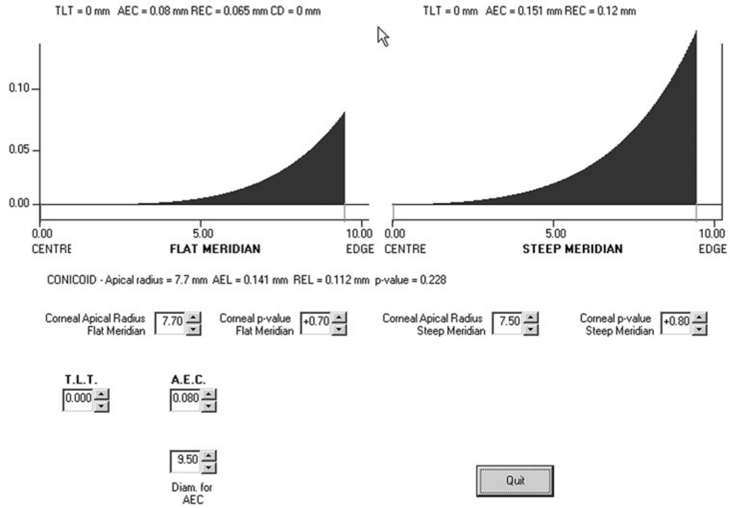

Figure 6.10 An ellipsoidal back surface contact lens on a typical cornea. The fit requested is for a TLT of zero and an AEC of 0.08 mm along the flat meridian

AEC of 0.08 mm. This is achieved by setting the p-value to 0.228, for a lens TD of 9.50 mm giving an AEL of 0.141 mm. The fluid lens profile of this lens is illustrated in **Figure 6.10**. The other possibility is to stick with the p-value of 0.8 and increase the contact lens apical radius. This will achieve a flat fit but it will be difficult to decide on the apical radius to select, from the results of the VK or keratometer. What is important to note is that the fitting relationship is determined by an interplay between the contact lens apical radius and the asphericity.

If we wish to achieve an apical clearance fit with a TLT of 0.02 mm and an AEC of 0.08 mm then the back surface specification derived by the program is

Conicoid apical radius = 7.12 mm p-value = – 0.408
AEL = 0.299 mm REL = 0.224 mm at diameter 9.50 mm

This gives

Flat meridian *TLT = 0.021 mm AEC = 0.081 mm REC = 0.065 mm*
CD = 5.58 mm
Steep meridian *TLT = 0.021 mm AEC = 0.152 mm REC = 0.12 mm*

This lens is illustrated in **Figure 6.11**. Note that the tear layer profile is very similar to that of the contralateral offset bicurve lens illustrated in **Figure 6.9**.

Figure 6.11 illustrates the fitting relationship, on a typical cornea where it can be noted that the contact diameter (CD) is 5.58 mm. The contact point is indicated by the short green vertical line. The negative p-value means that this surface is hyperboloidal and it must be increasingly obvious that if we intend to use a conicoidal surface, without extra peripheral curves, then the

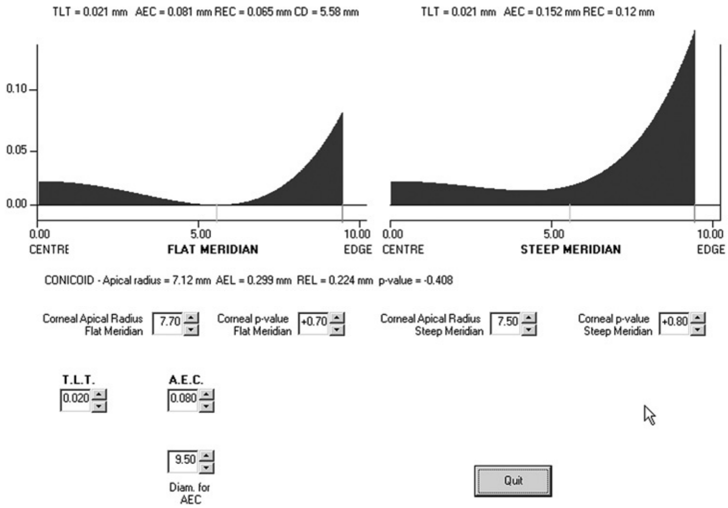

Figure 6.11 An ellipsoidal back surface contact lens on a typical cornea. The fit requested is for a TLT of 0.02 mm and an AEC of 0.08 mm along the flat meridian (Plate 2)

surface will need to flatten towards the limbus much more rapidly than the cornea. This is especially true for an apical clearance fit. There may be an optical problem with a hyperboloidal surface in the way that it will interfere with the aberrations of the contact lens. An aspheric surface will affect the amount of spherical aberration and will induce asymmetrical aberrations like coma when it decentres on the eye.

The conicoidal surface designs acquired above are subject to limitations imposed when writing the program. These are as follows:

(a) If a TLT of 0 mm is requested then the computer ascribes a contact lens back surface apical radius equal to the apical radius of the cornea. A p-value is then calculated to achieve the AEC requested.
b) If the TLT is greater than 0 mm, the computer initially assumes that the contact diameter is 5 mm in order to calculate the central sag of the contact lens. This is then substituted into the calculations that determine the contact lens apical radius and p-value. When this contact lens back surface is matched to that of the cornea the actual contact diameter is calculated.

The conicoidal back surface design described above is just one approach to the problem. There are other solutions. It is possible to achieve a particular TLT and AEC by a number of combinations of apical radius and p-value. The value of 5 mm for the contact diameter was taken because diameters larger than this will require surfaces that are even more aspheric than the one above if an AEC around 0.08 mm is required. A smaller contact diameter will result in a less aspheric surface.

In the case of the flat fit, where the TLT is zero, it seemed most obvious to work with a lens apical radius equal to the corneal apical radius. The p-value could be increased by opting for a lens apical radius that is flatter than the corneal apical radius.

6.12 The polynomial back surface

This has already been discussed in Chapter 4. The polynomial provides the lens designer with much more flexibility and it becomes possible to produce a back surface like the one derived in Chapter 4, where there is an apical clearance fit over the central region where the surface is effectively spherical, an alignment fit in the mid periphery and a flat peripheral fit that provides the required AEC.

If we take a cornea of apical radius 7.70 mm and p-value 0.8 for the flat corneal meridian, with the steep corneal meridian having an apical radius of 7.6 mm and a p-value of 0.85, and request a polynomial back surface that gives a TLT of 0.02 for a BOZD of 6 mm, a bearing zone width of 1 mm, a total diameter of 9.50 mm and an AEC of 0.08 mm along the flat meridian, then the computer program comes up with the design shown in **Figure 6.12**. The use of the term BOZD is thought to be appropriate here because the program will always produce a spherical central back surface curve. The polynomial equation coefficients for this surface are given in **Figure 6.12** as well as the TLT and AEC. It will be noted that the TLT and AEC are both 0.006 mm greater than requested. This, as explained in Section 4.7, is due to the fact that

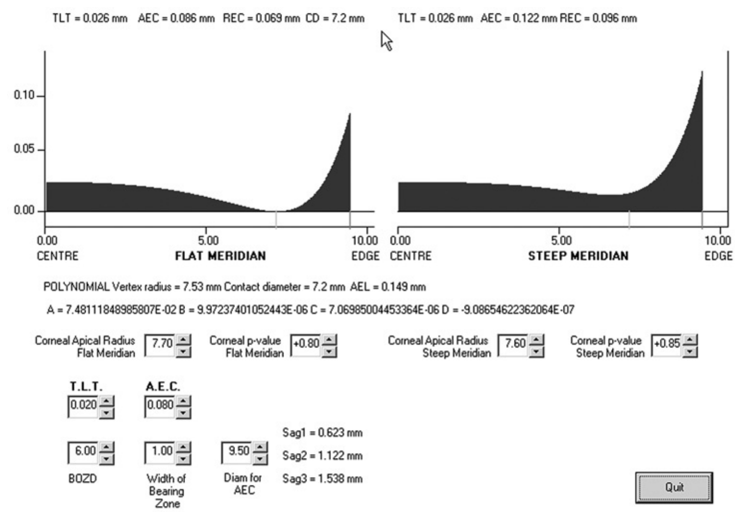

Figure 6.12 A polynomial back surface contact lens giving an apical clearance fit on a typical cornea with a 1 mm wide bearing zone (Plate 3)

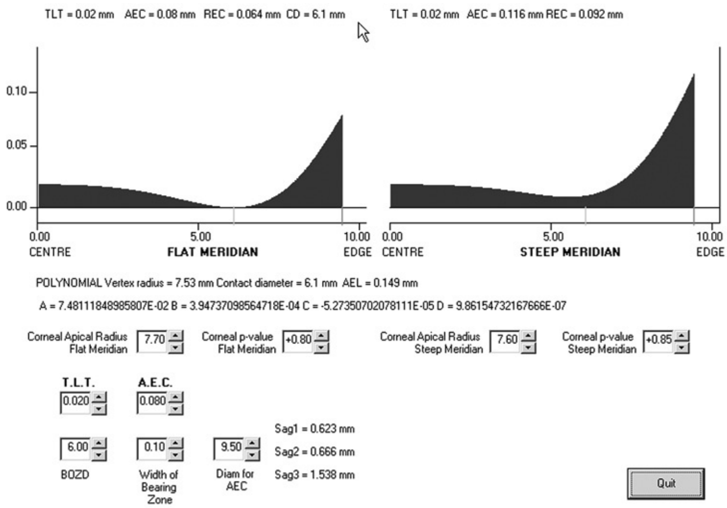

Figure 6.13 A polynomial back surface on the same cornea with the same requested TLT and AEC as in Figure 6.12. This time the bearing zone is only 0.1 mm wide (Plate 4)

the shape of the surface between diameters 6 and 8 mm does not result in a matching fit between the contact lens and the cornea. The computer has calculated the contact diameter to be 7.2 mm and the contact point is marked by the short green line. The 'bearing zone', therefore, lifts the lens off the cornea and is responsible for the increase in the TLT and AEC. This problem can be circumvented by requesting a very narrow bearing zone.

The same cornea was considered for fitting as above but with a bearing zone width of 0.1 mm. The resulting fit is as illustrated in **Figure 6.13**. The contact point in the graphical display is seen to be 6.10 mm and the TLT and AEC calculated by the computer are now exactly as requested. The contact diameter of 6.10 mm indicates that the contact point is in the middle of the requested bearing zone. The general shape of the tear layer profile is much the same in **Figures 6.12 and 6.13**.

6.13 The fitting relationship on a toric cornea

The computer displays illustrate the fluid lens profile for both the flat and the steep meridians. This chapter has described how the tear layer profile is derived for the flat meridian. The problem of calculating the TLT and AEC along the steep meridian is fully discussed in the next chapter where it will be seen that, as before, the calculations simply involve the various sagitta values.

In a typical cornea, where the corneal toricity is low, we concentrate on fitting the flat principal meridian (the near horizontal meridian in with-the-

rule astigmatism) and we also over-refract this flat meridian because we expect to order a BVP equal to the ocular refraction of the flat meridian (the sphere power of the negative sphere cyl form) in an alignment fitting. The steep principal meridian is left to look after itself. The contact lens will fit flat on this meridian and this will increase the edge clearance, which helps with tear exchange under the lens. The corneal astigmatism is neutralized by the back surface of the fluid lens, so if we have optically corrected the flat corneal meridian then the steep meridian will also be corrected where the astigmatism is corneal.

In the computerized approach to fitting, we maintain our concentration on the flat meridian and we need only be concerned with the steep corneal meridian with more pronounced degrees of corneal toricity. The programs display the tear layer profile and the edge clearance on the steep meridian in order to complete the picture. The fit on the flat meridian determines the TLT and this will also be the TLT for the steep meridian. The AEC can be calculated as described in Section 6.5 for the steep meridian.

6.14 The effect of back surface changes on the fluid lens profile

The computer programs can also be used to investigate the effects of proposed changes to the back surface specification of an RGP lens.

6.14.1 The relationship between diameter and radius

The first possible modification to the lens specification to be considered is one where the practitioner wishes to maintain the clinical fit but requires a larger or smaller contact lens. Optometrists will have come across the statement that *It has been found from clinical experience that if a given BOZR over a certain BOZD gives a satisfactory central fit, then 0.05 mm should be added to the BOZR for each 0.5 mm increase in BOZD in order to maintain the fit*. Textbooks indicate also that the same changes apply to the back peripheral radii. Intuitively, this recommendation looks reasonable because a larger lens will rest on the more peripheral (flatter) regions of the cornea and in consequence the surface radii of curvature will need to be increased a little. The computer programs will be used to check the validity of this rule of thumb that is part of the folklore of contact lens fitting.

Let us take our typical cornea with apical radii of 7.7 and 7.6 mm with p-values of 0.8 and 0.9 respectively and consider the following tricurve back surface

C3 *7.78:7.00/8.45:8.40/9.77:9.00*

which gives an alignment fit with:

TLT = 0 AEC = 0.08 mm REC = 0.066 mm along the flat meridian

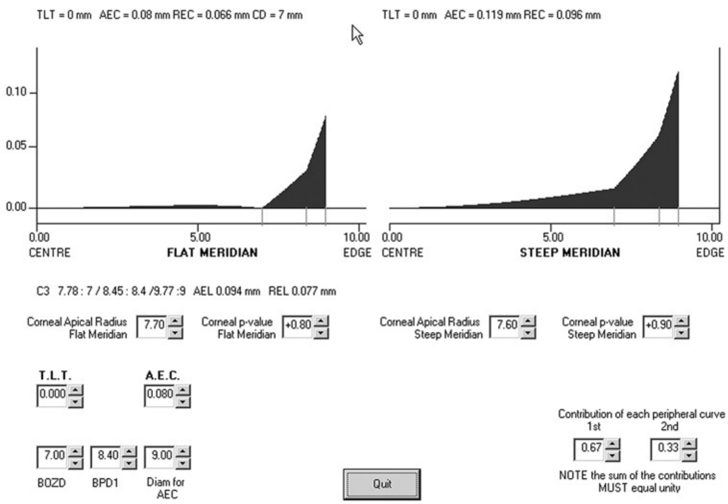

Figure 6.14 The tear layer profile of a tricurve lens giving an alignment fit with an AEC of 0.08 mm on the flat meridian of a typical cornea

See **Figure 6.14.**

If we now increase all three diameters by 0.5 mm. What radii do we require to maintain the fit? The answer is given by the *Design* program

C3 *7.80:7.50/8.40:8.90/9.55:9.50*

This gives a similar fit as shown by **Figure 6.15** with

TLT = 0 mm   AEC = 0.08 mm   REC = 0.064 mm along the flat meridian

Manufacturers offer radii in 0.05 mm steps. So we see that in our typical cornea the BOZR does not need changing when the BOZD is increased 0.5 mm. The peripheral curve radii in fact need decreasing.

It may be better to work on a single peripheral curve. Let us use the program *TLT AEC* and take the original lens and simply increase the TD by 0.5 mm and increase the BPR_2 by 0.05 mm as the rule of thumb suggests. Thus the lens becomes

C3 *7.78:7.00/8.45:8.40/9.82:9.50*

which alters the AEC from 0.08 to 0.126 mm along the flat corneal meridian. This is illustrated in **Figure 6.16**.

These examples serve to show that the rule should be discarded. A similar exercise with other designs and with steep and flat corneas supports the results above (Douthwaite, 1990). If the cornea is a typical one, then the surface radii are probably best left unaltered when the diameters are changed. However, the only way to be sure of maintaining an unaltered clinical fit is to calculate the required changes. The computer can do this quickly and conveniently.

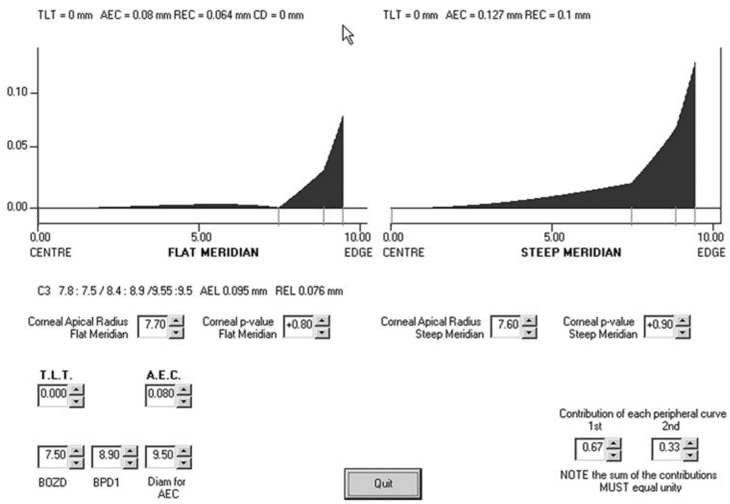

Figure 6.15 A tricurve design fitted to the same cornea with the lens diameters 0.5 mm larger than the lens in Figure 6.14. The radii have been adjusted to maintain the TLT at 0 mm and the AEC at 0.08 mm

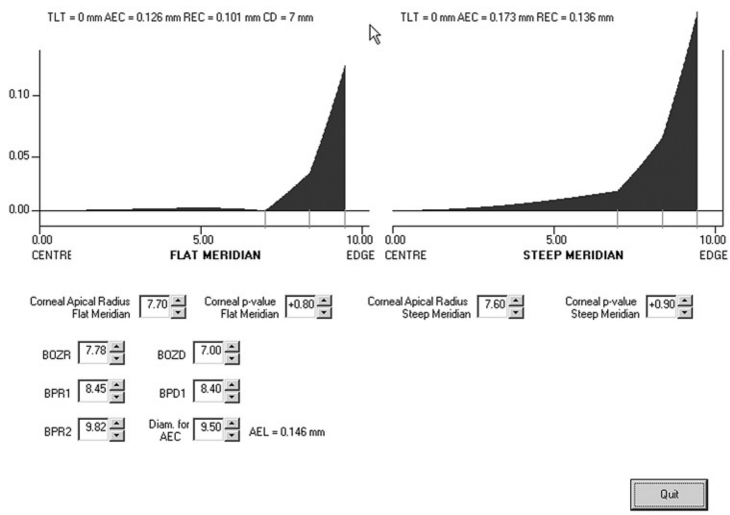

Figure 6.16 A tricurve design fitted to the same cornea with the TD increased by 0.5 mm and the BPR_2 increased by 0.05 mm compared to the lens in Figure 6.14

6.14.2 The change of the lens periphery to alter the axial edge clearance

It may be, that during an aftercare assessment, the optometrist decides that the fluorescein picture indicates that the edge clearance requires some adjustment. Let us suppose that the cornea has an apical radius of 7.70 and

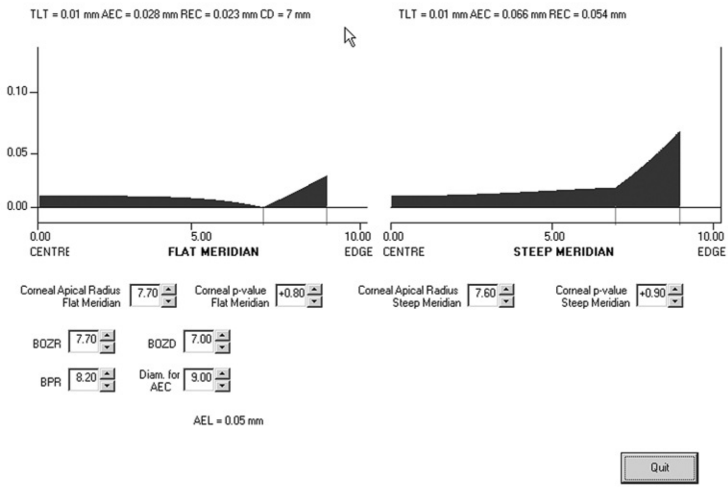

Figure 6.17 A bicurve design that produces an inadequate AEC along the flat corneal meridian

p-value of 0.8 for the flat meridian and apical radius 7.60 mm with a p-value of 0.9 for the steep meridian. The band of fluorescein around the edge of the lens is very narrow and we therefore decide to increase the edge clearance of the lens whose specification is at present

C2 7.70:7.00/8.20:9.00 AEL = 0.05 mm

This gives

Flat meridian *TLT = 0.01 mm AEC = 0.028 mm REC = 0.023 mm*
Steep meridian *TLT = 0.01 mm AEC = 0.066 mm REC = 0.054 mm*

See **Figure 6.17**.

The AEC can be increased to 0.08 mm by flattening the peripheral curve. If you were asked to guess at a new BPR what would you say? The new BPR, in the past, would be a guess based on experience. The computer program *Design* indicates that the new specification should be

C2 7.70:7.00/8.85:9.00 AEL = 0.102 mm

This gives

Flat meridian *TLT = 0.01 mm AEC = 0.08 mm REC = 0.066 mm*
Steep meridian *TLT = 0.01 mm AEC = 0.119 mm REC = 0.097 mm*

See **Figure 6.18**.

The large increase in the BPR is required because the peripheral band is only 1 mm wide. The computer program takes the guessing out of the exercise.

Figures 6.17 and 6.18 also demonstrate that:

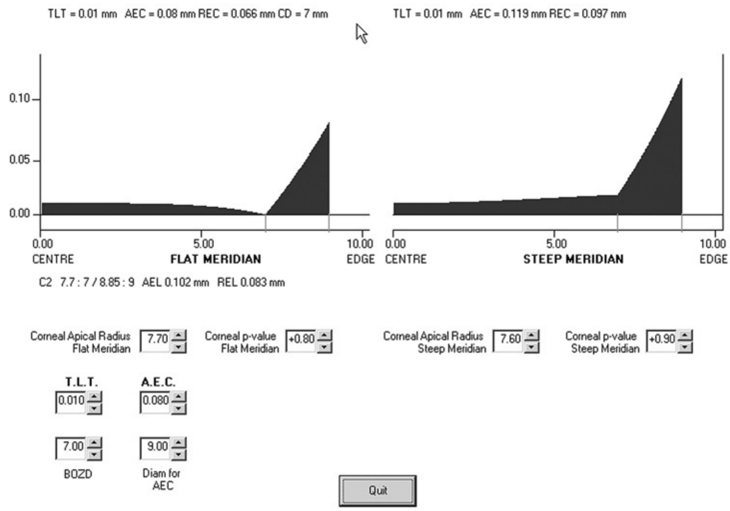

Figure 6.18 The bicurve lens in Figure 6.17 has been modified by flattening the BPR by 0.65 mm to achieve an AEC of 0.08 mm along the flat corneal meridian

- A change in AEC of 0.052 mm along the flat meridian also produces the same change in AEC along the steep meridian when the BPR is changed.
- A change in AEC of 0.052 mm will produce an identical change in the contact lens AEL when the BPR is changed.

An alternative solution to increase the AEC would be to decrease the BOZD. This widens the peripheral band and this will increase the AEC. The *TLT AEC* program indicates that with a BOZD as small as 1 mm the AEC along the flat meridian is still only 0.074 mm. This is obviously no solution for this particular lens design. If the peripheral radius had been longer to start with then a decrease in BOZD could have had the desired effect. The tear lens profile for a BOZD of 1.00 mm is shown in **Figure 6.19**.

The final possibility is to increase the TD in order to increase the AEC. The program indicates that a TD of 10 mm gives an AEC of only 0.042 mm along the flat meridian. See **Figure 6.20**.

The graphical display suggests that further increase in total diameter, even if it was desirable, will result in a decrease of the AEC due to the progressive flattening of the ellipsoidal cornea. Again a flatter peripheral curve at the outset would have helped to make this option work.

It is unlikely that we could have reached these conclusions intuitively.

6.15 Summary of the computerized fitting routine

If an autokeratometer or VK that will give the corneal apical radius and the p-value is used then the computer programs can be used to advantage. The use of computer programs will:

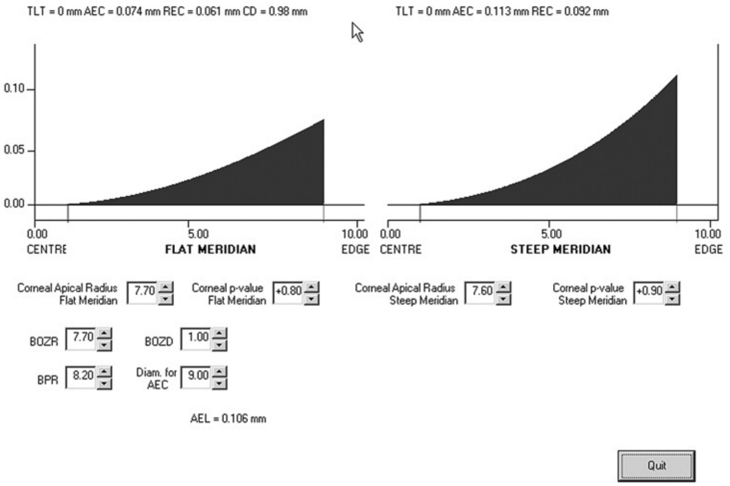

Figure 6.19 The bicurve lens in Figure 6.17 has been modified by reducing the BOZD to 1 mm in the hope of increasing the AEC to 0.08 mm along the flat corneal meridian

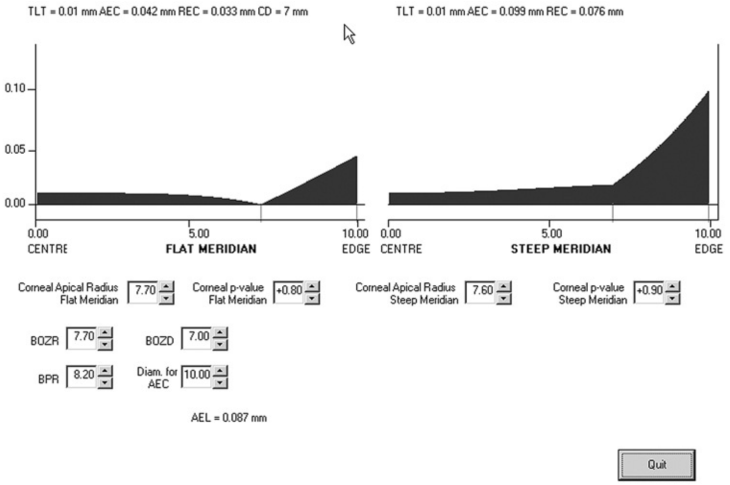

Figure 6.20 The bicurve lens in Figure 6.17 has been modified by increasing the TD to 10.00 mm in the hope of increasing the AEC to 0.08 mm along the flat corneal meridian

- speed up the fitting process
- probably improve the precision of the assessment
- provide a lens specification that is tailor made to the cornea being fitted
- allow the optometrist to dictate the specification to the laboratory rather than accepting proprietary designs from the laboratory
- take the guessing out of re-ordering lenses that currently do not fit as required.

These can all be seen as advantages.

The routine to follow is based on our experience with this approach at the University of Bradford:

(a) After the preliminary examination the optometrist will have some idea of which fitting set s/he intends to use. The readings taken with the VK enable the initial fitting to be performed on the computer. The optimum TLT and AEC are entered into the program, the lens type, transition diameters and total diameters are entered. The diameters entered will be the diameters of the fitting set lenses that the optometrist intends to use. The computer will calculate the radius of curvature of all the curves required to achieve the requested TLT and AEC. It has therefore decided what radii are required for matching the lens curves to the cornea when the lens is in a centred position. These radii should be recorded.
(b) A fitting set lens of the same BOZR as the one suggested by the computer is placed in the patient's eye to assess factors like patient reaction, lens position, lens movement, over-refraction, visual acuity etc. Fluorescein assessment will not normally be required.
(c) If all is well, the lens is ordered with the full back surface specification calculated by the computer, with radii rounded to the nearest 0.05 mm. This ensures that the peripheral curves are designed for the individual cornea under consideration.
(d) The lens BVP is deduced in the usual way from the results of the over-refraction with the fitting set contact lens.

There are computer programs in Chapter 11 that allow other features to be considered. The mean or the harmonic mean thickness of any design can be calculated and this can be useful information when considering the oxygen transmission of the soft or RGP lens options that are being considered. Also, the program that calculates the front surface curves in a lenticulated lens will allow the optometrist to instruct the laboratory on the front, as well as the back, surface specification.

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CHAPTER 7

Astigmatism and corneal toricity

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7.1 Corneal astigmatism

Taking the constants of the exact schematic eye as a basis for illustration and supposing that the anterior corneal radius is correct in the horizontal meridian (7.7 mm) but 15% shorter in the vertical meridian (6.55 mm), then the dioptric power of the anterior corneal surface will be +48.83 D along 180 and +57.40 D along 90; this gives 8.57 D of anterior surface astigmatism, with-the-rule, i.e. the vertical meridian is more positive (myopic) than the horizontal by 8.57 D. However, if the back surface of the cornea is also

assumed to be toroidal in form with the same principal meridians, the horizontal meridian radius will be 6.8 mm (exact eye) with the vertical meridian radius 15% shorter, giving a radius value of 5.78 mm. The dioptric power of the posterior corneal surface is deduced using the relationship

$$\text{power} = \frac{n_a - n_c}{r}$$

where n_a and n_c are the refractive indices of the aqueous and cornea respectively and r is the radius of curvature of the posterior corneal surface. The dioptric power of the horizontal meridian is –5.88 D and the vertical meridian is –6.92 D, giving 1.04 D of against-the-rule astigmatism, i.e. the vertical meridian is more negative than the horizontal by 1.04 D. This is illustrated in **Figure 7.1** which shows that some of the anterior corneal astigmatism is neutralized by the posterior cornea.

We can use the step along method to calculate the front vertex power (FVP) of the cornea for both meridians. The difference between the two meridians will give the power of the cylinder at the corneal front vertex. The corneal refractive index is taken as 1.376 and the corneal thickness is assumed to be 0.5 mm. **Figure 7.2** illustrates the path of the light rays.

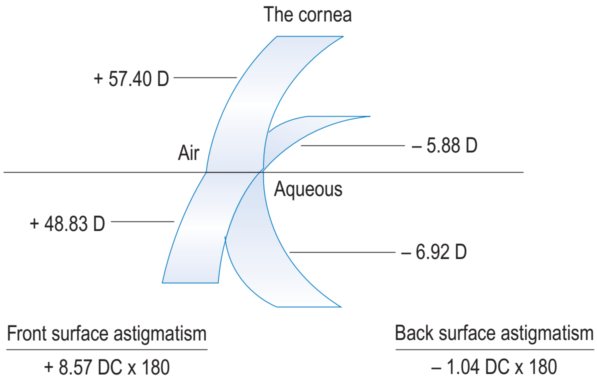

Figure 7.1 The surface powers of the cornea

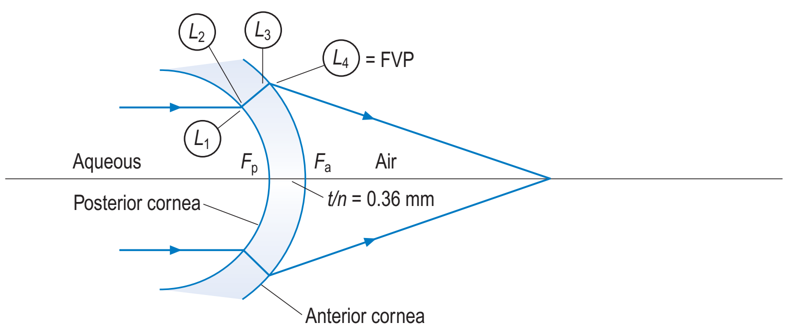

Figure 7.2 Paraxial light rays passing through the cornea. The numbers in the circles represent the incident and emergent vergences at the two surfaces

Anterior corneal power (F_a) is +48.83 D along 180 and +57.40 D along 90. Posterior corneal power (F_p) is –5.88 D along 180 and –6.92 D along 90.

Along 180

$$
\begin{array}{lll}
 & \textbf{Vergence (D)} & \textbf{Distance (mm)} \\
L_1 = & 0.00 & \\
F_p = & \underline{-5.88} & \\
L_2 = & -5.88 \longrightarrow & -170.07 \\
 & & \;\;-0.36 \quad t/n \\
L_3 = & -5.87 \longleftarrow & -170.43 \\
F_a = & \underline{+48.83} & \\
L_4 = & +42.96 = \text{FVP} &
\end{array}
$$

Repeating the above exercise for the 90 meridian, we can calculate

FVP along 90 = +50.50 D

Therefore cylinder at front vertex = 50.50 – 42.96 = 7.54 D with-the-rule. This is the total corneal astigmatism when the anterior surface astigmatism is 8.57 D. Thus the total corneal astigmatism is 88% of the anterior corneal astigmatism according to these calculations. This is in good agreement with measurements on the eyes of 60 young adult subjects by Dunne *et al.* (1991) that resulted in the conclusion that the total corneal astigmatism is 86% of the anterior surface astigmatism.

In the above example the total corneal astigmatism is 7.54 D. If a contact lens is placed on this eye then the cylindrical power of the posterior surface of the fluid lens can be calculated as follows

$$\text{Fluid lens posterior surface power} = \frac{336}{6.55} - \frac{336}{7.7}$$

$$= 51.30 - 43.64$$

$$\text{Fluid lens posterior surface cylinder power} = 7.66\ \text{D}$$

Thus the fluid lens back surface slightly over corrects (by 1.6%) the corneal astigmatism.

7.2 Alternative approach

When a spherical back surface hard or rigid gas permeable (RGP) contact lens rests on the eye, the toric cornea has some of its anterior surface astigmatism neutralized by the fluid lens. The fluid lens in these circumstances has a spherical front surface and a toric back surface and so it is obviously the back surface of the fluid lens that is responsible for the astigmatic neutralization. The amount of dioptric power, and therefore the amount of astigmatism neutralized, can be deduced from the relationship

F_c for the anterior cornea $= \dfrac{1.376 - 1}{r_c}$ (2)

F_f for the posterior surface $= \dfrac{1 - 1.336}{r_c}$ (3)
of the fluid lens

where r_c is the radius of the anterior cornea. The ratio of fluid surface power to corneal surface power F_f / F_c is

$$\frac{336}{376} = 0.89$$

Thus 89% of the anterior corneal surface astigmatism is neutralized by the back surface of the fluid lens. We have already noted that the total corneal astigmatism is 88% of the anterior corneal surface astigmatism and so we see that the back surface of the fluid lens over-corrects the total corneal astigmatism by 1%. We can therefore conclude that a spherical back surface contact lens fitted to a toric cornea will, by means of the accompanying fluid lens, slightly over-correct the corneal astigmatism. Note that this is independent of the refractive index of the contact lens.

From a clinical point of view we see that *all* the corneal astigmatism is corrected by the back surface of the fluid lens even when the astigmatism is as large as 7.50 D.

7.3 Astigmatic correction by spherical RGP contact lenses

Let us take a typical example.

An eye has an ocular refraction of

–3.00 DS/–1.00 DC × 15

The keratometry readings are

7.80 along 15 7.60 along 105

An RGP trial lens has the following specification

BOZR 7.80	TD 9.30	BVP	–2.50 DS
The over-refraction is			–0.50 DS
Therefore order			–3.00 DS

Along the flat meridian (k-reading 7.80), note the BOZR and the k-reading are both 7.80 mm which suggests an alignment fit. The contact lens BVP ordered to correct the refractive error and the ocular refraction of the flat meridian are both –3.00 DS. This also suggests an alignment fit because the fluid lens power must be plano along 15.

Along the steep meridian (k-reading 7.60), the contact lens is fitting 0.2 mm flat. This will give the fluid lens a power of approximately –1.00 D along

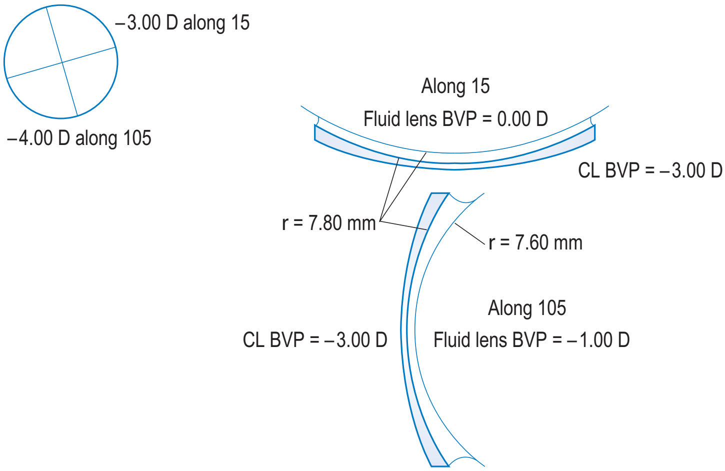

Figure 7.3 An illustration of how the back surface of the fluid lens corrects the corneal astigmatism

105 and plano along 15. In other words, the fluid lens has a power of –1.00 DC axis 15. This corrects the corneal astigmatism. All this is illustrated in Figure 7.3.

7.4 Soft lenses and corneal astigmatism

A soft lens (particularly a thin soft lens) will align itself to the anterior cornea and this means that its surfaces will take up the same toricity as the cornea on which the contact lens is resting. The back surface of the fluid lens still corrects the corneal astigmatism but unfortunately the front surface of the fluid lens will be toric with the same radii of curvature and this means that, for a thin fluid lens, the front surface neutralizes the refractive power of the back surface. This results in the fluid lens transmitting all the corneal astigmatism and this phenomenon will repeat for the front and back surfaces of the contact lens. Thus the corneal astigmatism will be transmitted through the fluid lens and contact lens resulting in an over-refraction cylinder identical to that of the ocular refraction. Soft lens manufacturers claim that, with thicker and lower water content soft lenses, some of the corneal astigmatism is neutralized due to the lens resisting the deformation by the cornea. This is disputed (Bernstein *et al.*, 1991) and it may be that thin soft lenses produce more variable vision than thicker lenses and, in consequence, give the impression of better correction of astigmatism with thick lenses. There seems to be general agreement that the total ocular astigmatism needs to be less than 1.00 D unless the wearer is prepared to accept some significant visual blur with a spherical soft lens.

Let us take an example and work through the fluid lens/contact lens system. We can, once again, consider that these components are separated by infinitely thin air films.

7.4.1 *Correction of corneal astigmatism by a thin soft lens*

An eye has keratometry readings

7.90 mm along 5 7.60 mm along 95

The ocular refraction is

–2.00 DS/–1.50 DC axis 5

The total corneal astigmatism can be calculated using the keratometer refractive index of 1.3375

$\frac{337.5}{7.9}$ along 5 $\frac{337.5}{7.6}$ along 95

This is

+42.72 D along 5 +44.41 D along 95

Total corneal astigmatism is 1.69 D more positive along 95

Fluid lens back surface power is

$\frac{-336}{7.9}$ along 5 $\frac{-336}{7.6}$ along 95

This is

–42.53 D along 5 –44.21 D along 95

Astigmatic correction is 1.68 D more negative along 95

However, the front surface of the thin fluid lens will have powers of

+42.53 D along 5 +44.21 D along 95

Astigmatic correction is 1.68 D more positive along 95

This neutralizes the fluid lens back surface correction and the corneal astigmatism is therefore still uncorrected.

The soft contact lens being worn has a refractive index of 1.444. The lens is afocal and can be considered as a thin lens. The lens back surface will take up the corneal curvature.

The contact lens back surface powers on the eye will be

$\frac{-444}{7.9}$ along 5 $\frac{-444}{7.6}$ along 95

This is

–56.20 D along 5 –58.42 D along 95

Astigmatic correction is 2.22 D more negative along 95

Note the contact lens back surface over-corrects the corneal astigmatism because the lens refractive index is greater than the refractive index of the cornea.

However, the front surface of this thin afocal lens will have powers of

+56.20 D along 5   +58.42 D along 95

Astigmatic correction is 2.22 D more positive along 95

Thus the contact lens back surface cylinder is neutralized by the front surface cylinder and this leaves the corneal astigmatism still uncorrected.

This example shows that all the corneal astigmatism will be transmitted through the system when a thin spherical soft contact lens is used.

7.4.2 *Correction of corneal astigmatism by a thick soft lens*

The flexibility of the soft contact lens inevitably means that optical calculations or calculations concerning the lens specification are open to question. The fluid lens formed behind a soft contact lens on the eye is likely to have a power around plano to –0.50 D. However, this cannot be reliably predicted and the changes occurring in the fluid lens power when the BOZR is changed are not predictable. If a soft lens is placed on a particular eye it will warp so that the back surface takes up the curvature of the cornea on which it rests. This lens flexure results in steeper radii for both the front and back surfaces of the contact lens and these induced changes may well affect the power of the contact lens. The change in the power of a soft contact lens due to flexure was discussed in Section 2.7.1 where equation (2.3) derived by Bennett (1976) was used to calculate the likely power changes.

$$\Delta F_v{}' = -300t\left\{\frac{1}{r_2{}'^2} - \frac{1}{r_2^2}\right\} \qquad (2.3)$$

where $\Delta F_v{}'$ is the change in BVP, t is the centre thickness, r_2 is the original back surface radius and $r_2{}'$ is the new back surface radius after bending the lens.

The equation indicates that when a soft lens is considered to be a thick lens then the power becomes more negative as the contact lens takes up a steeper radius of curvature when placed on the eye and that this power change is dependant on the radius change and the thickness of the contact lens.

7.4.2.1 The implications for corneal astigmatism

If we assume that the cornea, fluid lens and contact lens are separated by thin air films and that the lens back surface aligns perfectly with the anterior corneal surface, then the thin fluid lens will be afocal in all meridians. Along the steeper corneal meridian, the thick contact lens back surface will be steeper. The overall power will become more negative with a steepening of

the surface curves, as indicated by equation (2.3). A spherical lens fitted to a toric cornea will undergo the greatest radius change on the steeper (more myopic) corneal meridian. This will induce some negative power increase which will help to correct the corneal astigmatism. This may help to explain why thicker soft lenses appear to correct more of the corneal astigmatism than thin soft lenses.

If we take the previous example but consider that the eye is fitted with an afocal soft lens of centre thickness 0.5 mm and assume that the lens back surface aligns to the anterior corneal surface in both meridians then we can calculate the amount of uncorrected astigmatism for this very thick soft lens.

We can use equation (2.3) to calculate the cylinder. In this case r_2 will be the radius of curvature of the flat corneal meridian and r_2' will be the radius of the steep meridian. $\Delta F_v'$ will be the difference in power between the two meridians due to the extra steepening of the soft lens on the steeper corneal meridian. That is $\Delta F_v'$ will indicate the correcting cylinder

$$\Delta F_v' = -300 \times 0.5 \left\{ \frac{1}{57.76} - \frac{1}{62.41} \right\} \qquad (2.3)$$

$$\Delta F_v' = -0.19 \text{ DC}$$

The total corneal astigmatism for this eye is 1.69 DC. The calculation shows that this thick soft lens has only corrected a small amount (approaching 0.25 DC) of the ocular astigmatism.

7.5 Residual astigmatism

Residual astigmatism is best described as the astigmatism still uncorrected when a spherical contact lens is used with a spherical over-refraction.

When an eye is fitted with a spherical back surface RGP contact lens, the corneal astigmatism can be considered to be neutralized and the wearer can attain a high level of visual acuity with only a spherical power. However, in some cases the contact lens over-refraction reveals the presence of astigmatism. This astigmatism must arise almost exclusively from eye components other than the cornea and is therefore attributed to the crystalline lens. This astigmatism, still present during the contact lens over-refraction using spherical lenses, is called residual astigmatism and from the foregoing could also be called lenticular astigmatism. These terms can therefore be used synonymously.

Thus for rigid spherical corneal lenses

residual astigmatism = total ocular astigmatism – corneal astigmatism

Lenticular astigmatism, where it exists, is likely to be small and it is likely to be against-the-rule.

An astigmatic effect is created when the chief ray of an incident pencil does not coincide with the optical axis, and so a decentred contact lens may well

produce apparent residual astigmatism. It is therefore important to ensure a well-centred lens at the time of the over-refraction.

In the case of spherical soft contact lenses, we have seen that all the ocular astigmatism will be transmitted through the fluid lens/contact lens system. Thus for soft lenses the residual astigmatism is equal to the ocular astigmatism.

The practitioner may occasionally encounter a patient with a spherical refractive error but with a toric cornea. The theoretical implication here is that the corneal astigmatism is neutralized by the lenticular astigmatism. If this patient is fitted with a spherical back surface RGP lens, then the corneal astigmatism is neutralized, leaving the residual lenticular astigmatism manifest. A spherical correction worked onto this contact lens would result in a visual acuity poorer than that obtained with spectacles. A thin soft lens, on the other hand, will transmit the corneal astigmatism which will therefore maintain neutralization of the lenticular astigmatism, ensuring a good visual acuity. Thus patients with toric corneas and no spectacle lens astigmatism will theoretically attain a higher visual acuity with spherical soft lenses than with spherical RGP lenses.

7.6 Soft toric contact lenses

The general recommendation for soft lenses is that if the ocular astigmatism is over 1.00 D then the visual acuity may be unacceptably low or variable with spherical lenses. This recommendation applies to astigmatism where the principal meridians are near horizontal and near vertical. If the principal meridians are oblique then the astigmatic tolerance may have to be lowered to less than 0.50 D. If the lens is a thicker or lower water content lens then it may transmit less of the corneal astigmatism, although this point is disputed. As with RGP lenses, any lenticular astigmatism will still be present in its entirety. An astigmatic correction can be effected by using soft toric contact lenses.

N.B. Soft toric contact lenses are required to achieve correction of the eye's astigmatism. We need to resort to toric soft lenses to achieve an acceptable visual acuity. Toric RGP lenses are required to improve the fit of the lens.

7.7 Back surface soft torics

The typical conventional back surface toric fitting set consists of a series of spherical lenses, with prism ballast, a truncation or prism thinning (dynamic stabilization) to provide rotational stability. The lenses will also possess orientation markers that allow an assessment of the lens orientation. If such a trial lens is fitted to a toric cornea it will warp into a toric form and transmit the corneal astigmatism.

Let us suppose that the patient's keratometry readings are

8.30 mm along 5   7.70 mm along 95

with a spectacle correction of

–0.50 DS/–4.00 DC axis 5 at a vertex distance of 12 mm

This gives a refraction at the cornea rounded to the nearest 0.25 D of

–0.50 DS/–3.75 DC axis 5

Let us suppose that the best fitting lens from the fitting set (refractive index = 1.444) is marked with a radius of 8.60 and diameter 13.00 mm. It is assumed that the 8.60 mm radius curve will produce a near afocal fluid lens along the flatter corneal meridian which has an error of –0.50 D. The steeper meridian, being more myopic, will require extra negative power. In our example the steeper corneal meridian requires an extra – 3.75 D of power.

The power of the contact lens back surface, in its natural state is

$$\frac{1-n}{r} = \frac{-444}{8.6} = -51.63 \text{ D}$$

The 95 meridian must be 3.75 D more negative than the 5 meridian, so its power must be –55.38 D. Therefore the radius of the 95 meridian must be

$$\frac{1-n}{F} = \frac{-444}{-55.38} = 8.02 \text{ mm}$$

If we use a back surface radius of 8.02 mm along 95 then this results in a contact lens with 3.75 D more negative power along this meridian and this corrects the astigmatism.

Thus the back surface toric lens specification to order would be

$$\frac{8.60}{800} \quad 13.00 \quad \frac{\text{BVP} - 0.50}{\text{BVP} - 4.25} \quad \frac{\text{along } 5}{\text{along } 95}$$

These recommendations make the assumption that the fluid lens has the same power (afocal) for both meridians, which will only be the case if the lens back surface approximates to the corneal curvature. If we then picture the contact lens, tear lens and cornea all separated by thin air films we can see that the corneal astigmatism has been neutralized by the increased negative power of the contact lens along 95 (see **Figure 7.4**).

In **Figure 7.4**, the back surface toric lens on the eye will possess a front surface, which has warped into a toric form with the front surface toricity being less than is the case with spherical lenses. In our example the toric lens back surface vertical meridian will warp 3.75 D less than a spherical lens, which indicates that the vertical meridian of the lens front surface will be approximately 3.75 D less positive than is the case with a spherical lens and the lens therefore corrects the astigmatism. The above assumes thin lens theory which results in no power change induced by flexure and therefore

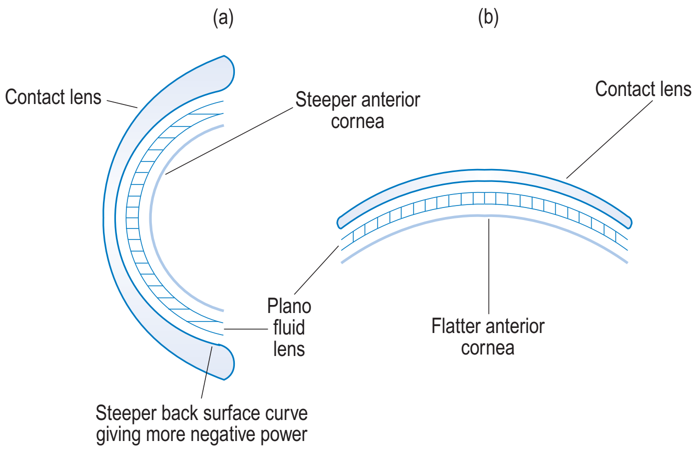

Figure 7.4 The soft contact lens/fluid lens/anterior cornea system separated by thin air films (a) along 95 and (b) along 5

maintenance of the BVP as the lens curvatures steepen on the eye. The contact lens specification above also assumes a plano fluid lens in both meridians. The precision and flexibility of the result will be improved by performing an over-refraction with the fitting set lens in place on the eye, in order to determine more accurately the powers required for the two meridians, and then proceed as above.

7.8 Front surface soft torics

The front surface toric conventional contact lens fitting set will consist again of spherical lenses with a truncation, prism or prism thinning. In the case of prism thinning, the lens is made rotationally stable by removing material from the lens in the upper and lower quadrants. These regions will normally be under the upper and lower lids making it extremely unlikely that the lens will rotate to allow the thicker regions to move under the lids. It is worth noting that the edge thickness will be greatest at 12 and 6 o'clock in a myopic eye with with-the-rule astigmatism. At the other end of the spectrum, a hyperopic eye with against-the-rule astigmatism will be corrected by a lens with the minimum edge thickness at 12 and 6 o'clock and there will, in consequence, be little or no possibility of thinning this region without first increasing the central thickness of the lens. These two extremes indicate that the thinning of a contact lens in the upper and lower quadrants may be limited by the type and degree of optical error. It is often necessary to make a toric soft lens undesirably thick in some regions in order to achieve thinning for rotational stability.

7.8.1 *Front surface toric correction of astigmatism*

If a conventional front surface soft toric fitting set lens (with spherical surfaces and a spherical BVP) is placed on a toric cornea it will warp into a bitoric form and transmit most if not all of the corneal astigmatism. If an over-refraction is performed with this lens, then the optical errors of both principal meridians are revealed. If we regard each meridian as an individual lens then we can see that the problem is no different from that of providing an appropriate soft lens for an eye which has no astigmatism. On the flat meridian the soft lens BVP to order is the algebraic addition of the trial contact lens BVP (which is spherical) and the effective power at the cornea of the lens in the refractor head along the flat meridian. This will be the sphere power of the negative sphere/cyl form of the prescription. The lens manufacturer simply calculates the front surface radius required for this BVP, starting out with a back surface power based on the specified BOZR of the lens and the lens BVP. The above also applies to the steeper corneal meridian where the BVP to order will be the addition of the trial lens BVP to the algebraic addition of the sphere and cylinder in the negative sphere/cyl form of the ocular refraction. The over-refraction result here will indicate more myopia and the extra negative power required will result in a front surface for the contact lens which is flatter than that of the less myopic meridian. The lens which is produced for this particular eye will have a spherical back surface and a toric front surface which makes the lens more negative in the most myopic meridian. When this lens is placed on the eye it is assumed that the back surface will warp into a toric form identical with that which was present with the trial lens on the eye and so the fluid lens power will be identical with that present at the time of the over-refraction, as will the amount of lens flexure.

An example may clarify the picture.

An eye with keratometry readings

8.30 mm along 5   7.75 mm along 95

and an ocular refraction of

–6.00 DS/–2.75   DC axis 5

is fitted with a spherical fitting set lens of BOZR 8.60 mm, TD 14.00 mm, BVP – 5.50 DS and central thickness 0.3 mm refractive index 1.444. The over-refraction result is

–0.50 DS/–2.50 DC axis 5

What are the front surface radii for the lens to be ordered assuming a central thickness of 0.3 mm for the ordered lens?

The lens to be ordered would be

BOZR 8.60/14.00 BVP – 6.00 DS/–2.50 DC axis 5

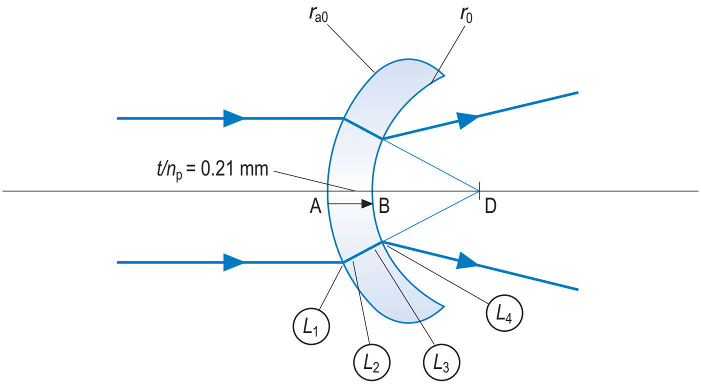

Figure 7.5 Vergence changes in the soft lens

The front surface radii can be calculated using the step along method and Figure 7.5.

Back surface power

$$F_0 = \frac{1-n}{r_0} = \frac{-444}{8.6}$$

$$= -51.63$$

Reduced thickness

$$\frac{t}{n} = \frac{0.3}{1.444} = 0.21 \text{ mm}$$

Along 5

Vergence (D)		Distance (mm)
$L_4 = -6.00 = \text{BVP}$		
$F_0 = \underline{-51.63}$ subtract		
$L_3 = +45.63 \longrightarrow$	$\frac{1000}{45.63} \longrightarrow$	$21.92 = \text{BD}$
	add	$\underline{0.21} = \frac{t}{n}$
$L_2 = +45.19 \longleftarrow$	$\frac{1000}{22.13} \longleftarrow$	$22.13 = \text{AD}$
$L_1 = \underline{0.00}$ subtract		
$F_{a0} = +45.19$		

$$r_{a0} = \frac{444}{45.19} = 9.83 \text{ mm}$$

Along 95

$$
\begin{array}{llcccl}
 & \textbf{Vergence (D)} & & & & \textbf{Distance (mm)} \\
L_4 = & -8.50 = \text{BVP} & & & & \\
F_0 = & \underline{-51.63} \text{ subtract} & & & & \\
L_3 = & +41.13 & \longrightarrow & \dfrac{1000}{41.13} & \longrightarrow & 23.19 = \text{BD} \\
 & & & & \text{add} & \underline{0.21} = \dfrac{t}{n} \\
L_2 = & +42.74 & \longleftarrow & \dfrac{1000}{23.40} & \longleftarrow & 23.40 = \text{AC} \\
L_1 = & \underline{\quad 0.00} \text{ subtract} & & & & \\
F_{a0} = & +42.74 & & & & \\
r_{a0} = & \dfrac{444}{42.74} = 10.39 \text{ mm} & & & &
\end{array}
$$

The lens orientation will be indicated by orientation markers for both front and back surface toric lenses.

7.8.2 *Lenticular astigmatism*

Since the front surface of the lens is being used to correct the astigmatic error, the lens will correct lenticular astigmatism. If a soft lens rests on an eye, which has a spherical cornea but possesses lenticular astigmatism, then it will correct the eye in a very similar way to a rigid front surface toric lens. This means that the soft lens front surface toric design can be used for both corneal and lenticular astigmatism or a combination of the two.

7.9 Specifying the orientation

In **Figure 7.6** we see a contact lens that has a central orientation marker that should be at 6 o'clock. The markers on either side of the central marker are displaced 15°. The lens in **Figure 7.6** has rotated 15° in an anticlockwise direction. The astigmatic correction orientation is specified in standard notation which uses an anticlockwise scale. The lens manufacturer will relate the principal meridians of the toric surface of the contact lens to the orientation markers. These are assumed to take up a position where the central line will run along the 90 meridian. The lens in **Figure 7.6** has rotated 15° in an anticlockwise direction from this ideal. This serves to increase the orientation value (using standard notation) of the markers and the toric surface principal meridians. The central orientation marker line is running along 105. If we wish to restore the toric surface back to a correct orientation for visual correction, we must reduce the axis orientation value in relation to the orientation markers.

If, for example, we required a lens of BVP

–2.00 DS / –1.50 DC axis 25

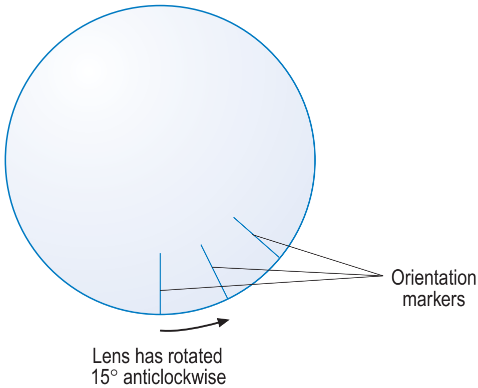

Figure 7.6 A toric lens that has rotated 15° anticlockwise from the expected orientation

then we would order the –1.50 DC at axis 10 to the orientation markers which have taken up an orientation 15° more anticlockwise from the assumed orientation. This reasoning results in an easily remembered rule.

If the lens rotates anticlockwise, then subtract the degree of rotation from the standard notation orientation of the prescription cylinder axis. If the lens rotates clockwise, then add the degree of rotation to the standard notation orientation of the prescription cylinder axis.

The only problem that remains is that of accurately assessing the orientation of the lens on the eye. This is easily achieved by placing a well adjusted trial frame on the patient's face and placing the patient on the slit lamp. The illuminating slit can be rotated until its orientation coincides with that of the contact lens orientation lines. The orientation can then be read off the protractor scale on the trial frame.

Alternatively we can use the two outer orientation markers compared to the central marker, to allow an orientation assessment to the nearest 5°.

Note that when the ordered lens is returned for patient collection, we expect the central orientation line of this lens to maintain its orientation along 105.

If the ordered lens takes up an unexpected orientation, then the over-refraction check will not produce the expected plano result. The presence of crossed cylinders will introduce an induced sphere and an induced cylinder that will manifest as the over-refraction result. The best way of illustrating this problem will be to take an example and use both a graphical and a trigonometric method for the solution. Note that the standard notation scale runs anticlockwise, so an anticlockwise rotation is a positive rotation.

We will assume that if the lens orientated correctly then the over-refraction result would be plano. Thus any other over-refraction result must be due to an unwanted rotation that produces crossed cylinders.

7.10 Stokes construction

7.10.1 Graphical method

Let us assume that we expected the orientation mark for the ordered soft toric lens to align along 90. The ocular astigmatism is 2.00 D with-the-rule axis 5 and the contact lens cylinder is –2.00 DC axis 5. Unfortunately when the lens is placed on the eye, the orientation marks take up an orientation along 100. This represents a 10° rotation in an anticlockwise direction. The induced sphere and cylinder can be deduced using Stokes construction as follows.

In our example the eye has an optical defect that includes a 2.00 D cylinder of with-the-rule astigmatism. Thus the eye is 2.00 D more positive along 95 and this is corrected by a –2.00 DC axis 5. The eye's optical error is thus +2.00 DC axis 5 and this is F_1. The correcting soft lens has a power of –2.00 DC axis 15.

1. The first thing to do is to transpose one of these cylinders in order that the two cylinders have the same sign. So the correction becomes

   F_2 = –2.00 DS/+2.00 DC axis 105

   When the soft lens is on the eye we have

   F_1 = plano/+2.00 DC axis 5

   combined with

   F_2 = –2.00 DS/+2.00 DC axis 105

   The axis is 105 because the lens has rotated 10° anticlockwise.
2. The spherical powers of the combination are isolated and added together algebraically. The combined spherical power of the eye and lens in this example is –2.00 DS.
3. **Figure 7.7** illustrates a parallelogram ABCD with side lengths equal to 2 units representing the 2.00 D cylinders for the eye and the lens. Side AB represents the 2.00 D cylinder axis 5. Side AD represents the 2.00 D soft lens cylinder axis 105. ∠BAD must be twice the angle between the two cylinder axes measured from line AB in an anticlockwise direction. In our example ∠BAD = 200°.
4. The length of the diagonal AC gives the power of the resultant cylinder. This measures 0.7 units in **Figure 7.7**.
5. ∠BAC measured from line AB in an anticlockwise direction and then divided by two gives the orientation of the resultant cylinder. In **Figure 7.7**, ∠BAC is 280° so the orientation of the resultant cylinder is 140° from axis 5 giving an orientation of 145.
6. The induced spherical power is calculated from the equation

$$\text{induced sphere} = \frac{F_1 + F_2 - C}{2} \tag{7.1}$$

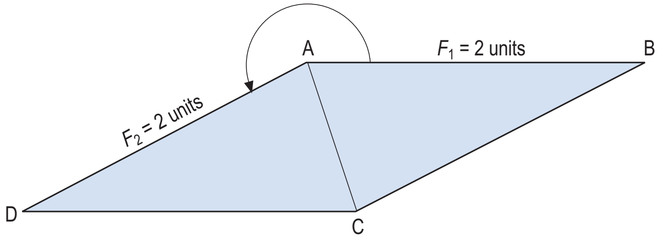

Figure 7.7 Stokes construction

where F_1 is the ocular Rx cylinder, F_2 is the soft lens cylinder and C is the induced resultant cylinder

$$\text{induced sphere} = \frac{2 + 2 - 0.7}{2} = 1.65 \text{ DS}$$

7. The induced sphere must be added to the spherical power of the combination deduced in 2

$$1.65 - 2.00 = -0.35 \text{ D}$$

Thus the expected remaining error is

–0.35 DS / +0.70 DC axis 145

Therefore the over-refraction result will be

+0.35 DS / –0.70 DC axis 145

7.10.2 *Result by calculation*

Figure 7.8 includes a line CE perpendicular to AB.

∠BAD is equal to twice the angle between the two cylinders = 200°. Therefore ∠ABC = 20°.

1. BC is equal to 2 units (the power of F_2). In triangle EBC

$$CE = \sin \angle ABC \times 2 = 0.6840$$
$$EB = \cos \angle ABC \times 2 = 1.8794$$
$$AE = AB - EB = 2 - 1.8794 = 0.1206$$

2. AC is the resultant cylinder. In triangle AEC

$$AC^2 = AE^2 + CE^2 = 0.4824$$
$$\mathbf{AC = 0.6946}$$

This is the power of the resultant cylinder.

3.

$$\tan \angle EAC = CE/AE = 5.6716$$
$$\angle EAC = 80°$$

This angle is measured from the line AB in a clockwise rotation. We need to measure the angle from line AB in an anticlockwise direction. This angle

will be 360 – 80 = 280°. The resultant axis is derived by dividing angle BAC (anticlockwise) by 2 and adding the result to the orientation of the ocular Rx axis.

In our example

$$\text{induced resultant cylinder axis} = \frac{280}{2} = 140° \text{ from the } F_1 \text{ axis}$$

The F_1 axis is 5. **This means the resultant cyl axis orientation is 145**.

4. The induced sphere is derived as before using equation (7.1). In our example

$$\text{induced sphere} = \frac{2 + 2 - 0.6946}{2}$$
$$= 1.6527 \text{ D}$$

This must be added to the spherical powers of the combination

$$1.6527 - 2.0000 = -0.3473$$

Thus

$$\text{induced sphere} = -0.3473 \text{ DS}$$

The expected error remaining uncorrected is

–0.35 DS/+0.69 DC axis 145

Therefore the over-refraction result is

+0.35 DS/–0.69 DC axis 145

7.11 Deducing the unwanted rotation from the over-refraction result

We will continue with the same example and use **Figure 7.8**.

Our over-refraction result indicates that the induced cylinder is

–0.69 DC axis 145

Thus the uncorrected cylinder on the eye is

+0.69 DC axis 145

The axis orientation is 140° from the ocular cylinder axis.

∠BAC measured in an anticlockwise direction must be 2 × 140 = 280. This means that ∠BAC measured in a clockwise direction from line AB is 80° and the length of the line AC is 0.69 units.

In triangle AEC

CE = sin 80× AC = 0.6795 AE = cos 80× AC = 0.1198
EB = AB – AE = 2 –0.1198 = 1.8802

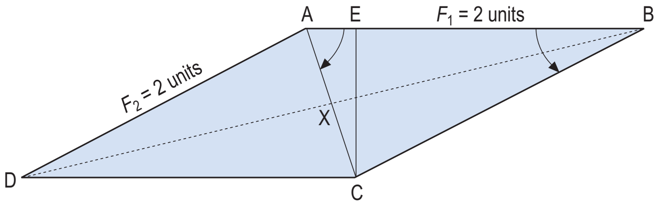

Figure 7.8 Stokes construction for calculation of induced cylinder

In triangle EBC

$$\tan \angle EBC = CE/EB = 0.6795/1.8802 = 0.3614$$
$$\angle EBC = 19.8697°$$
$$\boldsymbol{\angle EBC/2 = 9.9349°}$$

This angle is measured from the line AB in an anticlockwise direction and is therefore a positive angle.

We deduce that the soft lens has a 10° unwanted anticlockwise rotation. In the clinical situation (working to the nearest 0.25 D) the over-refraction result would be –0.75 DC ×145. Repeating the above calculation with a +0.75 D resultant cylinder gives an unwanted rotation of 11° anticlockwise.

7.12 Quick solution

7.12.1 To calculate the unwanted rotation

When the ocular Rx cylinder and the soft lens cylinder both have the same power then **Figure 7.8** illustrates that

$$\sin \angle ABX = \frac{\frac{1}{2}\text{ induced cylinder}}{\text{Power of } F_1 \text{ or } F_2} \qquad (7.2)$$

In our example

$$\sin \angle ABX = \frac{0.5 \times 0.6946}{2} = 0.1736$$

$$\boldsymbol{\angle ABX = +10° = \text{unwanted rotation}}$$

7.12.2 To calculate the over-refraction result

Also equation (7.2) can be used to determine the power and orientation of the induced cylinder where the amount of unwanted rotation is known

$$\frac{\text{power of induced cyl}}{2} = \text{power of cyl } F_1 \times \sin \angle ABX$$

$$\text{power of induced cyl} = 2 \times \sin 10° \times 2$$
$$\boldsymbol{= 0.6946 \text{ DC}}$$

$\angle BAX = \angle ABX - 90$
$\angle BAX = 10 - 90 = -80°$

This angle is measured in a clockwise direction from line AB and is therefore negative. ∠BAX is twice the angular difference between the F_1 axis and the resultant cylinder axis. Therefore the resultant cylinder axis is 40° clockwise from the F_1 axis. Thus axis 5 minus 40 gives an orientation of 145. **The resultant cyl axis is 145**.

7.13 Disposable soft toric lenses

The advent of disposable soft toric lenses has resulted in trial lenses that already possess a worked cylinder. Disposable soft toric lenses will possess either a front or a back toric surface. Ideally the practitioner will select a lens with the desired sphere, cylinder and axis orientation. In practice this ideal may be compromised and the practitioner will have to select a lens as close to the ideal as possible. Also when the lens is placed on the eye it may display an unwanted rotation. Once again we have a situation where crossed cylinders are present. We need to be able to order a lens that corrects the refractive error.

7.13.1 Calculating the theoretical over-refraction

If an eye is fitted with a disposable soft toric lens, then this lens will have a cylindrical power incorporated. In these circumstances the sphere power, the cylinder power and the axis orientation may not be what is required. The over-refraction in these circumstances will produce a result that is not easily converted to the lens power to order.

Let us take a typical example.

The eye being examined has an ocular refraction of

–0.75 DS/–2.00 DC axis 5

This eye is fitted with a toric soft lens which has a BVP of

–0.25 DS/–1.50 DC axis 10

This lens rotates 5° anticlockwise on the eye giving an axis orientation of 15. Thus, this lens has a total unwanted rotation of +10°.

On the eye, the soft lens is

–0.25 DS/–1.50 DC axis 15

(a) Considering the sphere power. **The soft lens corrects 0.25 D of myopia leaving 0.50 D of myopia uncorrected.** Thus the expected over-refraction result arising from the spherical powers will be –0.50 DS.
(b) Considering the crossed cylinders**.** The previously described method can be used to calculate the resultant from the crossed cylinders as follows.

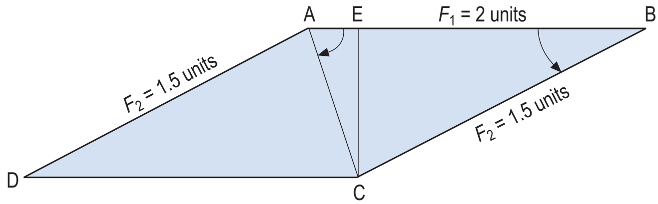

Figure 7.9 Stokes construction for the two cylinders

Figure 7.9 illustrates Stokes construction for this example. The eye's ocular cylindrical refraction result is –2.00 DC axis 5.

This means that the eye is more myopic in the near vertical meridian, i.e. the eye's error is +2.00 DC axis 5 This is F_1.

The soft lens cylinder is –1.50 DC axis 15 which must be transposed into the positive cylinder form –1.50 DS/+1.50 DC axis 105. The +1.50 DC axis 105 is F_2.

We now have two positive crossed cylinders.

The combined sphere power of the two lenses is –1.50 DS.

The angle between the two cylinder axes is 100° (105 – 5).

From **Figure 7.9**

$\angle$BAD must be 2 × 100 measured in an anticlockwise direction

$\angle EBC = \angle BAD - 180 = 200 - 180 = 20°$ (anticlockwise)

In triangle EBC

$EB = \cos \angle EBC \times BC = \cos 20 \times 1.5 = 1.4095$

$EC = \sin \angle EBC \times BC = \sin 20 \times 1.5 = 0.513$

$AE = AB - EB = 2 - 1.4095 = 0.5905$

In triangle AEC

$AC^2 = AE^2 + EC^2 = 0.5905^2 + 0.513^2$

$\mathbf{AC = 0.7822}$

This is the power of the resultant cylinder

$\tan \angle EAC = EC/AE = 0.513/0.5905$

$\angle$EAC is measured in a clockwise direction and is therefore negative

$\angle EAC = -40.9827° \quad \angle EAC/2 = -20.4913°$

This angle is measured from the F_1 axis which means it is –15.4913° from the horizontal meridian. Thus the axis orientation is

$180 - 15.4913 = 164.5087 \cong \mathbf{165}$

$$\text{The induced sphere} = \frac{(F_1 + F_2 - C)}{2} = \frac{2 + 1.5 - 0.7822}{2}$$

$$= +1.3589$$

This must be added to the combined sphere power of –1.50 DS giving

1.3589 –1.50 = –0.14 DS

Thus the remaining uncorrected error is

–0.14 DS/+0.78 DC axis 165

The over-refraction result will be

+0.14 DS/– 0.78 DC axis 165

From (a) we note that there is 0.50 D of myopia left uncorrected.

Therefore in our example the sphere will become 0.50 D more negative.

The theoretical over-refraction becomes

– 0.36 DS/– 0.78 DC axis 165

7.13.2 *Calculating the BVP to order from an over-refraction with crossed cylinders*

We will use the above example. The starting point for calculating the power to order will be the clinical information available to us when the over-refraction is completed. We will now know the following:

- the ocular prescription negative cylinder axis is 5.
- the power of the soft lens is –0.25 DS/–1.50 DC axis 10.
- the cylinder power of the soft lens is –1.50 DC transposed to –1.50 DS/+1.50 DC.
- the lens rotates 5° anticlockwise.
- the over-refraction sphere is –0.36 DS.
- the over refraction cylinder is –0.78 DC.
- the over-refraction axis is 165.

What BVP needs to be ordered for this eye? Using **Figure 7.9**

AC = 0.78

Resultant axis (165) is 160° from the ocular prescription (F_1) axis

∠BAC = (2 × 160) = 320 degrees (anticlockwise)
320 – 360 = –40 degrees (clockwise from AB)

In triangle AEC

AE = cos 40 × 0.78 = 0.5975

In triangle EBC

Soft lens cylinder = 1.50 DC axis 105

Therefore ∠EBC = 20 ° (anticlockwise)

this is twice the unwanted rotation

EB = cos 20 × 1.5 = 1.4095
AB = AE + EB = 0.5975 +1.4095
AB = 2.0007

Thus the cylindrical error of the eye is 2.0007 DC.

This positive cylinder that represents the error of the eye will require a –2.00 DC cylindrical lens for correction.

The cylinder axis will be the ocular prescription negative cylinder axis which is 5.

The sphere power of the crossed cylinders is calculated from

$$S = \frac{(F_1 + F_2 - C)}{2} = \frac{(2 + 1.5 - 0.78)}{2} = 1.36$$

This must be added to the combined sphere power. The combined sphere power will be the power of the trial lens cylinder arising from its transposition to the positive cylindrical form

1.36 – 1.50 = –0.14 DS

This is the remaining uncorrected error which will require a +0.14 DS lens to correct. The over-refraction sphere is –0.36 which is 0.50 D more negative than the result from the combination of the crossed cylinders on their own.
We must add to this the –0.25 DS of the soft contact lens giving a total sphere power of –0.75 DS.

Therefore the BVP to order for this eye is

–0.75 DS/–2.00 DC axis 5

which is the ocular refraction of this eye.

There is a computer program in Chapter 11 that uses the above approach to calculate:

- the expected over-refraction result.
- the unwanted rotation.
- the BVP to order based on the toric soft lens being used and the over-refraction result with this lens.

7.14 RGP toric lenses

We have seen that when any contact lens is in place on the astigmatic eye then the back surface of the fluid lens eliminates all the corneal astigmatism. This is true for both soft and RGP lenses. A toric soft lens is made toric in order to achieve full correction of the refractive error and in consequence provide the patient with a visual acuity as good as that provided by a spectacle correction. In the case of the RGP lens, a lens with spherical surfaces will produce a spherical front surface for the fluid lens. Thus all surfaces in the correcting system are spherical with the exception of the posterior fluid lens surface. This is toric because the cornea is toric. This back surface corrects the corneal astigmatism and so the astigmatic eye is fully corrected by a spherical RGP lens with a spherical power no matter how large the astigmatism.

The need for toric RGP lenses in the presence of corneal astigmatism is dictated not by concerns about uncorrected astigmatism but by fitting considerations. The use of a spherical lens on a toric cornea may result in a fit that is uncomfortable and unstable. A toric back surface will provide a better match to the toric cornea. Thus toric soft lenses are required for optical clarity but toric RGP lenses are required to achieve a satisfactory fit.

7.15 Back surface toric rigid corneal lenses

7.15.1 Induced astigmatism

Increasing corneal toricity ultimately leads to a situation where a spherical back surface corneal contact lens will not be tolerated by the wearer. In these circumstances the fitting relationship between the cornea and the contact lens may be improved by using a toric back surface contact lens. If the toric back surface was fitted on alignment to both the flat and the steep corneal principal meridians, then there would be minimum edge clearance maintained all around the lens edge. This would produce high capillary attraction forces resulting in a very thin fluid lens. Thus tear exchange behind the lens would be poor. The general approach to fitting toric back surface RGP lenses is to use a lens that aligns with the flatter corneal meridian and fits slightly flat on the steeper meridian. The fluorescein picture will be similar to that observed when a spherical back surface lens is fitted to a slightly toric cornea. The slightly flat fit on the steep corneal meridian will increase the edge clearance here and this will encourage a better tear exchange.

The disadvantage of a toric back surface contact lens is that the fluid lens will possess an anterior surface that is now toric. Essentially, on the steeper meridian, the anterior surface of the fluid lens becomes more positive and the back surface of the contact lens becomes more negative when a change is made from a spherical to a toric back surface contact lens. The contact lens refractive index is higher than that of the tears and this therefore results in over-correction of the corneal astigmatism because the system is becoming over-negative on the steeper (more myopic) corneal meridian. We can therefore conclude that the fitting of a toric back surface contact lens will result in an astigmatic over-correction called *induced astigmatism*. This induced astigmatism is present at the contact lens/fluid lens interface. It is induced by the contact lens and is completely independent of the corneal astigmatism which is still eliminated by the posterior surface of the fluid lens. The induced astigmatism can be neutralized by incorporating a toric front surface on the contact lens, which then becomes a compensated parallel bitoric lens. In these circumstances the toric front surface is there to correct the induced astigmatism and so when the contact lens rotates on the eye, the induced astigmatism (produced by the back surface) and its correction (worked on the front surface) maintain their alignment which results in no deterioration in the clarity of the retinal image. Thus a lens that corrects its

own induced astigmatism does not need to be rotationally stable to maintain optical clarity.

Induced astigmatism can be defined as

induced astigmatism	=	contact lens back surface astigmatism in air	–	fluid lens front surface astigmatism in air

7.15.2 The clinical procedure for prescribing toric back surface lenses

It may be useful to take a typical example.

Ocular refraction	–1.00 DS/–3.00 DC axis 180	
Keratometry readings	8.10 mm along 180	7.55 mm along 90

The keratometry readings suggest that the astigmatism is corneal. A contact lens fitting session indicates that a toric lens is required for comfort, however, only a spherical fitting set is available. The lens which provides an alignment fit along 180 is

C2 8.10:6.50/8.70:9.00 BVP –0.75 DS

The over-refraction result with this lens is –0.25 DS, giving a visual acuity of 6/5. The lack of any cylinder in the over-refraction and the high visual acuity support the notion that the astigmatism is corneal.

The practitioner decides to order a toric back surface with BOZRs of 8.10 mm along 180 and 7.7 mm along 90. This will give an alignment fit along the flat corneal meridian and a slightly flat fit along the steep meridian. The fluorescein picture is then very similar to that which occurs when a spherical surface lens is fitted to a slightly toric cornea.

The BVP for this lens is calculated as follows.

Along 180

The fitting relationship is illustrated in **Figure 7.10**. The over-refraction result indicates that the contact lens BVP must be –1.00 DS to correct the refractive error of this meridian. The specification for this meridian remains unchanged in the lens to be ordered since the BOZR is still 8.10 mm. Therefore the BVP to be ordered remains unaltered at –1.00 DS.

Along 90

In **Figure 7.11(a)** the over-refraction with the fitting set lens indicates that a BVP of

–0.75 –0.25 = –1.00 DS will be required if the BOZR is to remain spherical.

However, in the toric lens to be ordered **(Figure 7.11(b))** the BOZR is reduced from 8.10 mm to 7.70 mm. The diagrams illustrate that this makes the fluid

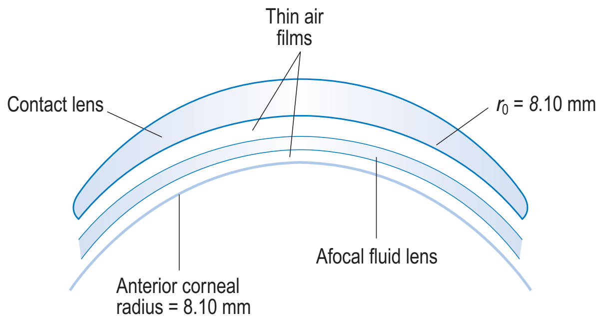

Figure 7.10 The fitting relationship along the flat meridian

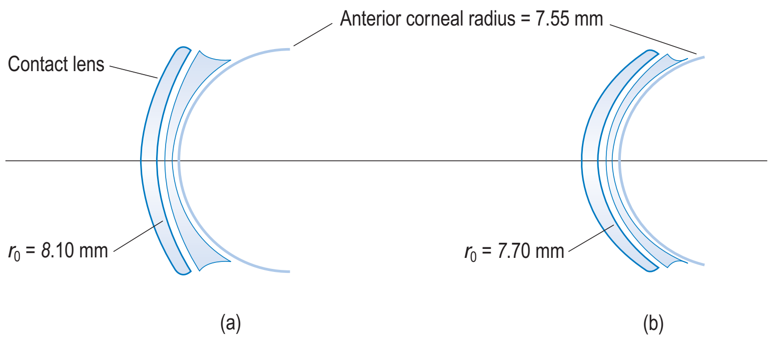

Figure 7.11 The fitting relationship along the steep meridian (a) using the spherical fitting set lens, (b) using the toric lens

lens less negative in power. Remember that we can consider the contact lens/fluid lens and cornea to be separated by infinitely thin films of air and this will not alter the track of the light rays. The contact lens must correct the refractive error of the eye *and* neutralize the fluid lens.

A BOZR of 8.10 mm gives an anterior surface power to the fluid lens in air of

$$\frac{336}{8.1} = +41.48 \text{ D}$$

A BOZR of 7.70 mm gives an anterior surface power to the fluid lens in air of

$$\frac{336}{7.7} = +43.64 \text{ D}$$

$$43.64 - 41.48 = +2.16 \text{ D}$$

We can conclude that along the 90 meridian for BOZR 7.70 mm, the fluid lens is 2.16 D more positive than at the time of the over-refraction, which also means that it is 2.16 D more positive than the 180 meridian.

Thus the fluid lens anterior surface astigmatism for the toric lens is

$$43.64 - 41.48 = +2.16 \text{ D axis } 180$$

If the contact lens is to maintain a correction identical with that of the spherical lens used for the over-refraction, then its power along 90 must be made more negative by 2.16 D at the back surface of the lens. The final prescription is therefore

C2 parallel bitoric $\dfrac{8.10 \text{ along } 180}{7.70 \text{ along } 90} : 6.50/\dfrac{8.70}{8.30} : 9.00$

BVP –1.00 DS/–2.16 DC axis 180

It is likely that the 2.16 DC would be rounded to the nearest 0.25 D making the cylinder –2.25 DC axis 180.

The lens manufacturer simply needs to know what powers to give each of the two principal meridians. S/he will obviously be unaware of how the lens will orientate on the eye. The lens order therefore might better be written as follows

C2 parallel bitoric $\dfrac{8.10}{7.70} : 6.50/\dfrac{8.70}{8.30} : 9.00 \quad \dfrac{\text{BVP } -1.00 \text{ D}}{\text{BVP } -3.25}$

If the contact lens material refractive index is 1.490 then the reduction in BOZR from 8.10 mm to 7.70 mm will induce a back surface cylinder on the contact lens which is 3.14 D more negative along 90, confirming over-correction of the astigmatism by the back surface of the contact lens. Thus the contact lens must have a front surface with the vertical meridian steeper than the horizontal in order to achieve the BVP requested.

7.15.3 *The use of the approximate rule that links radius to power change*

If we consider the approximate rule (see Section 2.6) on change of fluid lens power with change of BOZR, i.e. *0.1 mm radius change induces a 0.50 D power change*, we can consider its application to the above problem. We know that the horizontal meridian requires a BVP of –1.00 D. This was deduced above from the over-refraction result. It could also have been deduced from the spectacle prescription as follows.

The ocular refraction along 180 is –1.00 DS.
The k-reading of the flat meridian is 8.10 mm.
The BOZR to be ordered along 180 (flat meridian) is 8.10 mm.
This means that the fluid lens power along 180 is plano.
Thus the contact lens BVP required along 180 is –1.00 D.

The lens radius is to be reduced by 0.4 mm along 90.

The approximate rule tells us that this will induce a power change in the fluid lens of +2.00 D.

The contact lens power must be made more negative by 2.00 D along 90 to compensate.

Therefore the BVP requested would be

–1 .00 DS/–2.00 DC axis 180

The approximate rule serves us very well and the use of simple mental arithmetic means that this approach is ideal for clinical work.

However, if we require an accurate result then we only need to consider the relationship

$$F = \frac{(n-1)}{r} \quad \text{for the two BOZRs}$$

where n = 1.336 (refractive index of the fluid lens). In other words, we need to use Heine's scale.

If a contact lens is to correct the eye, it must correct the refractive error *and* neutralize the power of the fluid lens. In a spherical RGP lens, the back surface of the fluid lens corrects the corneal astigmatism and all the other surfaces anterior to this are spherical. Thus astigmatism is corrected.

In the case of a toric RGP lens, the front surface of the fluid lens is now toric and this will require an astigmatic correction.

In our example

$$F_{\text{steep}} = \frac{336}{7.70} \qquad F_{\text{flat}} = \frac{336}{8.10}$$

$$= +43.64 \qquad = +41.48\text{ D}$$

43.64 –41.48 = 2.16 D

The anterior surface of the fluid lens is more positive along 90. So we need a –2.16 D cylindrical power with the power meridian along 90 (the steep meridian) to correct the eye. This means the cylinder axis must be along 180 (the flat meridian).

7.15.4 *Corneal astigmatism combined with lenticular*

When a spherical RGP lens is used for the over-refraction, any manifest astigmatism is called residual astigmatism. This will arise from the crystalline lens in the eye. Thus the terms residual astigmatism and lenticular astigmatism are synonymous in these circumstances.

If in our example there had been some residual astigmatism that became apparent at the over-refraction, then the approach to the problem remains unaltered. For example, the trial lens is

C2 8.10:6.50/8.70:9.00 BVP – 1.00 DS

suppose the over-refraction result was

–0.50 DS/+0.75 DC axis 180

The +0.75 DC axis 180 is required to correct lenticular astigmatism.

Start out by ignoring the lenticular astigmatism. The spherical trial lens requires a BVP of

–1.00 –0.50 = –1 .50 D

We intend to order a toric back surface with BOZRs of 8.10 and 7.70 mm.

We have seen that the BVP required for correction of the induced astigmatism results in

–1.50 DS/–2.16 DC axis 180

But there is now also a +0.75 DC axis 180 in the over-refraction. This now needs adding to our result. Therefore

BVP required = –1.50 DS/–1.41 DC axis 180

Thus, if lenticular astigmatism is present, it can be corrected by suitable modification of the cylindrical power of the contact lens. Where residual astigmatism is present, either alone or in combination with corneal astigmatism, then the contact lens must maintain its orientation if it is to fully correct the refractive error. It must be pointed out that on the few occasions where lenticular astigmatism is present, the amount is small. Therefore in the vast majority of patients, lenticular astigmatism is of no clinical significance.

7.15.5 Compensated toric rigid corneal lenses

In the absence of lenticular astigmatism, a lens with a toric back surface will induce astigmatism into the system and this induced astigmatism will need to be neutralized by the toric front surface worked onto the lens. It was noted that if this type of lens rotates on the eye then the induced astigmatism (produced by the toric back surface) and its correction (the front toric surface) maintain their alignment with each other (parallel bitoric) and the retinal image remains clearly in focus. This type of lens is described as a compensated toric lens. A compensated toric lens is a parallel bitoric that corrects its own induced astigmatism. Toric RGP fitting set lenses could be produced as compensated torics where an over-refraction will require only spherical powers because the lens corrects its own induced astigmatism and the corneal astigmatism is neutralized by the back surface of the fluid lens.

If we take the same back surface toric lens used in Section 7.15.2 for the purposes of illustration we have

C2 PMMA parallel bitoric $\dfrac{8.10}{7.70} : 6.50/\dfrac{8.70}{8.30} : 9.00\ \dfrac{-1.00\text{ D}}{-3.16\text{ D}}$

This is a compensated toric lens because the 2.16 D cylinder is there to correct the astigmatic element introduced by the toric front surface of the fluid lens.

7.15.5.1 The three, two, one rule

If we continue with our example and assume that the lens material refractive index is 1.5

(a) The contact lens back surface cylinder in air is deduced from

$$\left(\frac{500}{8.1}\right) - \left(\frac{500}{7.7}\right) = 61.73 - 64.94 = -3.21 \text{ DC axis } 180$$

(b) The cylinder required for the correction of the induced astigmatism in air is deduced from

$$\left(\frac{336}{8.1}\right) - \left(\frac{366}{7.7}\right) = 41.48 - 43.64 = -2.16 \text{ DC axis } 180$$

(c) The induced astigmatism on the eye is deduced from

refractivity of the interface = 1.500 – 1.336 = 0.164

$$\left(\frac{164}{8.1}\right) - \left(\frac{164}{7.7}\right) = 20.25 - 21.3 = -1.05 \text{ DC axis } 180$$

The approximate ratios are

back surface cyl/cyl required for correction/induced astigmatism

3 / 2 / 1

Thus a toric back surface RGP lens has a back surface cylinder that is larger than the cylinder required for correction in air. The induced astigmatism is approximately half of the cylinder power in air required for correction.

In our example the toric back surface cylinder power in air is 3.21 D, the contact lens requires a 2.16 D cylinder in air to correct the induced astigmatism, but the induced astigmatism on the eye is only 1.05 D.

The 3/2/1 rule gives an approximate comparison between the three cylindrical powers. An accurate comparison can be made as illustrated in the following example.

The contact lens toric back surface is 8.40 mm along 180 and 7.60 mm along 90 and the refractive index of the material is 1.490.

The contact lens back surface cylinder is

$$\frac{-490}{7.6} - \frac{-490}{8.4} = -64.47 + 58.33 = -6.14 \text{ DC}$$

The fluid lens front surface cylinder is

$$\frac{336}{7.6} - \frac{336}{8.4} = 44.21 - 40.00 = 4.21 \text{ DC}$$

The ratio of these powers is

$$\frac{4.21}{6.14} = 0.686$$

The difference in the two cylindrical powers arises from the differences in the refractivities of the fluid and the contact lens. It must therefore be no surprise to find that the ratio of the refractivities is

$$\frac{336}{490} = 0.686$$

Therefore given either the contact lens back surface cylinder power or the contact lens cylindrical power in air required to correct the induced astig-

matism, then the other power can be calculated by either multiplying or dividing by the ratio of the refractivities

$$6.14 \times 0.686 = 4.21 \quad \frac{4.21}{0.686} = 6.14$$

The difference between the contact lens power in air required to correct the induced astigmatism and the induced astigmatism on the eye can be calculated in a similar way.

The induced astigmatism on the eye in this example will be derived from

$$\text{power} = \frac{n_f - n_{cl}}{r}$$

where n_f is the refractive index of the fluid lens and n_{cl} is the refractive index of the contact lens

$$1.336 - 1.490 = -0.154$$

$$\text{Induced astigmatism} = \frac{-154}{7.6} - \frac{-154}{8.4}$$

$$= -20.26 + 18.33 = -1.93 \text{ DC}$$

The ratio of the induced astigmatism on the eye and the cylinder in air required to correct the induced astigmatism is

$$\frac{1.93}{4.21} = 0.458$$

The ratio of the refractivities is

$$\frac{154}{336} = 0.458$$

$$4.21 \times 0.458 = 1.93 \quad \frac{1.93}{0.458} = 4.21$$

We can therefore conclude that an accurate comparison between the three cylindrical powers can be made using:

3 Lens back surface cylinder power

2 Power in air required to correct induced astigmatism

$$= \text{lens cylinder power} \times \frac{336}{490}$$

1 Induced astigmatism on the eye

$$= \text{power in air} \times \frac{154}{336}$$

7.15.5.2 Clinical use of the compensated toric trial lens

The big advantage of using a compensated toric fitting set is that the lens will behave optically like a spherical contact lens because the posterior surface of

the fluid lens is correcting the corneal astigmatism and a compensated toric contact lens corrects its own induced astigmatism. Thus, in the absence of lenticular astigmatism, the over-refraction will be performed using spheres only. The lens BVP to order will be deduced by adding the over-refraction sphere to the spherical component of the trial contact lens BVP as follows

Trial contact lens BOZRs	$\dfrac{\text{8.40 along 180}}{\text{7.60 along 90}}$
BVP	–2.00 DS/–4.21 DC axis 180
over-refraction	–1.50 DS
order BVP	–3.50 DS/–4.21 DC axis 180

Note that

$$\left(\frac{336}{8.4}\right) - \left(\frac{366}{7.6}\right) = 4.21\text{ D}$$

confirming that the lens is a compensated toric.

Also note that this lens does not need to maintain its orientation to maintain optical clarity.

7.15.5.3 Misalignment of the surfaces of a compensated toric lens

One problem with a parallel bitoric is that of manufacturing the lens with the principal meridians of the front and back surfaces precisely aligned.

Consider the following RGP compensated toric lens:

BOZR 8.40 mm along 180 and 7.60 mm along 90
BVP –2.00 DS/–4.21 DC axis 180
Refractive index of material 1.490
This lens fully corrects the eye of the wearer

The back surface cylinder in air is

$$\frac{490}{8.4} - \frac{490}{7.6} = 58.33 - 64.47 = -6.14\text{ DC axis 180}$$

This back surface cylinder is partially neutralized by the contact lens front surface cylinder in order to produce the –4.21 DC axis 180 requested. Therefore the effective power of the front surface cylinder at the contact lens back surface will be

–4.21 + 6.14 = +1.93 DC axis 180

However, if the lens is manufactured with the principal meridians of the front surface at 5 and 95 when the back surface principal meridians are at 180 and 90, then we have

+1.93 DC axis 5

in combination with

–6.14 DC axis 180

which is the same as

–6.14 DS / +6.14 DC axis 90

The combination of the two surfaces is illustrated in **Figure 7.12**.

We have a problem of obliquely crossed cylinders and we need to calculate the power of the combination.

7.15.5.4 Stokes construction

Figure 7.13 illustrates Stokes construction for our example. A parallelogram is drawn with side AB representing the cylinder power nearest the horizontal meridian (1.93 units) and side AD representing the other cylindrical power (6.14 units). The two cylinder powers are represented as before as vectors.

$$2\alpha = \angle BAD = 2 \times 85 = 170°$$

The length of the diagonal AC indicates the power of the resultant cylinder arising from the two crossed cylinders.

The procedure using Stokes construction is as follows

(a) Transpose the cylinders, if necessary, so that they are both the same sign. Note the combined power of any spheres. The combined sphere power in our example is –6.14 D.

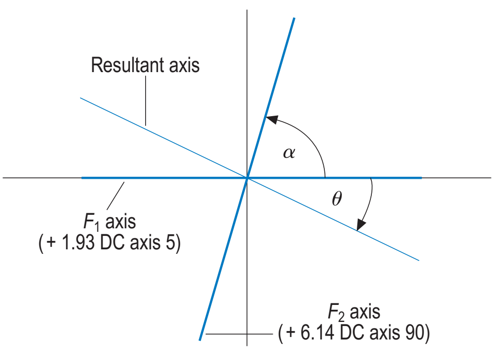

Figure 7.12 The combination of the crossed cylinders. Angle α is 85°

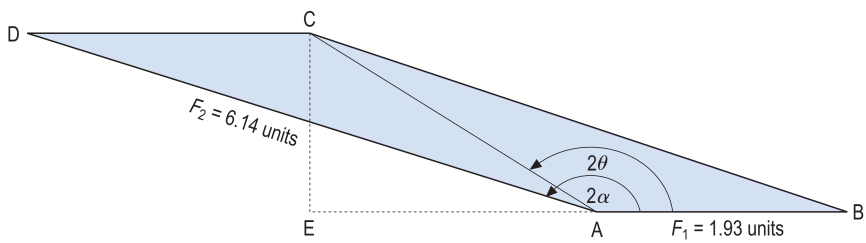

Figure 7.13 Stokes construction for the bitoric contact lens with misaligned surfaces in air

(b) Choose the cylinder with the smallest numerical axis and call its power F_1. The axis will be represented as a horizontal line. F_1 in this example is +1.93.

(c) Find angle α from

$$\alpha = \text{axis } F_2 - \text{axis } F_1$$

In our example

$$\alpha = 90 - 5 = 85°$$
$$2\alpha = 170°$$
$$\angle DAE = 10°$$

(d) In triangle BCE

$$\angle EBC = \angle DAE = 10°$$
$$CB = 6.14$$
$$\sin \angle EBC = CE/CB$$
$$CE = \sin \angle EBC \times CB = \sin 10 \times 6.14$$
$$= 1.0662$$
$$EB^2 = CB^2 - CE^2$$
$$EB = 6.0467$$
$$AE = EB - AB = 6.0467 - 1.93 = 4.1167$$

In triangle ACE

$$CA^2 = CE^2 + AE^2$$

CA = 4.2525 this is the resultant cyl power (*C*)

(e) $\tan \angle EAC = CE/AE$

$$\angle EAC = 14.5202°$$
$$2\theta = 180 - 14.5202 = 165.4798$$
$$\theta = 82.7399 \cong 83° \text{ anticlockwise from AB}$$

The resultant cyl orientation is 83 + 5 = 88.

(f) Find the induced sphere from

$$S = \frac{(F_1 + F_2 - C)}{2}$$

$$S = \frac{(1.93 + 6.14 - 4.2525)}{2}$$
$$= +1.9088 \text{ DS}$$

This must be added to the combined sphere –6.14 +1.9088 = –4.2312 DS

(g) Therefore the resultant power arising from the two crossed cylinders is

–4.23 DS/+4.25 DC axis 88

or

+0.02 DS/–4.25 DC axis 178

The original BVP included a sphere of –2.00 D giving a final BVP of

–1.98 DS/–4.25 DC axis 178

The power changes induced by the misalignment of the front and back surfaces of the contact lens are modest and it is unlikely that they would be detected when checking the lens BVP which should be

–2.00 DS / –4.21 DC axis 180

On the eye

On the eye, the effective power of the front surface cylinder at the back surface (+1.93 DC axis 5) remains the same as in air. The back surface of the contact lens is now in contact with the tears (refractive index 1.336). The power of the back surface on the eye is given by

$$\frac{(1336 - 1490)}{7.6} - \frac{(1336 - 1490)}{8.4} = -20.26 + 18.33$$

$$= -1.93 \text{ DC axis } 180$$

In this situation

F_1 is +1.93 DC axis 5
F_2 is –1.93 DC axis 180

which is

–1.93 DS / +1.93 DC axis 90

If we follow the same routine as before using **Figure 7.14**. The line CE is perpendicular to AB.

(a) The two cylinders have been transposed to give the same sign. The combined sphere is –1.93 DS.
(b) The F_1 cylinder is +1.93 DC axis 5.
(c) $\alpha = 90 - 5 = 85°$

$2\alpha = 170°$

$\angle EBC = 10°$

(d) In triangle BCE

$$\sin \angle EBC = CE/CB$$
$$CE = \sin \angle EBC \times CB = \sin 10 \times 1.93$$
$$= 0.3351$$
$$EB^2 = BC^2 - CE^2$$
$$EB = 1.9007$$
$$AE = AB - EB = 1.93 - 1.9007 = 0.0293$$

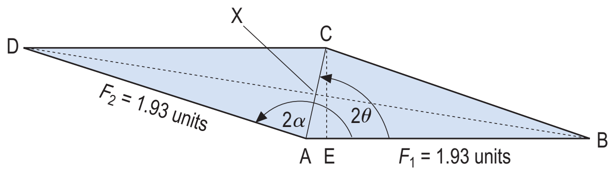

Figure 7.14 Stokes construction for the bitoric contact lens with misaligned surfaces on the eye

In triangle ACE

$$CA^2 = CE^2 + AE^2$$

CA = 0.3364 this is the resultant cyl power (C)

(e)

$$\tan \angle EAC = CE/AE$$
$$\angle EAC = 85.003°$$
$$2\theta = 85.003°$$
$$\theta = 42.5015°$$

The resultant cyl orientation is 42.5 + 5 = 47.5.

(f) Find the induced sphere from

$$S = \frac{(F_1 + F_2 - C)}{2}$$
$$S = \frac{(1.93 + 1.93 - 0.3364)}{2}$$
$$= +1.7618 \text{ DS}$$

This must be added to the combined sphere

–1.93 + 1.7618 = –0.1682 DS

(g) Therefore the resultant power arising from the two cylinders is:

–0.17 DS / +0.34 DC axis 47.5

This represents the extra (unwanted) power incorporated into the prescription due to the misalignment because, if the front and back surfaces were correctly aligned, then the resulting power, and hence the over-refraction, would be zero. The theoretical over-refraction that would be obtained when this misaligned lens was placed on the eye for which it was manufactured is

+0.17 DS / –0.34 DC axis 47.5

We see that there is a residual cylinder on the eye when the lens may have appeared to be correct when checking its BVP on the focimeter.

7.15.5.5 The over-refraction used to assess the degree of misalignment

The geometry of Stokes construction in **Figure 7.14**, where F_1 is equal to F_2, leads to the conclusion that for a misaligned compensated toric lens on the eye

$$\alpha = 2\theta$$

If we draw the diagonal BD. It will bisect line AC at point X

$$\cos \alpha = \frac{AX}{AB}$$

AX is half the over-refraction cylinder. AB is the induced astigmatism. 2α is twice the angle between the two surface cylinder axes.

$$\cos\alpha = \frac{\text{over-refraction cylinder power}}{2 \times \text{the induced astigmatism}} \tag{7.3}$$

In our example

$$\cos\alpha = \frac{0.34}{2 \times 1.93}$$

$$\alpha = 85°$$

Therefore the rotation of the front surface when the back surface F_2 axis is along 90 will be 90 – α

rotation = 90 – 85 = 5°

If the resultant negative cylinder axis found at the time of the over-refraction is between 0 and 90, as in our example (47.5°), the front surface has rotated out of alignment in an anticlockwise direction. If the resultant negative cylinder axis is between 90 and 180 the front surface has rotated clockwise.

7.15.6 Single toric back surface

An uncorrected cylinder of 1.00 D produces about the same retinal image blur as an uncorrected sphere of 0.50 D. Many patients would happily accept this small amount of under-correction. If we were dealing with such a patient then the best solution would be to order the toric back surface and request a spherical front surface for the contact lens. This lens will be cheaper and quicker to make.

Let us suppose that we used a trial lens with a BOZR 8.10 BVP –1.25 DS and over-refraction of –1.75 DS.

This lens gives an alignment fit along the flat corneal meridian and would require a BVP of –3.00 DS to correct the eye.

We decide to order a toric back surface RGP lens with radii of 8.10 and 7.70 mm and refractive index 1.490. The cylinder produced by the back surface of this lens will be

$$\frac{490}{7.7} - \frac{490}{8.1} = 63.64 - 60.49 = 3.15 \text{ D}$$

more negative along the steep meridian.

The cylinder required to correct the induced astigmatism will be

$$\frac{336}{7.7} - \frac{336}{8.1} = 43.64 - 41.48 = 2.16 \text{ D}$$

more negative along the steep meridian.

Thus the lens back surface provides a cylinder that is 0.99 D too negative along the steep meridian. This is the induced astigmatism. This extra 1.00 D of negative cylinder power will push the circle of least confusion 0.50 D away from the eye's refracting surfaces and thus induce 0.50 D of accommodation. This extra accommodative effort can be eliminated by making the sphere power 0.50 D more positive than the sphere derived at the time of the over-refraction. In this example, we need a lens with a spherical front surface that gives a BVP of –2.50 D along the flat meridian of the lens.

This lens will be easier to produce because only one toric surface needs to be generated and there is in consequence no problem of ensuring accurate alignment of the front and back toric surfaces. The one dioptre of uncorrected induced astigmatism will produce a small, but probably acceptable, amount of retinal blur.

7.15.6.1 The influence of the lens refractive index

If we decide to order a lens with a toric back surface and a spherical front surface then the induced astigmatism will be left uncorrected. The refractive index of the lens above is 1.490 and the refractive index of the tears is 1.336. We have seen that the induced astigmatism is calculated using the relationship

$$\left(\frac{154}{8.1}\right) - \left(\frac{154}{7.7}\right) = -0.99 \text{ DC axis } 180$$

The 154 arises from the difference of the refractive index of the fluid lens and that of the contact lens material. It follows that if we can use a low refractive index lens material then the induced astigmatism will be reduced. In our example, let us assume that we change the lens material to one of refractive index 1.4. The induced astigmatism then becomes

$$\left(\frac{64}{8.1}\right) - \left(\frac{64}{7.7}\right) = -\ 0.41 \text{ DC axis } 180$$

It is therefore advisable to consider using a low refractive index material to minimize the induced astigmatism when going for the toric back-spherical front surface option.

7.16 The fitting relationship of a spherical RGP lens on a toric cornea

If a practitioner is attempting to fit a moderately toric cornea with a spherical back surface hard or RGP contact lens, then a commonly encountered recommendation is to use a BOZR somewhere between the radii of the two principal meridians of the cornea as measured by the keratometer. The usual recommendation is to go for a BOZR one-third of the way from the flatter

meridian, two-thirds from the steeper. For example, if the keratometer readings were

7.80 mm along 180 and 7.20 mm along 90

then a contact lens of BOZR 7.60 mm may represent the best compromise fit. This lens is a little steep along 180 and is flat along 90 but not as flat as would be the case if a BOZR of 7.80 mm (aligning with the 180 meridian) is used. It is claimed that the 7.60 mm radius will produce a reduced edge clearance at the top and bottom of the lens in this example. Some workers have disputed this, claiming that since the lens is steep along 180 this lifts the lens back vertex off the corneal apex, which restores the edge clearance to much the same value as a lens which is fitted on alignment with the 180 meridian. Let us take an example to attempt to resolve the dispute.

A cornea with apical radii of 7.90 mm along 180 and 7.30 mm along 90 and a p-value of 0.8 for both meridians is fitted with a bi-curve lens of the following specification

C2 7.90:7.00/10.50:8.50

(a) What is the axial edge clearance (AEC) along 90?
(b) What would the AEC be along 90 if the contact lens BOZR was changed to 7.70 mm with the other lens parameters unaltered?

The BOZR 7.90 mm lens will fit with apical clearance on the flat corneal meridian and the contact lens will touch the cornea at the BOZD transition. The fit is illustrated in **Figure 7.15**

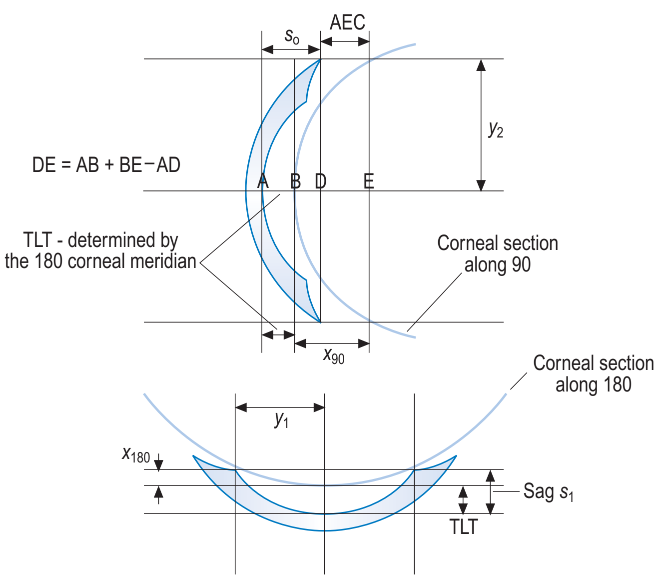

Figure 7.15 A C2 spherical contact lens and the two anterior corneal principal meridians. The TLT is determined by the fitting relationship along 180

(a)

C2 7.90 : 7.00/10.50 : 8.50

Along 180

TLT

The cornea

$r_o = 7.90$ mm $p = 0.8$ $y_1 = 3.5$ mm

$$x_{180} = \left(\frac{r_o - \sqrt{(r_o^2 - py^2)}}{p}\right) \tag{4.9}$$

$x_{180} = 0.808$ mm for diameter 7.00 mm

The contact lens

BOZR (r_0) = 7.90 mm contact semi-diameter (y_1) = 3.5 mm

$$\text{sag } s_1 = r_0 - \sqrt{(r_0^2 - y_1^2)} \tag{3.1}$$

$s_1 = 0.818$ mm for diameter 7.00 mm

Tear Layer Thickness (TLT)

Figure 7.15 illustrates that

$\text{TLT} = s_1 - x_{180}$

$\text{TLT} = 0.818 - 0.808 = 0.01$ mm

This TLT value applies also to the 90 meridian.

Along 90

The TLT is 0.01 mm.

AEC

The cornea

$r_o = 7.30$ mm $p = 0.8$ $y_2 = 4.25$ mm

$$x_{90} = \frac{r_o - \sqrt{(r_o^2 - py_2^2)}}{p} \tag{4.9}$$

$x_{90} = 1.335$ mm for diameter 8.50 mm

The contact lens

The overall sag (s_0) of the contact lens is $s_o = s_1 + s_2 - s_3$.

$s_1 = 7.9 - \sqrt{(7.9^2 - 3.5^2)} = 0.818$ mm

$s_2 = 10.5 - \sqrt{(10.5^2 - 4.25^2)} = 0.899$ mm

$s_3 = 10.5 - \sqrt{(10.5^2 - 3.5^2)} = 0.601$ mm

Therefore

$s_o = 1.116$ mm for diameter 8.5 mm

AEC along 90

$$\begin{aligned}\text{AEC} &= \text{TLT} + x_{90} - s_o \\ &= 0.01 + 1.335 - 1.116 \\ &= \mathbf{0.229\ mm}\end{aligned}$$

(b)

C2 7.7:7.00/10.50:8.50

Along 180

TLT

The cornea
As before

$$r_o = 7.90 \text{ mm} \quad p = 0.8 \quad y_1 = 3.5 \text{ mm}$$
$$x_{180} = 0.808 \text{ mm for diameter 7.00 mm}$$

The contact lens

$$s_1 = 0.841 \text{ mm for diameter 7.00 mm}$$

TLT

$$\text{TLT} = 0.841 - 0.808 = 0.033 \text{ mm}$$

Thus the TLT has increased from 0.01 mm with a BOZR of 7.90 mm to 0.033 mm with a BOZR of 7.70 mm.

Along 90

The TLT is 0.033 mm.

AEC

The cornea
As before

$$r_o = 7.30 \text{ mm} \qquad p = 0.8 \qquad y_2 = 4.25 \text{ mm}$$
$$x_{90} = 1.335 \text{ for diameter 8.5 mm}$$

The contact lens
The overall sag of the contact lens is $s_o = s_1 + s_2 - s_3$.

$$\begin{aligned} s_1 &= 7.7 - \sqrt{(7.7^2 - 3.5^2)} && = 0.841 \text{ mm} \\ s_2 &= 10.5 - \sqrt{(10.5^2 - 4.2^2)} && = 0.899 \text{ mm} \\ s_3 &= 10.5 - \sqrt{(10.5^2 - 3.5^2)} && = 0.601 \text{ mm}\end{aligned}$$

Therefore

$$s_o = 1.139 \text{ mm for diameter 8.5 mm}$$

AEC along 90

$$\begin{aligned}\text{AEC} &= \text{TLT} + x_{90} - s_o \\ &= 0.033 + 1.335 - 1.139 \\ &= \mathbf{0.229\ mm}\end{aligned}$$

Thus the AEC along 90 remains unchanged when the BOZR is decreased from 7.90 to 7.70 mm.

This example illustrates that fitting steeper than alignment on the flatter meridian does not in fact reduce the edge clearance on the steeper meridian. It could be argued that this approach should be discouraged since it produces a lens which is flat on one meridian and steep on the other with no advantage as far as edge clearance is concerned.

If we consider, for a moment, a spherical bicurve contact lens sitting on a spherical cornea with an apical clearance fit, it must be obvious that decreasing the BOZR will increase the TLT but will leave the AEC unaltered if the peripheral radius is not changed because the lens contacts the cornea at the BOZD transition in both cases. The situation is exactly the same on a toric cornea, as shown by the example above. **Figure 7.15** illustrates that the increase in the TLT is of no consequence to the AEC because the lens is resting on the transition along the flat meridian in an apical clearance fit. The only way to alter the AEC is to change the contact lens peripheral radius. It will be instructive to repeat the example with an alternative lens design where the peripheral radius is steepened by the same amount as the BOZR.

A cornea with vertex radii of 7.90 mm along 180 and 7.30 mm along 90 and a p-value of 0.8 for both meridians is fitted with a bicurve lens of the following specification

C2 7.90:6.50/8.90:9.00

(a) What is the AEC along 90?
(b) Also calculate the AEC along 90 for the contact lens

C2 7.70:6.50/8.70:9.00

(a) Using **Figure 7.15**

C2 7.90:6.50/8.90:9.00

Along 180

The cornea

x_{180} = 0.693 mm for diameter 6.50 mm

The contact lens

s_1 = 0.699 mm for diameter 6.50 mm

TLT

$$\begin{aligned}\text{TLT} &= 0.699 - 0.693\\ &= 0.006 \text{ mm}\end{aligned}$$

Along 90

The cornea

$$x_{90} = 1.512 \text{ mm for diameter } 9.00 \text{ mm}$$

The contact lens overall sag

$$s_o = 1.306 \text{ mm for diameter } 9.00 \text{ mm}$$

AEC

$$\begin{aligned}\text{AEC} &= 0.006 + 1.512 - 1.306\\ &= \mathbf{0.212\ mm}\end{aligned}$$

(b) Using **Figure 7.15**

C2 7.70:6.50/8.70:9.00

Along 180

The cornea

$$x_{180} = 0.693 \text{ mm for diameter } 6.50 \text{ mm}$$

The contact lens

$$s_1 = 0.720 \text{ mm for diameter } 6.50 \text{ mm}$$

TLT

$$\begin{aligned}\text{TLT} &= 0.720 - 0.693\\ &= 0.027 \text{ mm}\end{aligned}$$

Along 90

The cornea

$$x_{90} = 1.512 \text{ mm for diameter } 9.00 \text{ mm}$$

The contact lens overall sag

$$s_o = 1.344 \text{ mm for diameter } 9.00 \text{ mm}$$

AEC

$$\begin{aligned}\text{AEC} &= 0.027 + 1.512 - 1.344\\ &= \mathbf{0.195\ mm}\end{aligned}$$

In this second example there is a reduction of AEC along the steeper corneal meridian when fitting steeper than alignment with the flat meridian. If the

steeper lens possesses a BPR, which is also steeper, then the edge clearance will decrease in both the flat and the steep meridians compared with that of the flatter lens. This decrease in edge clearance is obviously due to the decrease in the peripheral curve radius only. Therefore the recommendation, that the BOZR should be selected to be one-third from the flattest and two-thirds from the steepest in order to reduce the AEC on the steep meridian, cannot be supported. The BOZR might best be left as an alignment or slightly steep fit on the flat corneal meridian, with **only the peripheral radii steepened by an appropriate amount**. The central alignment fit along the flat corneal meridian will result in a larger area of contact in the central region of the lens. This is more satisfactory than the situation where the lens is a steep fit in the central region of the flat corneal meridian and is a flat fit on the steep corneal meridian where the lens will rest on two relatively small crescents situated on the flat corneal meridian.

Contact lens laboratories may not wish to release information concerning the specification of their peripheral radius/radii and diameters. The use of their proprietary design trial lenses will still allow independent fitting of the central and peripheral curves. When ordering we can request a lens with a BOZR that gives an alignment fit along the flat meridian for the central region of the lens but request that this is combined with the peripheral radius/radii of the trial lens that gives the best overall AEC.

There are two further points of information that can be illustrated with the following examples

B&L k-readings	7.80 mm along 180	7.20 mm along 90
	p-value 0.8 for both principal meridians	

The apical radii for the two corneal meridians are

7.769 mm along 180 7.171 mm along 90

If this cornea is fitted with the contact lens

C5 7.72:7.50/8.16:7.90/8.64:8.30/9.15:8.60/9.56:9.30

This lens will rest on the first transition so the contact diameter is equal to the BOZD and results in

AEL	= 0.109 mm	TLT	= 0.02 mm
AEC_{flat}	= 0.08 mm	AEC_{steep}	= 0.234 mm

The lens specification required to halve the AEC along the flat meridian without altering the TLT is

C5 7.72:7.50/7.99:7.90/8.26:8.30/8.53:8.60/8.73:9.30

This results in

AEL	= 0.069 mm	TLT	= 0.02 mm
AEC_{flat}	= 0.04 mm	AEC_{steep}	= 0.193 mm

These two lenses illustrate

(a) The decrease in AEC along the flat and steep meridians is the same when the peripheral curves are steepened.

AEC change along 180 = 0.08 – 0.04 = 0.04 mm
AEC change along 90 = 0.234 – 0.193 = 0.041 mm

This means that the decrease in AEC is limited to the point where the AEC is zero on the flat corneal meridian and if it was decreased to this point the lens would almost certainly be uncomfortable and unwearable.

(b) The decrease in AEL is the same as the decrease in AEC.

AEC change between the two lenses = 0.08 – 0.04 = 0.04 mm
AEL change between the two lenses = 0.109 – 0.069 = 0.04 mm

(c) The peripheral curve steepening required to halve the AEC along the flat meridian, in this design, is not a simple matter of decreasing the radii by some proportion of the corneal astigmatism. In the example above BPR_1 was decreased by about 0.2 mm, BPR_2 by 0.4 mm, BPR_3 by 0.6 mm and BPR_4 by 0.8 mm. This applies only to the design considered here but the result illustrates that the radius changes required are unlikely to be derived intuitively.

7.17 The relationship between a toric cornea and a toric back surface RGP lens

The fitting relationship between a toric cornea and a toric back surface RGP lens can be calculated in exactly the same way as illustrated in the previous section. The only thing to remember is that the TLT must be calculated using the meridian that gives the steepest fit (which should be the flat corneal meridian). The AEC can then be calculated for both meridians as above. If the fit is flat on both meridians then the TLT will be zero.

7.18 Front surface toric RGP lenses

If we consider an eye with a spherical cornea which is fitted with a spherical back surface RGP contact lens and is then subjected to an over-refraction, we may find that the best visual acuity is only achieved when a cylindrical element is added to the over-refraction. We have discovered the presence of residual or lenticular astigmatism which will need correcting.

If the back surface of the contact lens is made toric then this will serve to make the lens less comfortable, and it is difficult to imagine any reason for fitting a spherical cornea with a toric back surface RGP lens. The most obvious way of overcoming the problem is to order a toric front surface for the contact lens. Let us take an example:

Ocular refraction	–2.50 DS/–1.50 DC axis 90
Keratometry readings	7.80 mm along 180 7.80 mm along 90
Trial contact lens:	
C2 PMMA	7.80:6.50/8.30:9.00 BVP –2.00 DS
Over-refraction	–0.50 DS/–1.50 DC axis 90
Visual acuity	6/5

In this case the contact lens BVP to be ordered will be

–2.50 DS/–1.50 DC axis 90

A toric front surface will be required, and so it may be more appropriate to consider the positive sphere cyl form

–4.00 DS/+1.50 DC axis 180

This tells us that the front surface of the contact lens must incorporate a positive cylinder axis 180 and its surface power will be a little less than 1.50 DC (the contact lens is thick). Let us suppose the central thickness of the lens to be ordered is 0.2 mm and the refractive index of the material is 1.450. What front surface radii are required for this lens?

We can deal with the two principal meridians as though they were separate lenses and use the diagram in **Figure 7.16**.

Along 180

The BVP required for the contact lens for this meridian is –4.00 D. Thus in **Figure 7.16** the vergence L_4 must be –4.00 D.

$$\text{Back surface power } F_0 = \frac{-450}{7.8} = -57.69 \text{ D}$$

$$\text{Reduced thickness } t/n = \frac{0.2}{1.45} = 0.138 \text{ mm}$$

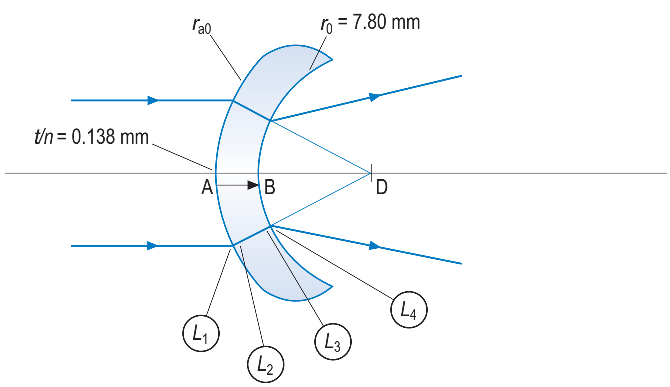

Figure 7.16 Diagram required for the calculation of the front surface radius

Vergence (D)			Distance (mm)
$L_4 = -4.00$			
$F_0 = \underline{-57.69}$ subtract			
$L_3 = +53.69 \longrightarrow$	$\frac{1000}{53.69}$	$\longrightarrow$	$18.625 = BD$
		add	$\underline{0.138} = t/n$
$L_2 = +53.30 \longleftarrow$	$\frac{1000}{18.763}$	$\longleftarrow$	$18.763 = AD$
$L_1 = \underline{0.00}$ subtract			
$F_{a0} = +53.30$			

$$r_{a0} = \frac{450}{+53.30} = 8.44 \text{ mm}$$

Along 90

The BVP required for the contact lens for this meridian is

$-4.00 +1.50 = -2.50$ D

Thus in **Figure 7.16** the vergence L_4 must be –2.50 D. The back surface power F_0 is, as before, −57.69 D and the reduced thickness is 0.138 mm.

Vergence (D)			Distance (mm)
$L_4 = -2.50$			
$F_0 = \underline{-57.69}$ subtract			
$L_3 = +55.19 \longrightarrow$	$\frac{1000}{55.19}$	$\longrightarrow$	$18.119 = BD$
		add	$\underline{0.138} = t/n$
$L_2 = +54.77 \longleftarrow$	$\frac{1000}{18.257}$	$\longleftarrow$	$18.257 = AD$
$L_1 = \underline{0.00}$ subtract			
$F_{a0} = +54.77$			

$$r_{a0} = \frac{450}{+54.77} = 8.22 \text{ mm}$$

Therefore the front surface radii for this lens are

8.44 mm along 180 and 8.22 mm along 90

This toric front surface is correcting the residual astigmatism and it is therefore imperative that the lens maintains its orientation. We must incorporate an element in the lens, which endows it with rotational stability. The popular solutions to this problem are to use:

(a) A ballast principle, which results in the heaviest part of the lens rotating to the 6 o'clock position. This is usually achieved by working a prism into the lens, which will then rotate to a vertical base/apex line orientation under the effects of the lids and of gravity. It is suggested by some that

the main influence on rotational stability arises from the lids and not from gravitational forces.

(b) Truncation, either single or double, which results in the straight edge of the truncation aligning with the general line of the lid margin. This solution works best with small palpebral apertures and tight lids particularly where the lower lid margin is higher than the lower limbus.

In the prism-ballasted lens, the maximum prism power likely to be comfortable is 3^{Δ} and it may be preferable to go for a slightly lower value of 2^{Δ}. It may also be necessary to consider combining ballasting with truncation in order to achieve an acceptable rotational stability. The double truncation may be useful with small palpebral apertures. Even then the lens may not behave as expected, i.e. the lens may not adopt an orientation with the prism base apex line vertical, or the truncations horizontal. It is therefore advisable to insert appropriate lenses in order to observe the likely orientation adopted by the lens on any particular eye.

7.18.1 *Specifying the orientation*

The specification of the orientation follows the same principles as those required for the orientation of soft toric lenses. Let us assume that we observe a contact lens that has settled to a truncation orientation of 15. The toric front surface of this lens will be calculated from the over-refraction measurements which are given in standard notation. However, the lens manufacturer must relate the principal meridians of the front surface to the truncation line which is expected to take up a horizontal orientation. Our lens has rotated 15° in an anticlockwise direction from this ideal. This serves to increase the orientation value (using standard notation) of both the truncation and the axis orientation of the prescription. If we wish to restore the toric surface back to a correct orientation we must reduce the axis orientation value in relation to the truncation line by 15°. If, for example, we required a lens of BVP

–2.00 DS / +1.50 DC axis 25

then we would order the +1.50 DC at axis 10 to the truncation line, since this itself is already at 15. Thus the rule for dealing with undesirable rotation is the same as that for soft toric lenses.

If the lens rotates anticlockwise, then subtract the degree of rotation from the standard notation orientation of the prescription cylinder. If the lens rotates clockwise, then add the degree of rotation to the standard notation orientation of the prescription cylinder.

The only problem that remains is that of accurately assessing the orientation of the truncation on the eye. This is best achieved by placing a well-adjusted trial frame on the face with a low powered full aperture cylindrical trial case lens clipped into the cylindrical cell. This type of lens is clearly marked with

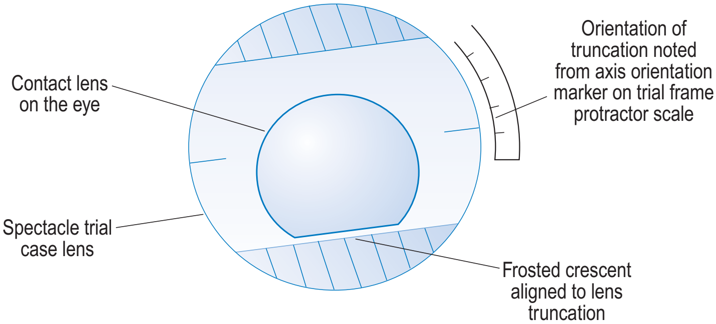

Figure 7.17 The method for assessing the orientation of the contact lens truncation

its axis, and also has two crescent-shaped frosted glass areas at the lens periphery whose straight edges are parallel to the cylinder axis. The cylinder is rotated until these straight edges are parallel to the truncation and the cylinder axis orientation can be read off the protractor scale on the trial frame. This is illustrated in **Figure 7.17**.

The above rule is also applied to a ballasted lens which will be marked either with the vertical ballast line or a line at right angles to this. The orientation of this line may be best observed using a table slit lamp and then rotating the slit until it is parallel to the marked line. The orientation of the slit can be assessed from the protractor scale of an empty trial frame, which the patient wears during this exercise.

Many modern slit lamps incorporate a protractor scale for slit rotation. The empty trial frame will not be required with this type of slit lamp.

Any unwanted rotation that occurs when the ordered lens is placed on the eye can be dealt with in exactly the same way as for soft lenses (see Section 7.10 onwards).

7.19 Summary

1. Corneal astigmatism is neutralized by the back surface of the fluid lens.
2. Residual astigmatism and lenticular astigmatism are synonymous for spherical RGP lenses.
3. The approximate rule linking a 0.1 mm change in radius to an 0.50 D change in fluid lens power can be used to advantage in the clinical situation, when you need to decide what power to order for a toric RGP lens.
4. RGP back surface toric lenses induce astigmatism and this will require a front toric surface to compensate for the induced astigmatism. The lens is then called a compensated toric.
5. The induced astigmatism in an RGP toric lens is a characteristic of the lens and is completely unconnected to the corneal astigmatism which is neutralized by the back surface of the fluid lens.

6. If we assume a refractive index for the contact lens material of 1.5, then the 3/2/1 rule gives the approximate ratios between
   (a) back surface cylinder in air.
   (b) cylinder in air required to correct the induced astigmatism.
   (c) induced astigmatism.
7. If a compensated RGP toric lens rotates during wear, the retinal image will remain clear.
8. Lenticular astigmatism requires a front surface toric and the lens must be rotationally stable.
9. If a fitting set lens rotates in a clockwise direction from the assumed orientation then add this unwanted rotation to the standard notation of the prescription cylinder axis found in the over-refraction, in order to ensure an appropriate cylinder axis orientation on the eye.
10. When the ocular Rx cylinder and the soft lens cylinder both have the same power but the lens takes up an unwanted orientation

$$\text{power of the induced cylinder} = \text{power of the ocular cylinder} \times \text{unwanted rotation} \times 2$$

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CHAPTER

8

Correction of presbyopia by contact lenses

Tony Hough

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8.1 Presbyopia

Presbyopia will start to produce symptoms associated with an inability to perform near work in the fifth decade of life. It has already been noted in Chapter 1 that myopic patients will require more accommodative effort when wearing contact lenses with hyperopes requiring more accommodative effort with a spectacle correction. Thus myopic patients in their early forties may be perfectly happy with the visual performance when wearing spectacles but could well have difficulty performing near work in contact lenses that fully correct their distance vision.

In the early stages of presbyopia it may be worth considering 0.50 D of extra positive power added to the distance prescription. This will produce a slight blur in distance vision but will significantly improve near vision performance in the early presbyope.

Some patients may be happy to accept full distance correction by contact lenses with a pair of reading spectacles provided for near work. The reading spectacle lenses will have a power equal to the reading addition. This solution always works but some patients object to the use of a pair of spectacles in combination with their contact lenses. In these patients it may be worth considering the monovision option.

8.2 Monovision

The monovision concept is a simple one. The patient is corrected for distance in one eye and for near in the fellow eye. Usually the dominant eye is corrected for distance with the non-dominant eye corrected for near. The patient will need to adapt to the new situation where the brain must learn to suppress the non-dominant eye in distance vision and to suppress the dominant eye in near vision. There is little problem with adaptation if monovision is instigated in early presbyopia when the power of the reading addition is low. There may be significant difficulties in attempting a monovision solution for the first time when the reading addition required is over 2.00 D.

It must be noted that the normal variation in vision associated with contact lens movement on the eye will be more of a problem due to eyes operating monocularly. There will be no cover from the fellow eye as occurs in a normal binocular correction. This means that the over-refraction result must be performed with the utmost care to ensure that the distance prescription is as accurate as possible. The near prescription may benefit from a slightly higher reading addition due to the non-dominant eye having to perform near work monocularly.

If neither a pair of reading spectacles nor monovision are acceptable to the patient then bifocal contact lenses will need to be considered.

8.3 The fundamental working principles for bifocal contact lenses

The fundamental working principles for rigid gas permeable (RGP) bifocal lenses are the same as those for the soft lens. The only important difference between the behaviour of the two lens types is that RGP lenses are more likely to move around on the anterior eye more than a well-fitting soft lens. RGP lenses will be used in this section to illustrate the principles. Any exceptions or additions that apply to soft lenses will be noted as appropriate.

8.3.1 Front surface solid bifocals

8.3.1.1 The centre distance bifocal

Figure 8.1 illustrates the form of a front surface solid bifocal contact lens. The central area of the front surface consists of a curve of greater radius of curvature (flatter) than that of the peripheral front surface. This results in a more positive power for the contact lens periphery. If the lens centres well on the eye when the primary position is adopted, then the central area can be used for distance vision. When the eye is depressed and converged in order to perform close work, then the visual axis of the eye is likely to pass through the more positive lens periphery. Thus the lens is expected to perform in a

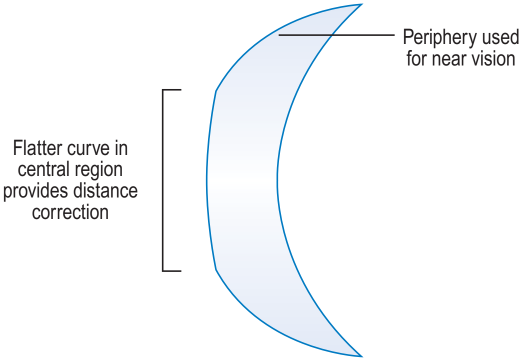


Figure 8.1 The front surface solid bifocal. The Wesley–Jessen design. This is a centre distance bifocal

manner similar to the conventional spectacle lens bifocal and this type of contact lens is referred to as an alternating (or translating) bifocal design. Translating designs work best with RGP lenses but do not generally work well with soft lens designs due to their lack of movement (lag) on the eye.

Let us suppose that a reading addition of +2.00 D is required for a particular eye and that the radius of curvature of the central area of the front surface of the contact lens is 8.45 mm and the material refractive index is 1.490.

$$\text{Power of central front surface} = \frac{490}{8.45} \text{ for a PMMA lens}$$
$$= +58.00\text{ D}$$
$$\text{power of peripheral front surface} = +58.00 + 2.00$$

(this assumes that the addition is measured on the surface that carries the segment)

$$= +60.00\text{ D}$$

Therefore

$$\text{radius of the peripheral front surface} = \frac{490}{60}$$
$$= 8.17\text{ mm}$$

This assumes that paraxial theory can be applied to the lens periphery. We can use thick lens theory to determine the power of the addition at the lens back vertex. If we assumed that the finished lens had a central thickness of 0.2 mm then the BVP of the addition would be +2.06 D. We can conclude from this that, for typical corneal contact lens parameters, the effect of lens thickness can be ignored. There are, however, two significant problems associated with this type of bifocal. The first is that the small difference in radius of curvature between the two portions of the front surface makes this lens difficult to manufacture. The second is that tear fluid on the front surface of the lens may alter the power of the addition as shown in **Figure 8.2**, where a negative tear

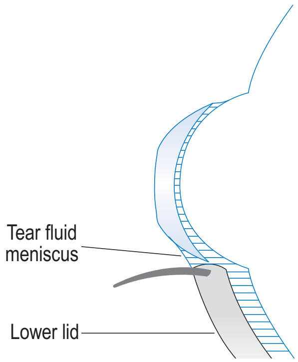


Figure 8.2 A fluid meniscus lens on the front surface of the contact lens reduces the power of the addition on the eye

meniscus has formed between the contact lens and the lid margin when the eye is depressed for near work.

Some practitioners therefore request a larger reading addition, and the extra positive power recommended can be as high as 1.00 D. The greatest problem associated with this feature is that the effect is variable and dependent on the characteristics of the tears and the palpebral aperture, which makes prediction of an optimum addition unreliable. This bifocal can be described as a centre distance bifocal.

8.3.1.2 The centre near bifocal

An alternative approach to front surface solid bifocal design is seen in **Figure** 8.3 where the central area of the lens has the most positive power. This is a centre near bifocal. In this design the lens is fitted to centre well for all positions of gaze. When the eye accommodates and converges for near, these changes will be accompanied by constriction of the pupil. It is likely in these circumstances that the pupil will be restricted mainly to the central area of the contact lens. In distance vision the larger pupil allows images to be formed by both the distance and near portions of the lens. The wearer must learn to ignore the out-of-focus image which arises from the central (near) portion in distance vision. The simultaneous presence of both the distance and near images has given rise to the term simultaneous (or bivision) bifocal. The Williamson–Noble design, illustrated in **Figure** 8.3, is a simultaneous vision bifocal. As already stated, the Wesley–Jessen design is described as an alternating bifocal but in reality there will probably be a simultaneous vision element in any alternating design.

The problems of manufacturing the lens and the requirement of a higher reading addition due to tear meniscus formation are much the same for the Williamson–Noble design as for the Wesley–Jessen. A tear meniscus will form around the anterior surface transition in the Williamson–Noble lens.

The centre near design of bifocal lens requires a lens that centres well on the cornea and does not move from this centred position when the eye looks

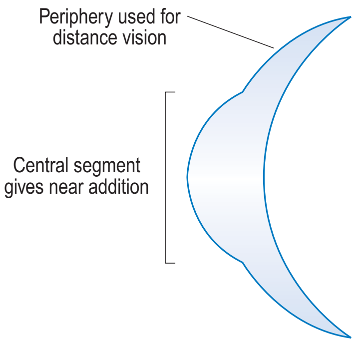


Figure 8.3 A centre near bifocal design. The Williamson–Noble bifocal

in secondary and tertiary positions. Soft lenses are more likely to fulfil this design requirement.

8.3.2 *Back surface solid bifocals*

An alternative to the front surface bifocal is to work the segment on the back surface of the lens. This design avoids the problem of addition power variation due to tear meniscus formation by generating the segment on the contact lens back surface. Thus the segment is in contact with fluid and not with air when on the eye. The difference in radius of curvature between the distance and near portions required to achieve the addition is considerably greater than with front surface bifocals, making the manufacture of the lens a less daunting proposition. The back surface solid bifocal is an alternating (centre distance) design. **Figure 8.4** illustrates the form of a typical back surface design. It is immediately apparent that the central segment is providing the distance correction.

Let us take a typical example to see how the lens is constructed.

The contact lens specification is

C2 7.90:7.00/8.90:9.00 BVP –5.00 D

We require a back surface solid bifocal with an addition of +3.00 D. The refractive index of the material is 1.490. What radius of curvature is required for the segment? What is the power of the addition in air?

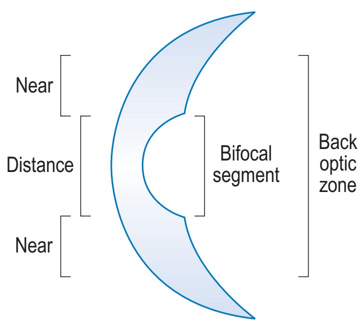


Figure 8.4 A back surface solid bifocal. The De Carle design

This lens would be manufactured as a single vision lens with a BVP equal to the power of the near prescription (–2.00 D in our example). The lens would then be changed to a bifocal by working the distance segment into the back surface.

On the eye

The power on the eye of a surface of radius 7.9 mm is

$$\frac{(1336 - 1490)}{7.9} = \frac{-154}{7.9} = -19.49\text{ D}$$

Since we need a +3.00 D addition, the bifocal segment must be more negative by 3.00 D. Therefore

power of bifocal segment surface on the eye $= -22.49$ D

radius of curvature of this surface $= \dfrac{(1336 - 1490)}{-22.49}$

$= 6.85$ mm

In air

power of surface of radius 7.9 mm in air $= \dfrac{-490}{7.9} = -62.03$ D

power of surface of radius 6.85 mm in air $= \dfrac{-490}{6.85} = -71.53$ D

therefore power of the negative addition in air $= 9.50$

The large difference in the two powers makes lens checking easier than with the front surface solid bifocals.

We have, in this treatment, ignored the fact that the back vertex of the distance and near curves do not exactly coincide but this does not influence the answer in any clinically significant way. If we considered the bifocal segment diameter in our example to be 5 mm, then the back vertex position will differ by 0.0665 mm between the distance and reading surfaces. The segment on the eye will be around 0.10 D more negative than the result calculated above where the shift of the back vertex of the segment was ignored.

We can deduce that the power of the addition in air will be approximately three times that on the eye. When checking these lenses the power of the addition in air will have to be calculated. This can be simplified to the relationship derived below.

The power of the back surface of the contact lens in air $= \dfrac{1 - n_p}{r_0}$

where n_p is the refractive index of the contact lens. If this lens is placed on the eye then the back surface interfaces with the tears and the back surface power is reduced.

$$\text{The power of the back surface of the contact lens on the eye} = \frac{n_t - n_p}{r_0}$$

where n_t is the refractive index of tears.

$$\text{Ratio of these two powers} = \frac{1 - n_p}{n_t - n_p} = \frac{n_p - 1}{n_p - n_t}$$

$$\text{Power of add in air} = \frac{n_p - 1}{n_p - n_t} A \qquad (8.1)$$

where A is the power of the add required on the eye. If we take n_p as 1.490 and n_t as 1.336, the relationship reduces to

power of add in air = 3.18 × power of add required on the eye

The bifocal types described so far can all be called concentric bifocals because in every case the segment is a circular one with its geometric centre at the geometric centre of the contact lens.

8.3.3 *Fused bifocals*

The fused bifocal is an alternating design which has the appearance of a spectacle bifocal lens in that the segment is a D, B or crescent shape, positioned in the lower part of the lens, with the segment material possessing a higher refractive index than that of the main lens. **Figure 8.5** illustrates the form of the fused bifocal. The manufacturer must calculate the radius of curvature of the depression curve r_{dc}, i.e. the curve of the interface between the segment and the main lens.

Let us take an example. The distance specification of the contact lens is

C2 7.90:7.00/8.90:9.00 BVP –5.00 D

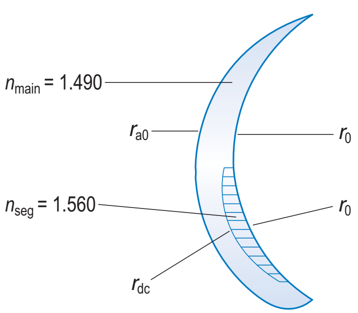


Figure 8.5 The fused bifocal contact lens

We require a fused bifocal with an addition of +3.00 D. What is the radius of curvature of the depression curve? What is the power of the contact lens in air? Assume the refractive index of the main lens is 1.490 with the refractive index of the segment being 1.560.

The distance BVP is of no direct consequence in this problem. All we need to concern ourselves with is the contact lens BOZR (r_0 = 7.90 mm).

On the eye

Power of the segment back surface $= \dfrac{(1336 - 1560)}{7.9} = -28.35$ D

Power of the main lens back surface $= \dfrac{(1336 - 1490)}{7.9} = -19.49$ D

Difference between these two $= 28.35 - 19.49 = 8.86$ D

Thus, the back surface power of the segment is 8.86 D more negative than the main lens, and we require the segment to be 3.00 D more positive than the main lens. Therefore the power of the bifocal depression curve must be

$8.86 + 3.00 = +11.86$ D to give the addition requested

radius of the depression curve $r_{dc} = \dfrac{(1560 - 1490)}{+11.86} = 5.90$ mm

N.B. The segment has been assumed to be thin.

In air

The difference between this lens and the back surface solid bifocal is that, for the fused bifocal, the back surface curve r_0 is the same for both the distance and near portions. Therefore the power of the fluid lens is the same for both distance and near. If we consider the back surface powers in air

power of the main lens back surface $= \dfrac{-490}{7.9} = -62.03$ D

power of the segment back surface $= \dfrac{-560}{7.9} = -70.89$ D

difference between these two $= 70.89 - 62.03 = 8.86$ D

This is exactly the same power difference as that found on the eye. Thus the power difference between distance and near portions has not changed from that on the eye. The depression curve radius r_{dc} will be the same as before, providing a power of +11.86 D, and so the addition in air is

$+11.86 - 8.86 = +3.00$ D

We can therefore see that the power of the add on the eye is the same as the power of the add in air for this type of bifocal. This makes focimeter checking of the power of the addition in air straightforward.

The segment is usually shaped something like an ophthalmic spectacle bifocal lens D segment. This means that the contact lens must be rotationally stable in order to maintain a correct orientation on the eye, and this is usually achieved by prism ballasting. The segment shape ensures that the optical centre of the segment is very close to the segment top. There is, therefore, little or no jump at the segment top in this design. The curved top of a crescent-shaped segment helps to maintain an acceptable performance even if the lens orientation varies and also reduces the image jump still further.

8.3.4 The diffractive bifocal

The diffractive bifocal was developed by Professor Mike Freeman (1986) and was marketed as both RGP and soft lens designs. In order to deviate light by diffraction, we need a structure on the optical surface which is fine enough to interact with the wavelength of the incident light. The finer the structure, the greater the deviation of the light. Note that this will result in red light being deviated more than blue (blue is deviated more by refraction) because the longer wavelength of red light means that the structure is finer by comparison. To deviate light of wavelength 555 nm through an angle of 20°, a furrow which is 1.5 μm in depth is required. In a diffractive lens the furrows take the form of circles or zones. The design manipulates the light into two images to create a simultaneous vision bifocal contact lens. The zero order (undeviated) image is used for distance vision and the first-order (deviated by a single wavelength phase difference at each zone boundary) image is used for near vision. In the RGP design, an addition value of +1.00 D requires 6 zones for the 5 mm aperture. This increases to 11 zones for an add of +2.00 D and 17 zones for an add of +3.00 D. The manufacture of such fine structures onto the surface of a contact lens is not a simple matter. As with the refractive bifocal designs already described, a more consistent optical performance is obtained when the diffractive surface is the posterior contact lens surface. The basic appearance of the diffractive bifocal is as shown in **Figure 8.6**.

In any simultaneous vision concentric bifocal it is essential that the central region covers the pupil at all times and this is relatively easily achieved in the soft lens design. For the RGP lens, the recommendation was to go for an apical clearance fit (slightly steep fit) in order to avoid contact between the diffractive ridges and the cornea and to encourage the lens to centre well. It must be noted that the circular ridges are only a few microns high and so represent less disturbance to the surface than a light scratch.

8.3.4.1 Comparisons between the diffractive and the refractive bifocal

(a) The distance and near images in the diffractive bifocal share the incident light equally no matter what the pupil size. This does not occur in the

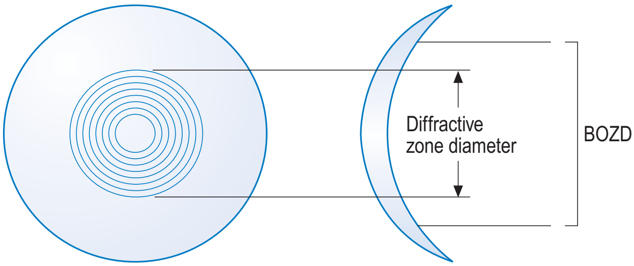


Figure 8.6 The diffractive bifocal

alternating refractive bifocal, where the ideal pupil will cover the distance region of the lens over perhaps ¾ of the pupil diameter in distance vision and around ¾ of the near vision region in near vision. This means that the out of focus image is formed by light passing through the remaining part of the pupil (¼ of the pupil diameter). When the pupil diameter changes from this hypothetical ideal, the proportions of light forming the clear and out-of-focus images will change, making the out of focus image easier or more difficult to ignore. This state of affairs inevitably results in a compromise in that if the distance vision is poor, due to the out of focus near image being difficult to ignore, then this could be improved by arranging for the distance region of the contact lens to cover more of the pupil. However, this improved distance performance will inevitably be accompanied by a deterioration of the near performance due to the near region of the lens now covering less of the pupil. If we consider the centre near simultaneous vision refractive bifocal, a small pupil diameter for near allows the near image to dominate and gives excellent near vision. However, problems arise for distance vision due to the narrow annulus used to form the distance image. An eye with a larger pupil will give better distance vision but the near vision will be poorer. Thus refractive bifocal lens performance results in a series of compromises that change with pupil size variation. The lens performance will, therefore, inevitably vary from eye to eye and in any given eye as the lighting conditions change.

In the diffractive bifocal, the distance and near images are derived from a 50:50 split of the incident light, no matter what the pupil size, providing the pupil is totally contained within the central diffractive region. It is, therefore, worth noting that this is an important criterion in the fitting requirements. Thus, bifocal vision extends over a wide range of illumination levels because the even split is maintained for all normal pupil sizes.

(b) The fact that the image for both distance and near vision, in the diffractive design, is formed by light passing through the entire area of the pupil means that the out of focus image is more out-of-focus than in the

refractive bifocal. The refractive bifocal uses part of the pupil aperture for distance and part for near, which means that the out-of-focus image is produced by a bundle of light rays passing through an aperture smaller than the pupil diameter. This leads to a pinhole effect (greater depth of focus) which helps to maintain the clarity of the unwanted image. Also, it is well known that the conventional diffraction limited image is degraded for small aperture sizes and this includes narrow annular apertures. The diffractive bifocal has been described as a full aperture bifocal and this means that not only is the out-of-focus image easier to ignore but the in-focus image detail resolution is better due to the larger aperture being utilized. The retinal image quality with different bifocal designs has been discussed by Charman and Walsh (1986). In a conventional concentric bifocal the out-of-focus image points may have an annular shape which changes to a crescent with decentration. In the diffractive bifocal, all out of focus image points are discs even with decentration. If the lens centration is good, the clear image will mask the blurred image. If the lens decentres, a ghost image may become apparent, particularly at near, and the position of the ghost image indicates the direction of the decentration. If the image is in a low position then the contact lens is riding low and this applies to all lens powers. If there is a binocular lateral displacement, e.g. temporal decentration in both eyes when the eyes converge to perform near work, then the two displaced images may be perceived as a single image further away than the near work.

(c) In the diffractive bifocal, the reading image is created by the diffractive interaction of the incident light and inevitably suffers from considerable chromatic aberration. This is reversed chromatic aberration (red light deviated more than blue) and is of the order of about –1.00 D for a +3.00 D addition. This tends to correct the intrinsic chromatic aberration of the eye which is about +1.00 D. However, a few patients have complained about coloured fringes with these lenses.

8.3.5 *Aspheric designs*

There is one final general design type that utilizes the aspheric surface in order to produce a lens with a progressive power variation from the centre of the lens to the edge. This type can be subdivided into front surface aspheric and back surface aspheric designs.

8.3.5.1 Front surface aspheric

If the front surface of a contact lens is a prolate (flattening) ellipsoid, then the most positive power will be present at the lens centre and the power will become increasingly negative towards the lens periphery. This design assumes that the small pupil induced by accommodation and convergence

will ensure that the most positive part of the lens is used for near vision. For distance vision, the increase in pupil size ensures that the more negative regions of the contact lens play an increasingly important role in retinal image formation. Thus this lens is a simultaneous vision, progressive lens not dissimilar to the centre near concentric bifocal already described. This type will be more suited to soft lenses which centre well on the eye and do not decentre with eye movements.

One suggested explanation of their use with the presbyope is associated with reducing the spherical aberration of the eye thereby improving the visual acuity of the presbyope. It has been estimated that the tolerance to blur circles for the human eye, where spherical aberration has been removed, is around 2.00 D. Therefore, if the lens power was 1.00 D more positive than the distance correction, the best focus with no blur will be for objects at 1 m from the eye, with the range from infinity down to 50 cm representing the limits of acceptable focus dictated by the tolerance to blur circles. This explanation is unlikely to be satisfactory when all the influences on ocular aberrations are considered.

8.3.5.2 Back surface aspheric

Once again the prolate ellipsoidal surface is used but on the back surface where the central part of the lens aperture will possess the most negative power. The lens periphery will be more positive. It will be necessary for this lens to ride high when the visual axes are depressed for close work and this is unlikely to happen with a soft lens. This lens is similar to the centre distance concentric bifocal but should provide a progressive increase in the reading addition. Typically the addition power offered by manufacturers is around 1.50 D.

8.4 Soft bifocal contact lenses: the limits of performance

It will now be useful to consider the problems of modern soft bifocal lens design in more detail. All bifocal contact lenses must take account of on-eye movement and lens position in order to function successfully when worn. With the exception of diffractive designs, all current soft and rigid bifocals rely on lens position in order to obtain their on-eye bifocal, multifocal or varifocal functionality. The on-eye dynamics of rigid lenses are quite different to soft lenses. Typically, rigid lenses will move both during normal eye excursions and on down gaze by up to 2 mm while soft lenses will move by perhaps only 0.25 mm (Little and Bruce, 1994; Young, 1996). The predictable, normal movement of rigid lenses enables bifocals to be designed which have a reading zone that is separate from the distance zone, even with aspheric designs. Although recent work by Chauhan and Radke (2001) suggest that the current generation soft lenses may move significantly during the closed

lid phase of blinking, they confirm that during open eye and, by implication, during down gaze, that soft lenses remain relatively static. Certainly, all current designs for bifocal products use the fact that modern, thin soft lenses drape onto the cornea and remain relatively static to obtain their bifocal functionality.

This presents lens designers with a seemingly irresolvable dilemma. All of the optical functionality for soft bifocals must be included within the central optic zone and must be related to current pupil diameter. This basic fact may mean that it will never be possible to develop soft bifocal contact lenses which will match spectacles or rigid contact lenses in terms of their visual functionality.

In order to illustrate comparative visual performance we will compare the *Acuvue Bifocal* and *Focus Progressives* with a leading rigid lens bifocal.

8.4.1 The market for presbyopic correction

The design and product development of soft bifocal contact lenses is driven by a relatively small number of American based corporations who control the finance and product research and development. Evidently, the burgeoning number of 'baby boomers' in the USA who are now presbyopic is central to the development process for this product range. The target end-user for bifocal soft lenses is probably aged 50–65 and probably residing in technologically developed countries like the USA, the original 15 countries of the European Union ('EU15') or Japan. A review of population trends in the UK and EU15, shown in **Figure 8.7**, indicates that people in the age group 50–65 currently constitute a significant market – about 10 million in the UK and 66 million in the EU15. European data confirm that this number is set to grow significantly over the next 20 years to levels of some 13.5 million in the UK and 85 million in the EU15 (population data obtained from *Eurostat Yearbook*, Edition 2001).

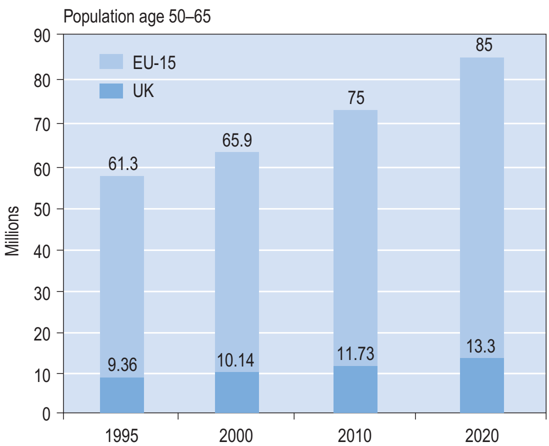


Figure 8.7 Predicted population of 50–65-year-olds in the UK and EU15 to 2020. (*Source: Eurostat Yearbook* 2001)

This is a large and growing market which the contact lens industry clearly needs to think about when it is developing new products. Also, many surveys which ask practitioners what they would most like to see from the contact lens industry identify *good bifocal contact lenses* as the main item on practitioners' wish lists.

Despite the large and growing market, the current reality is that only a tiny proportion of presbyopic vision correction is by means of contact lenses. The Eurolens Research annual survey (Morgan and Efron, 2001) of wearing trends in the UK for 2001 indicated that about 5% of new soft lens fits were multifocal with a similar percentage for soft lens refits. New fits of rigid lens multifocals accounted for 2% of new fits and 10% of refits. Overall, if we take soft and rigid lens multifocals together it is estimated that this modality accounts for less than 1% of vision correction in the UK. Data from other markets are no more encouraging. We must ask why the usage of multifocal contact lenses is so low. Probably the primary reason is what might be described as the prescribing culture. Practitioners lack the confidence and the competence to deliver this modality to their patients. Additionally, however, it is clear that although product pricing, delivery and reproducibility are currently good, the visual performance of current products is lacking compared to spectacles.

8.4.2 *Understanding the product capability*

Before fitting complex contact lenses it is essential to understand their clinical capabilities. We need to know what this product can do for patients, how it achieves this and how best to apply the product. If a product works well for a domestic worker, will it work equally well for someone who works with computers? This level of understanding is skilled and requires experience in order to deliver it to a mixed group of patients.

A clear grasp of power characteristics is central to understanding all bifocal contact lens performance. Even lenses having similar characteristics may well have markedly different clinical visual performance characteristics. For example, both the *Essilor Rhythmic* and *CIBA Vision Focus Progressives* could correctly be described as centre near varifocal devices but their visual performance and clinical management/delivery are very different.

In almost all cases the focimeter is of little or no use in determining the power characteristics of these lenses. In most cases it is impossible to credibly determine the power even at the centre using a focimeter. It is essential to use a power mapping device, of which there are just two commercially available at this time: the *Rotlex Contest*, which uses a Moiré fringe method, and the *Visionix 2001* which utilizes the Hartmann–Schack method in which a matrix of 'micro lenses' is used to determine the power at many points on the lens being measured and then mathematical algorithms are used to reconstruct a power map of the complete lens.

Both instruments are capable of measuring soft lenses immersed in saline with a fair degree of credibility. To date, there are no data from the international standards group dealing with contact lenses on the acceptability of these instruments when used to measure bifocal soft lenses. Both instruments can provide maps and profiles of multifocal contact lenses. As an example, **Figure 8.8(a)** shows the power map for a *QuasarPlus* rigid multifocal lens and **Figure 8.8(b)** shows the power profile along a selected meridian.

The poor reproducibility and problematic service records of these instruments has led to their usage being very limited in the manufacturing industry; typically they are used to measure samples or for quality management. The relatively high cost (the cost of 20 focimeters), difficulty of use and relative skill of interpretation of the results mean that their current usage in clinical practice is just about nil. It is therefore a practical impossibility to measure with any confidence the powers of current soft bifocal contact lenses in a normal practice setting.

8.4.3 Spherical aberration

A good understanding of spherical aberration of the cornea and typical lenses is at the core of understanding the clinical performance of most current bifocal contact lenses. It is, therefore, appropriate to begin this analysis with a review of some basic principles. The direct calculation of longitudinal spherical aberration involves the comparison of the paraxial image distance for an object point to the image distance for the entire lens aperture. In its simplest form, this is shown schematically in **Figure 8.9**. In this case, the system of lens, tears and cornea is more positive marginally than paraxially.

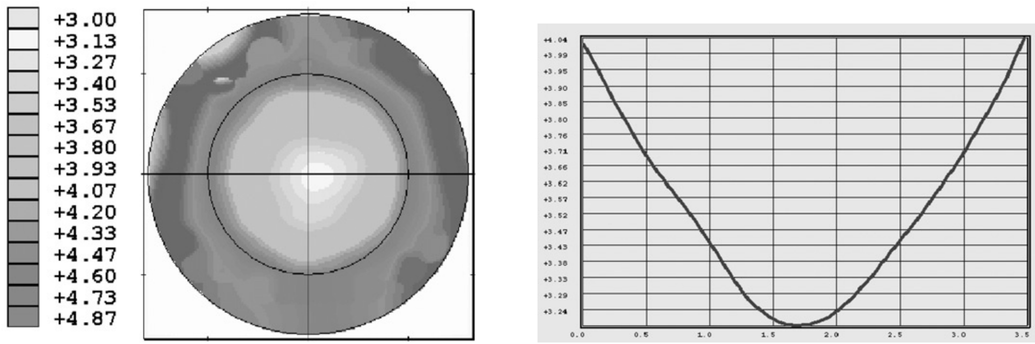
In all cases where contact lenses are worn, the cornea itself forms an important part of the optical system of lens, tears and cornea. A basic understanding of corneal spherical aberration is essential to understand the likely clinical performance of:

(a) rigid aspheric multifocals
(b) soft aspheric bifocals (i.e., most current products)
(c) rigid aberration control lenses
(d) soft aberration control lenses which are nowadays often, rather oddly, referred to as *aberration blocking* lenses.

This latter category is assuming increasing importance in the development of current soft contact lenses.

The shape of the cornea will determine its level of longitudinal spherical aberration. There are many studies relating the longitudinal spherical aberration of typical corneas to contact lens wear (Charman *et al.*, 1978; Charman and Saunders, 1990; Cox, 1990; Cox and Holden, 1990; Thibos et al., 1997).

In order to represent and set up reasonably accurate models of the longitudinal spherical aberration of the cornea and related contact lens systems, a computer model which will be referred to as a ‘differential power map’



(a) (b)

Figure 8.8(a) The power map of a *QuasarPlus* rigid varifocal contact lens measured on a *Visionix 2001* (Plate 5). (b) The power profile along a single meridian

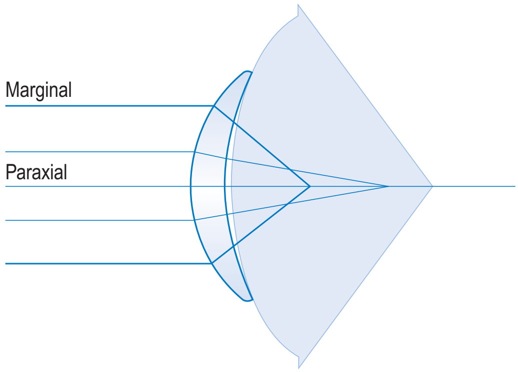


Figure 8.9 Schematic representation of the longitudinal spherical aberration of the optical system comprising lens, tears and cornea

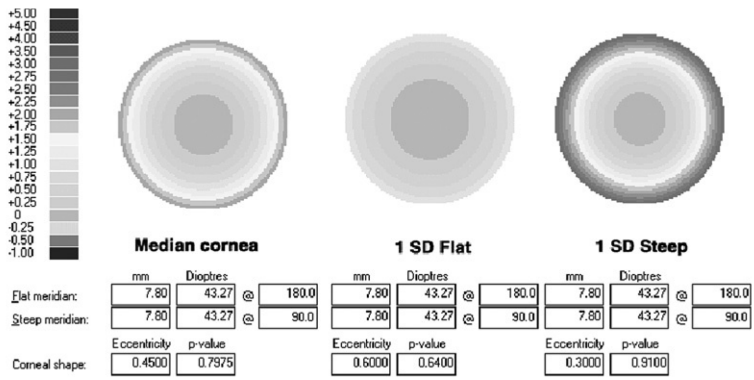


Figure 8.10 The spherical aberration of the cornea varies dependent on the corneal asphericity. All three corneas have the same apical radius. (a) The median cornea (p-value 0.79) shows moderate spherical aberration. (b) This cornea has a p-value one standard deviation less than the median cornea. The increased asphericity has reduced the spherical aberration. (c) This cornea has a p-value one standard deviation greater than the median cornea. The decreased asphericity results in a greater amount of spherical aberration than that present in the other two corneas (Plate 6)

has been developed. This model enables the power at each point on a target optical system to be plotted relative to the power at the centre of the element or system. The model system can be a single element, such as a lens or a corneal surface, or a combined system, such as the lens, tears and cornea. A typical scale is shown on the left hand side of **Figure 8.10**, remembering that we are primarily interested in positive spherical aberration as we are discussing bifocal correction.

Examples of differential power maps depicting corneal spherical aberration are shown in **Figure 8.10(a–c)**. These show how the spherical aberration of corneas having the same vertex radius will vary depending on the corneal shape. **Figure 8.10(a)** shows a cornea having a median value of numerical

eccentricity (e = 0.45, p = 0.8) and **Figure 8.10(b)** shows a cornea that is more aspheric (e = 0.60, p = 0.64). The increased asphericity means that the corneal curvature will flatten more rapidly as we move from the apex to the limbus and this reduces the positive spherical aberration seen in **Figure 8.10(a)**. **Figure 8.10(c)** shows a cornea having a numerical eccentricity one standard deviation less aspheric (e = 0.30, p = 0.91). This almost spherical cornea shows more positive spherical aberration than either of the other two corneas. In all cases, the vertex radius is the same but the level of spherical aberration varies significantly between the three corneas.

Then, remembering that positively powered contact lenses exhibit positive longitudinal spherical aberration while negatively powered contact lenses exhibit negatively powered longitudinal spherical aberration, it is obvious that any rigid bifocal which limits itself to a single aspheric geometry will behave anomalously when worn on a representative population of eyes. Similarly, any *aberration blocking* soft lens which is designed to correct or reduce the longitudinal spherical aberration of the lens in air using, say, an aspheric front surface, will also behave anomalously when worn.

For example, if one takes a standard spherical rigid lens having a nominal power of –5.00 D and places it on a normal cornea, the resulting system of lens, tears and cornea will have uniform power, to within 0.25 D, across the optic zone, as shown by the differential power map in **Figure 8.11**. The negative spherical aberration of the lens has cancelled the positive spherical aberration of the eye. If the contact lens had already been *aberration controlled* then we would have introduced spherical aberration to a system which would ordinarily have none. It is therefore vital to understand the underlying optics in order to understand the potential applications of aberration controlled lenses and to be able to apply rigid lens bifocal products sensibly.

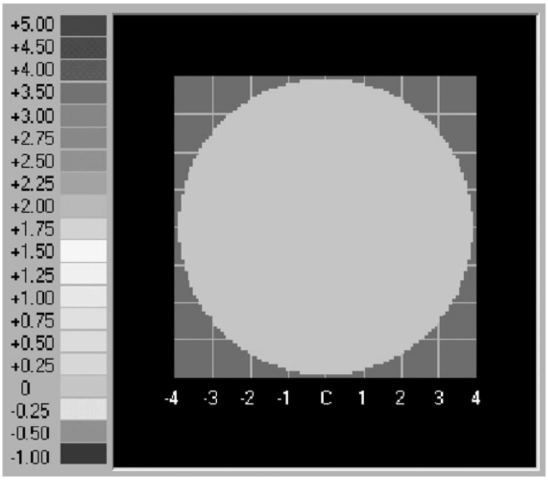


Figure 8.11 Differential power map of a normal cornea wearing a standard spherical rigid contact lens having a nominal power of –5.00 D. The power of the lens, tears, cornea system is uniform to within 0.25 D across the optic zone (Plate 7)

8.4.4 *Rigid lens bifocals: QuasarPlus – a case study*

In order to set down a reference for soft bifocal performance it is appropriate to quantify the clinical performance and expectations for a current rigid lens bifocal. The purpose of this short case study is to identify the essential differences in optical performance as compared to soft bifocals and to clarify our expectations as to what we can reasonably achieve with soft bifocal contact lenses.

The *QuasarPlus* bifocal, which is manufactured by No. 7 Contact Lens Laboratory Limited, has been chosen because market data have shown that it is a popular rigid lens bifocal design in the UK. Additionally, the product design is patented (United States Patent 6,390,624 B1. Hough, Contact Lens) so it is possible to obtain a detailed and clear description of the lens.

To order a *QuasarPlus*, the practitioner provides details of keratometry and spectacle prescription to the laboratory which then manufactures lenses based on these data. *QuasarPlus* is a varifocal device having no distinct distance or reading zones. The design strategy is to use a realistic predictive computer model of the clinical data to devise an aspheric back surface geometry which will modify the spherical aberration of the combined optical system of lens, tears and cornea to give a predictable varifocal effect when worn by the specified patient.

A typical example is shown in **Figures 8.12** and **8.13**. Here, the patient has a distance power of –3.00 D and a reading addition of +1.50 D. The pupil position in forward gaze and down gaze is simulated in the model and the

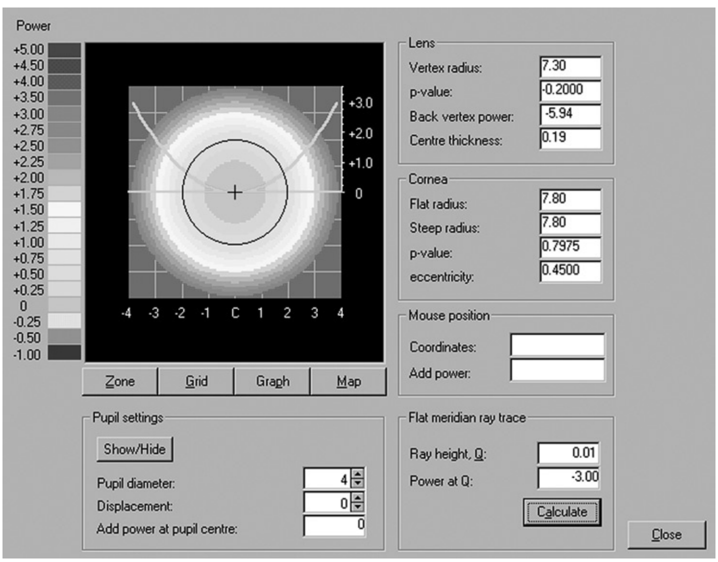


Figure 8.12 The *QuasarPlus* design process showing the power map when the pupil is centred in forward gaze (Plate 8)

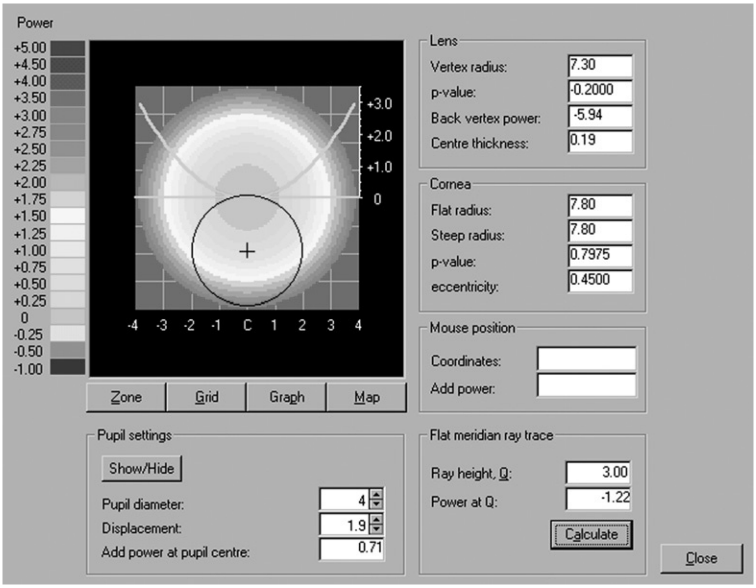


Figure 8.13 The same *QuasarPlus* lens showing the power map when the pupil is displaced on downgaze (Plate 9)

back surface geometry is calculated to give the required reading addition. A formal clinical trial has shown that the success of this approach is excellent (Woods *et al.*, 1999). Also, it is now established that typical wearers can expect all day wear for a variety of visual tasks. However, the product is effectively limited to rigid lens wearers and to practitioners who have the necessary skill, understanding and market orientation to deliver the product so it is likely to remain a small percentage of the overall contact lens market.

8.4.5 *Soft bifocal lenses*

A quick glance at any product guide will confirm that there are numerous bifocal products in the soft lens market. According to the UK *ACLM Contact Lens Yearbook* 2001, it is possible to categorize available products as planned replacement (up to one month) and conventional lenses.

Table 8.1 lists the leading planned replacement products and Table 8.2 lists a selection of the conventional products.

The market-leading products may be classified into three main functional groups:

(a) centre near aspheric, for example, *Essilor UV Rhythmic* Multifocal;
(b) adapted centre near aspheric, for example, *CIBA Vision's Focus Progressives*; and
(c) multiple zones, for example, *Acuvue Bifocal*.

Table 8.1 Planned replacement (up to 1 month) soft bifocal lens products available in the UK, 2001

Product	Design
Acuvue Bifocal	Multiple zones
Focus Progressives	Central power peak
Lifestyle MV2	Discrete zones
Occasions	Centre distance aspheric
Rhythmic UV Multifocal	Centre near aspheric
Source: ACLM Contact Lens Yearbook 2001	

Table 8.2 A selection of conventional soft bifocal lenses available in the UK, 2001

Product	Design
Echelon	Diffractive
ennovy Elite	Centre near/centre distance aspheric
N5	Centre near aspheric
PS45	Centre near aspheric
PSD	Centre near aspheric
Variations	Centre near aspheric
Source: ACLM Contact Lens Yearbook 2001	

The traditional centre near aspheric group now represents a somewhat historical design approach. There have been many embodiments, ranging from the ill-fated *PS45* to the *Essilor Rhythmic*. This design strategy is now at the end of its product life cycle. The performance/marketing difference between the many similar lenses is the shape of the power gradient – the balance of power distribution across the optic zone. No such product has or will offer genuine binocular vision; all rely on modified monovision to obtain their functionality, as even a brief review of the relevant fitting guides will confirm.

The *Acuvue Bifocal* and *Focus Progressives* lenses will be considered as currently popular designs. The *Focus Progressives* product is a single design which is available in two modalities; daily disposable and monthly disposable. The *Acuvue Bifocal* is available as a monthly disposable product.

It is important to note that the delivery system plays a vital part in the success of these products. Both feature a relatively complex design which is manufactured in very high volumes at low unit cost so that the cost throughout the supply chain is manageable and reasonable. Both processes demonstrate excellent reproducibility, a feature frequently lacking in previous generations of lathe cut products. A clear message emerges at this early stage for manufacturers. Bifocal soft lenses, as other soft lenses, must be deliverable within the current patient management culture which mandates the frequent replacement of lenses.

8.4.6 The design dilemma

The case study of the *QuasarPlus* rigid lens bifocal clearly demonstrated that with rigid lens designs – even aspheric designs – we have a separate definable zone which is accessed by the pupil when reading. In the case of soft lenses which drape onto the cornea and remain relatively static, all of the visual functionality must be contained within the current pupil diameter. Adding to this challenge, we find that pupil diameter varies as a function of light levels, age and visual tasks. Research has shown that in a medium lighting level of 50 candelas/m^2, pupil diameter for a 20-year old is typically 4.35 mm and that for a 60-year-old it is 3.45 mm (European Patent Application no. EP 0 756 189 A2).

The development of complex designs has highlighted the need to understand the relationship of pupil diameter to age and lighting levels. First, it was necessary to establish a reference set of lighting levels. A relevant European patent application identifies the levels set out in **Table 8.3** as a suitable reference.

As bifocal lenses will be targeting the presbyopic age group, the same patent application identifies the pupil size for the 40–60 age group in these lighting levels, as set out in **Table 8.4**.

We need to consider the basic question: ‘What percentage of the pupil do we need to cover with reading power in order for the wearer to read comfortably?’ Sadly, the literature does not have an answer. Gasson and Morris (2003), discussing segmented rigid lens bifocals, estimate that the coverage

Table 8.3 Lighting levels for common visual tasks

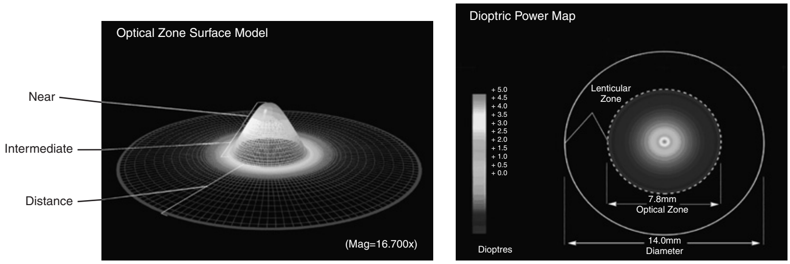
Light level (candelas/m^2)	Visual activity
2.5 (‘low’)	Distance: outdoors at night/driving at night
50 (‘medium’)	Near and distance: especially reading and office work
250 (‘bright’)	Distance: outdoors, bright sunshine

Source: European patent application no EP 0 756 189 A2

Table 8.4 Pupil diameters/lighting levels for the 40–60 age group

Light (candelas/m^2)	Pupil size (mm)	Std Dev
2.5 'low'	5.45	0.97
50 'medium'	3.65	0.57
250 'bright'	3.03	0.43

Source: European patent application no. EP 0 756 189 A2



Focus Progressives power characteristics (promotional literature)

Figure 8.14 *Focus Progressives* power characteristics as described in the company promotional material (Plate 10) (Source: CIBA Vision, Switzerland. Reproduced with permission.)

should be 75%. In a personal communication, Charman (2001) suggested that 40% is sufficient because at that level the brain could suppress the unwanted 'noise'. In the absence of published work, we will assume that pupil coverage of about 40% with reading power is sufficient in typical wearers for comfortable reading.

8.4.7 Current product analysis 1: Focus Progressives

Focus Progressives is a single design which features a central power peak – a modern adaptation of the earlier widespread centre near aspheric designs. The company promotional literature depicts the power distribution as shown in Figure 8.14.

It is difficult to obtain credible and usable power measurements of this lens. Using the *Visionix 2001* (0.25 mm resolution matrix), a typical power map for the *Focus Progressives* is shown in **Figure 8.15**. This identifies some of the many difficulties with this task. Because this thin lens is immersed in saline in a curvette during the measurement it is common to see 'islands' of power variation. These are best ignored.

Looking carefully, and discounting the islands, it is evident that the power map is consistent with the product description. Repeated measurements of the power profile identify the dimensions of the power peak for this product,

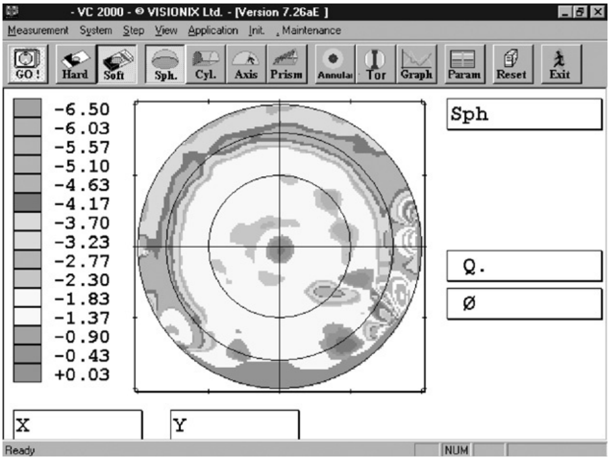


Figure 8.15 A Visionix 2001 power map of *Focus Progressives* lens (Plate 11)

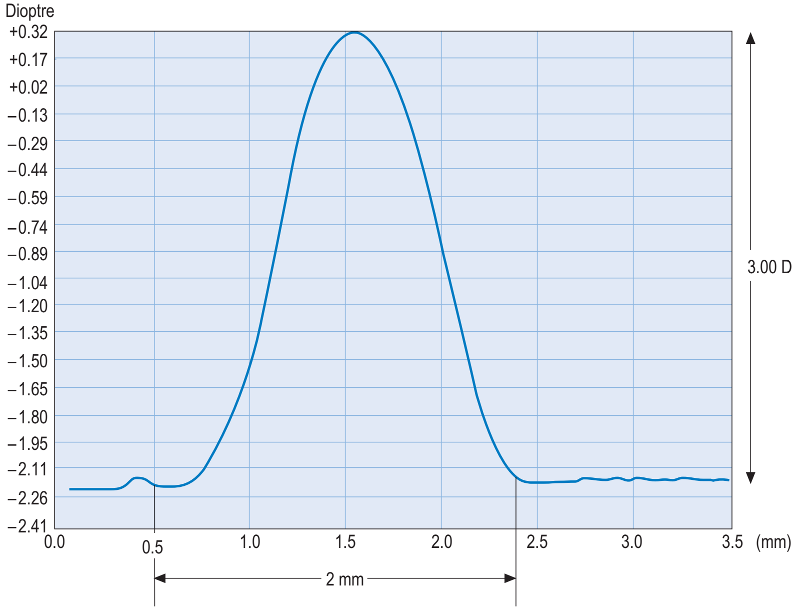


Figure 8.16 Measured dimensions of the *Focus Progressives* power peak

as shown in **Figure 8.16**. It is 2 mm wide and 3 dioptres high and has a shape which approximates closely to a normal bell curve distribution.

Using these data, a mathematical model was constructed to enable theoretical calculations of the visual performance of the lens when worn. The primary application of the mathematical model will be to establish the pupil coverage by distance and reading powers for a range of pupil diameters.

The model applied to a typical lens is shown in **Figure 8.17** from which it can be noted that it conforms very well with both the manufacturer's published data and the measured data.

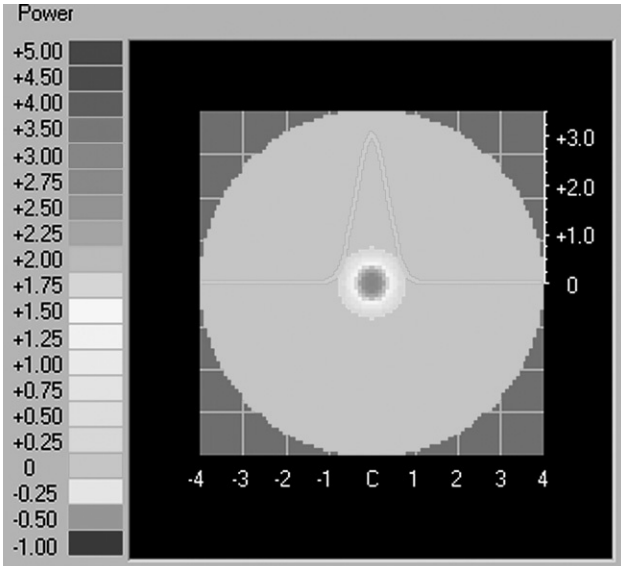


Figure 8.17 Mathematical model of the *Focus Progressives* power characteristics, showing both the simulated power map and power profile (Plate 12)

Table 8.5 Tolerances used to test the power distribution over a selected range of pupil diameters

Pupil power coverage is described as	If the power is within this tolerance of the nominal value
'Distance'	+0.25 D
'Reading'	–0.25D, +0.75 D
'Intermediate'	Neither of the above

An interactive computer model was developed which could calculate the power distribution across a pupil of any specified diameter. For this, and also for the subsequent study of the *Acuvue Bifocal*, the tolerances set out in Table 8.5 were used.

The specification of the tolerance for distance power was aligned with International Standards but the tolerance for reading power was generously set to one-quarter dioptre under the nominal value but up to +0.75 D more positive. Any power on the pupil which did not fall into this definition of reading or distance was identified in the pie charts as 'intermediate'. It is worth noting that some of this intermediate may be very useful for observing intermediate work.

To test the model, the following prescription was used

Distance power = –3.00D, reading addition = +2.00 D

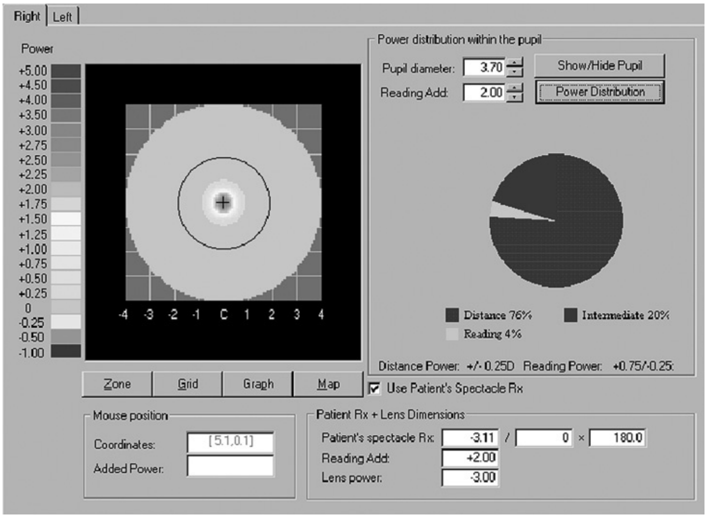


Figure 8.18 Modelling the on-eye power characteristics for *Focus Progressives* – the lens selected is the wearer's distance power (Plate 13)

The following pupil sizes were tested:

6.50 mm, corresponding to low light levels
3.70 mm, corresponding to medium light levels
2.40 mm, corresponding to bright light levels

It is important to note that *Focus Progressives* are not available in a nominal selection of reading additions; each lens is labelled with the distance power, then the central power peak is used to provide the reading addition. In the first case, the lenses assessed corresponded to the patient's nominal distance power, in this case –3.00 D. In other words, the hypothetical patient was using a lens where the labelled power corresponded exactly to their distance prescription (fully understanding that this is not what the manufacturer recommends).

A typical display from the model for *Focus Progressives* is shown in **Figure 8.18**. The results for this distance prescription lens are summarized in **Table 8.6**. It is evident that lenses selected in this way will not provide adequate reading power coverage at any light level.

This is recognized by the lens manufacturers who provide a nomogram to select the likely best first choice lens for a given prescription. When the nomogram lens is selected, a typical display of the model calculations is shown in **Figure 8.19**. The results for our selected light levels/pupil diameters are summarized in **Table 8.7**.

Evidently, if the nomogram lenses are used, the wearer's distance vision is effectively blurred to provide a variation on monovision. This is clearly a 'lifestyle' product.

Table 8.6 Analysis of power distribution for a range of pupil diameters using the *Focus Progressives* lens which is selected to match the wearer's distance correction

	Percentage of pupil covered by		
Pupil diameter (light level)	**Distance**	**Reading**	**'Intermediate'**
6.50 mm (dim)	93%	1%	6%
3.70 mm (med)	76%	4%	20%
2.40 mm (bright)	44%	9%	47%

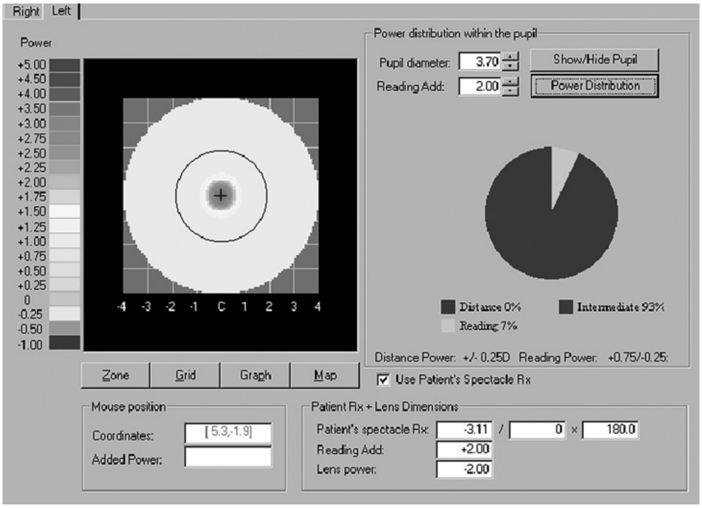


Figure 8.19 Modelling the on-eye power characteristics for *Focus Progressives* – the lens selected is according to the nomogram provided by the manufacturer (Plate 14)

Table 8.7 Analysis of power distribution for a range of pupil diameters using the *Focus Progressives* lens which is selected according to the nomogram provided by the manufacturer

	Percentage of pupil covered by		
Pupil diameter (light level)	**Distance**	**Reading**	**'Intermediate'**
6.50 mm (dim)	0%	2%	98%
3.70 mm (med)	0%	7%	93%
2.40 mm (bright)	0%	16%	84%

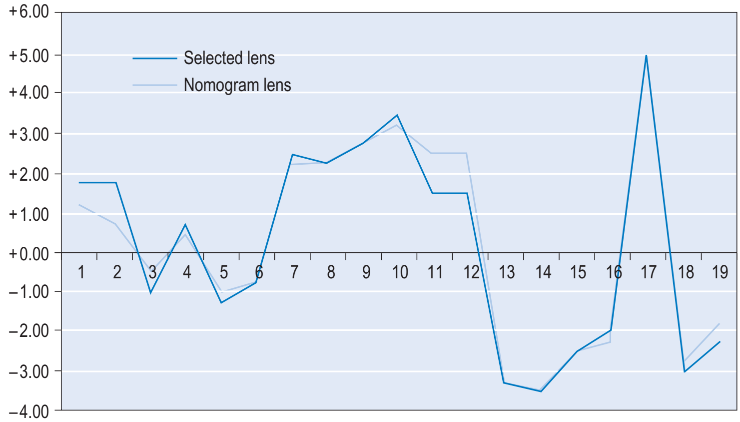


Figure 8.20 *Focus Progressives*: actual lens power worn in 19 eyes compared to the nomogram selected lens

8.4.8 *Real world micro study: Focus Progressives*

A group of 10 patients (19 eyes) who normally wear *Focus Progressives* in their daily lives were analysed to establish the relationship between the nominal lens power and their spectacle prescription. Of particular interest was the relationship between the actual lens worn and the lens identified by the nomogram as being most likely to succeed.

Remembering that the *Focus Progressives* design does not have a choice of reading additions, **Figure 8.20** compares the nomogram lens power to the power of the lens being worn. The actual lenses being worn show a remarkable degree of conformity to the nomogram power. However, if one looks at the distance power which would be reasonably expected to be used we note that in almost all cases the wearing eye is corrected with significantly excessive positive power as illustrated in **Figure 8.21**.

In summary, it is noted with *Focus Progressives* that the lens powers measure very consistently with the labelled values but that the central power peak does not provide sufficient reading addition in most cases. This is compensated for by the use of a nomogram which adapts the lenses worn to provide a modified form of monovision where the eye wearing the bifocal is blurred for distance.

8.4.9 *Current product analysis 2: Acuvue Bifocal*

The *Acuvue Bifocal* employs multiple concentric zones of alternating distance and reading power to create a bifocal device when worn. The dimensions and layout of the zones is described in marketing literature by the manufacturer as 'pupil intelligent' which is intended to convey the impression that the product is designed to provide a reasonable balance of distance and near

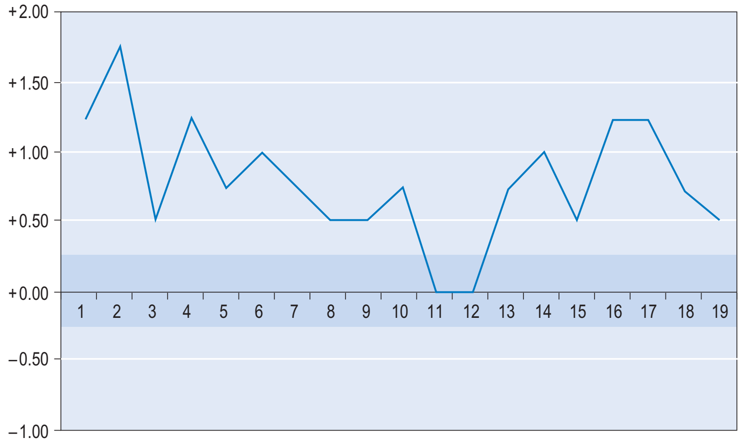


Figure 8.21 *Focus Progressives*: the difference between the wearer's distance prescription and the power of the contact lens in the 19 eyes. The positive values indicate that the distance vision will be blurred

powers over a range of lighting levels and pupil diameters. The central zone is intended to provide distance vision then zones alternate reading and distance. The pupil surface area is said to be sufficiently covered by distance or near power depending on pupil size (light levels) and the wearer's visual activity. The proposed functionality of the *Acuvue Bifocal* presumes this relationship between pupil size and visual activity but if the wearer needs to read small print in bright sunshine, for example, this lens will not work very well.

It is worth noting that underlying this design strategy (and that of the *Focus Progressives* product) is the notion that soft lenses when worn remain relatively static on the eye. If these lenses were to move continuously the wearer would experience very unstable, disturbed vision.

In the promotional literature, the *Acuvue Bifocal* identifies a series of pupil sizes related to the lighting levels just discussed and states the ratio of distance to near correction for each pupil size. These values are summarized in Table 8.8.

8.4.10 *Acuvue Bifocal: A mathematical model of power distribution*

Based on the values given in the promotional literature and relevant patent literature, a model of the *Acuvue Bifocal* zone diameters was calculated and is shown in **Figure 8.22**.

Further, this model was extended in a similar fashion to the model for *Focus Progressives* to enable the theoretical power distribution to be calculated for a given pupil size. A view of this model is shown in **Figure 8.23**. It is important to note that in the model the power of marginal ring zones is

Table 8.8 Ratio of distance to near power coverage given in the *Acuvue Bifocal* promotional literature

Light	Pupil size (mm)	Ratio D:N	Example activities
Dim	6.5	65:35	Night driving
Medium	3.7	50:50	Mixture/indoor
Bright	2.4	70:30	Outdoor

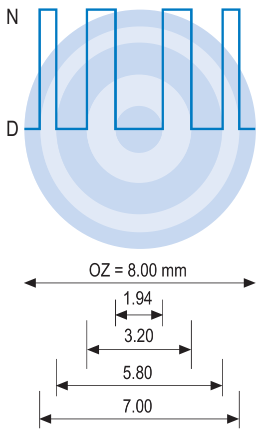


Figure 8.22 *Acuvue Bifocal*: layout of distance/near zones and the calculated dimensions of those zones based on public domain information

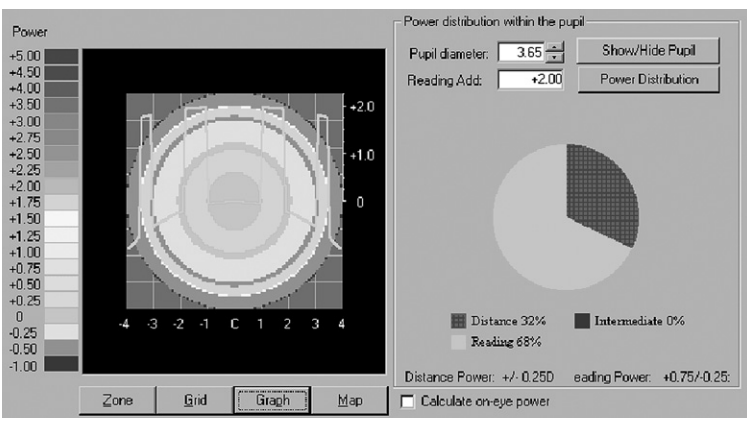


Figure 8.23 Computer model of the *Acuvue Bifocal* power distribution. Note the impact of spherical aberration on the marginal zones (Plate 15)

Table 8.9 Analysis of power distribution for a range of pupil diameters with the *Acuvue Bifocal*

Pupil diameter (light level)	Percentage of pupil covered by		
	Distance	Reading	'Intermediate'
6.50 mm (dim)	46%	35%	19%*
3.70 mm (med)	50%	50%	0%
2.40 mm (bright)	69%	31%	0%

*The intermediate power coverage is due to the spherical aberration of the outer power zones.

influenced by spherical aberration – the distance power steps exhibit an increasing negativity away from the paraxial region.

The degree of spherical aberration in the outer zones of the power model will obviously be influenced by the nominal lens power. To assess the power distribution over a variety of pupil sizes, the tolerances set out in **Table 8.5** were again used. Sometimes, depending on the lens nominal power, there will be some residual intermediate power caused by the influence of spherical aberration on the outer optic (power) zones. To test the theoretical optical performance of the *Acuvue Bifocal*, the same prescription (–3.00 D, +2.00 add) was tested at the same three pupil sizes as for *Focus Progressives*.

The results of the simulation are given in **Table 8.9**. In this case the simulated performance is very good, providing appropriate reading and distance cover at all three of the selected pupil diameters.

8.4.11 *Real world micro study:* Acuvue Bifocal

The case records of a group of 21 patients who normally wear *Acuvue Bifocals* were analysed to compare the powers of the actual lenses worn with the wearer's spectacle prescription. Unlike the *Focus Progressives* product, the *Acuvue Bifocal* is supplied in a range of reading additions so that in this case it is appropriate to compare both distance and reading powers with the wearer's prescription.

The relationship of nominal distance power to the wearer's prescription is shown in **Figure 8.24** and the reading power in **Figure 8.25**. Both show dramatic scatter which we will try to understand and explain.

The fitting guide for the *Acuvue Bifocal* is not so much a nomogram as is the case with *Focus Progressives* but more a trouble-shooting guide which sets out a series of remedial and adaptive steps that can be used by the practitioner to improve the visual performance of the product for each wearer. Undoubtedly, there will be wearer-related reasons for the scatter shown in **Figures 8.24 and 8.25** caused by the individual response to simultaneous vision and the variety

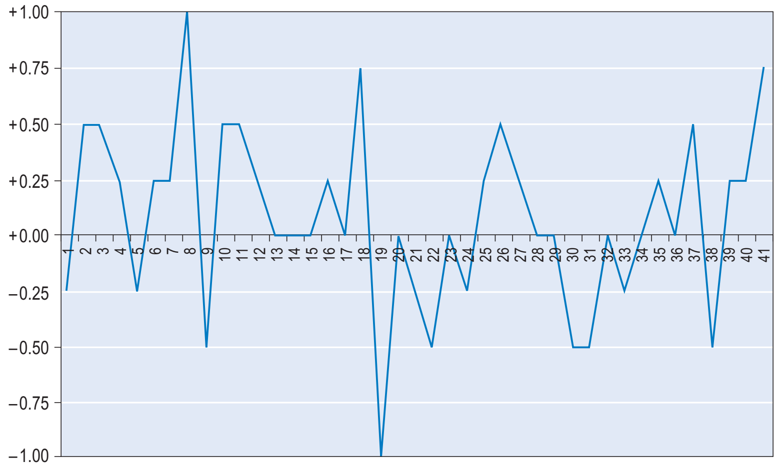


Figure 8.24 *Acuvue Bifocal*: the difference between the wearer's distance prescription and the power of the contact lens for a group of 21 wearers (42 eyes)

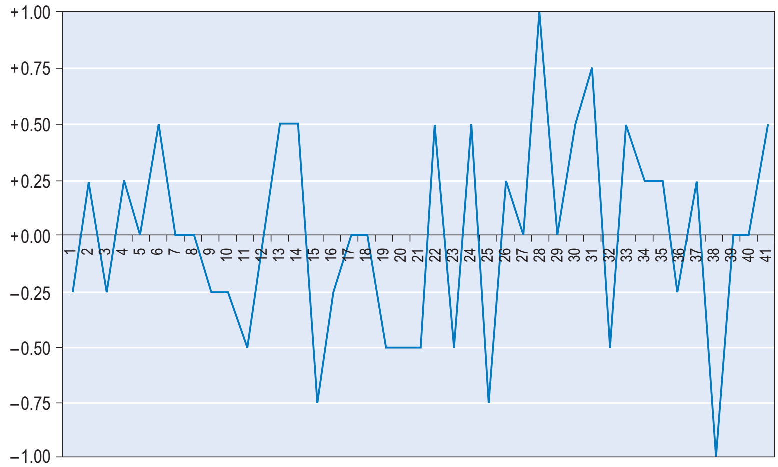


Figure 8.25 *Acuvue Bifocal*: the difference between the wearer's reading prescription and the power of the contact lens for a group of 21 wearers (42 eyes)

of visual tasks undertaken. Managing the personal preferences of each wearer is certainly a significant feature of successful fitting for this product. However, any objective technical review would reasonably conclude that the dramatic scatter in the nominal power values has more than wearer-related reasons – that it is also related to the power profiles of the real, as distinct from the theoretical, product. In other words, even a short study would reasonably conclude that the product being delivered has power profiles which are different to the theoretical values.

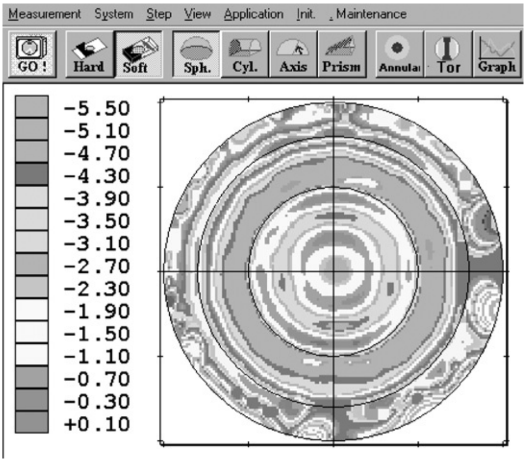


Figure 8.26 Power map of an *Acuvue Bifocal* lens having nominal powers of –3.00 D for distance with an add of +2.00 D (Plate 16)

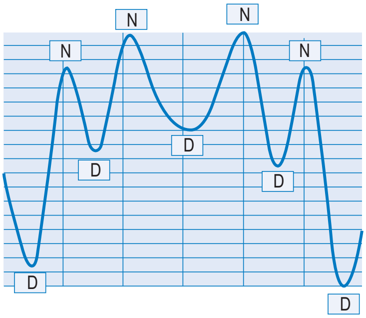


Figure 8.27 Power gradient of the *Acuvue Bifocal* lens shown in Figure 8.26 with distance (D) and near (N) zones identified

This is unsurprising when one looks at the requirements of the manufacturing relative to the dimensions involved. The *Acuvue Bifocal* is manufactured by means of moulding, in this case a patented process described as stabilized soft moulding. In order to obtain the very precise power distribution, zone dimensions and especially the transitions between zones would have to be near perfect. Any moulding process is subject to a degree of imprecision so we must conclude that the manufacturing process causes the transition quality to degrade by comparison to the theoretical model.

Again this is a difficult product to measure in a credible and consistent fashion. A power map of an *Acuvue Bifocal* having nominal powers of –3.00 D for distance with an Add of +2.00 D is shown in Figure 8.26. It would be very difficult for a practitioner to make any sense of this image, underlining the notion that it is pointless to try to measure these lenses in normal practice. Careful, repeat and independent measurements allow a skilled and experienced technician to derive a typical power profile which is shown in Figure 8.27. Here, the distribution of distance and near zones is evident but it is apparent that at this one-quarter millimetre resolution the step transitions of the theoretical product are not present on the end product.

The moulding process has the following steps:

(a) production of a mould master;
(b) production of individual moulds;
(c) forming of the stabilized soft lens;
(d) completion of the hydration process.

Each step will have an impact on the transition quality so that the final lenses will have transitions which are relatively imprecise by comparison to the theoretical model. Junction management is acknowledged to be difficult by lens manufacturers; there is at least one patent devoted to the design of transitions for this type of product. The scatter of powers worn is partly explained by the variability of transitions between power steps in the delivered product. This is shown graphically in **Figure 8.28**.

The influence of zone transition on pupil power coverage may be analysed by examining the loss of power in typical cases. Let us assume that we have a pupil size of 3.65 mm. If we have power zones exactly as in the *Acuvue Bifocal* theoretical model (in this case the relevant diameters are 1.94 mm and 3.20 mm) then this would provide reading power over 49% of the pupil area and distance power over 51% of the pupil area.

Let us now assume that due to the manufacturing process we lose an annulus around each of these zones. The impact of this for 'blur bandwidths' of 0.2 mm and 0.3 mm are shown in **Table 8.10**. It is therefore evident that small errors or blur bandwidths around the zone transitions can cause significant loss of pupil power coverage.

In summary, the measured powers of the *Acuvue Bifocal* have a different (but related) characteristic to the designed power profiles. The visual response to the product is complex and is adapted by a fitting to optimize for individual preferences. It is clear that the manufacturing process impacts on the intended visual performance.

8.4.12 Discussion

The visual performance of current soft bifocal contact lenses is lacking by comparison to both spectacles and rigid lens bifocals. To a significant extent,

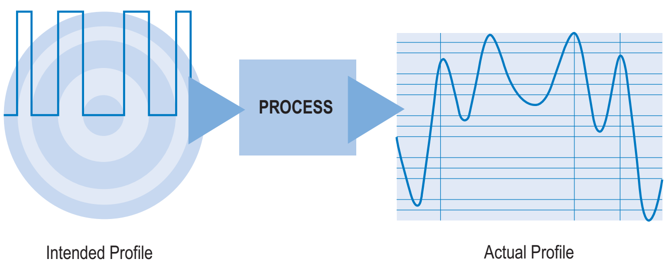


Figure 8.28 Comparing the *Acuvue Bifocal* theoretical power profile with the measured power profile

Table 8.10 The influence of transition blurring on power coverage for a pupil having a diameter of 3.65 mm

Pupil diameter = 3.65 mm	
Acuvue Bifocal zone diameters included in this pupil size: 1.94 mm, 3.20 mm	
Ratio of distance: near with no transition blur: 51:49	
Blur bandwidth	**Percentage of pupil coverage which is 'lost' by the blurring**
0	0%
0.20 mm	15%
0.30 mm	23%

this is due to the fact that modern soft contact lenses are virtually static during open eye wear; there is no predictable on-eye movement. This means that all of the bifocal functionality must be located inside a small central zone related to pupil size; this is naturally difficult to do. Current manufacturing technologies cannot cope with instantaneous transitions which will lead to some loss of pupil power coverage by comparison to the intended product specification.

From a manufacturer's point of view, any new design of bifocal soft lens, if it is to succeed in the mass market, must be capable of manufacture at very low unit costs and must be highly reproducible. These factors combine to suggest that new designs will emerge only very slowly over the next years, limited by current investment levels, stock level requirements and market conditions. It is evident that a very large investment would be required in marketing and practitioner training in order to deliver genuinely new designs to the market. This is a major deterrent to green fields research and development; large corporations will want to see the returns on their current investments and then review the position.

From a wearer's point of view, the current generation of soft bifocals have limited application by comparison to spectacles: '20 / happy' is probably as good as it gets!

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CHAPTER 9

Miscellaneous features

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9.4 Orthokeratology 302

There are a number of topics which have not yet been covered that are not extensive enough to warrant a separate chapter. These topics are included in this chapter which therefore inevitably contains sections which have little or nothing in common with their neighbours. The chapter starts by considering underwater lenses for SCUBA divers. This is followed by a discussion of the Galilean telescope, produced when a negative contact lens is combined with a positive spectacle lens to produce magnification for the visually impaired patient. The penultimate section deals with the correction of the aphakic/high hyperopic eye. Finally the topic of orthokeratology is discussed.

9.1 Underwater lenses

There are two basic approaches to the correction of vision under water. The first is to equip the scuba diver with a scleral lens on which is mounted an air cell. This is in effect a miniature face mask. The second approach consists of manufacturing a bifocal scleral lens which is sighted with one power for surface vision and the other for underwater vision.

9.1.1 The air cell lens

Figure 9.1 illustrates a typical air cell design. The flat plate and air cell ensure that light is travelling in air before being refracted by the contact lens optic. The lens therefore performs in exactly the same way as a conventional face mask. The flat plate is angled as shown in order to help make upper lid

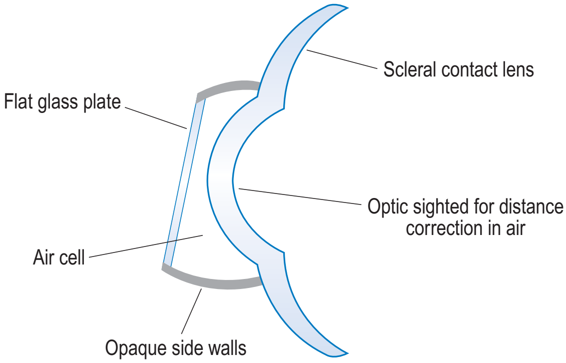

Figure 9.1 The air cell underwater contact lens

movements more comfortable and derive more lens support from the lower lid.

The wearer sees everything at the apparent distance under water, owing to refraction, from water to air, at the flat plate. The opaque side walls restrict the field of vision (but not the field of fixation). The field of view is further reduced under water by refraction at the flat plate. These changes are illustrated in **Figure 9.2** where the underwater eye is in an air cell provided by a face mask. **Figure 9.2(a)** illustrates the fact that images are formed closer to the eye under water than is the case when viewing objects in air. This produces the impression of everything looking larger under water. Note that the reduced apparent distance means that presbyopic problems are greater when viewing under water objects with a face mask. **Figure 9.2(b)** illustrates the reduced field of fixation for a face mask. A typical face mask will give a field around 50° from the primary position for both the nasal and temporal field in air. This is reduced to around 40° under water. The air cell contact lens will have a very small flat glass plate which moves with the eye and so the field of fixation is unchanged. However, the refractive effect illustrated in **Figure 9.2(b)** will produce a reduction in the field of view through the air cell contact lens.

In the case of the air cell contact lens, the plate angle must be very similar in the two eyes if diplopia is to be avoided under water. If either lens rides lower or higher than its fellow, then this will induce a tilt of the flat plate which will result in vertical diplopia under water.

The air in the air cell must be absolutely dry if internal condensation is to be avoided. Also the air cell may collapse or leak at depth due to the increase in water pressure. The pressure increases by one atmosphere for every 10 m of depth.

9.1.2 The bifocal lens

This was originally suggested by Bennett (1965). **Figure 9.3** illustrates the Douthwaite (1971) design which is fitted in order to discourage lens lag. The

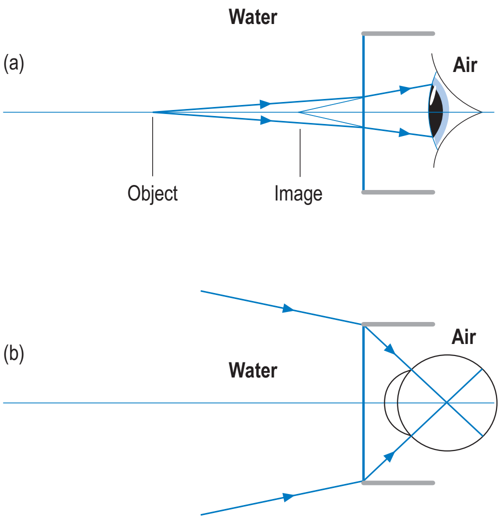

Figure 9.2 (a) The effects of refraction at the water/air boundary result in images being formed closer to the eye than the distance of the objects. (b) The effects of refraction at the water/air boundary serve to decrease the field of view of the air cell face mask

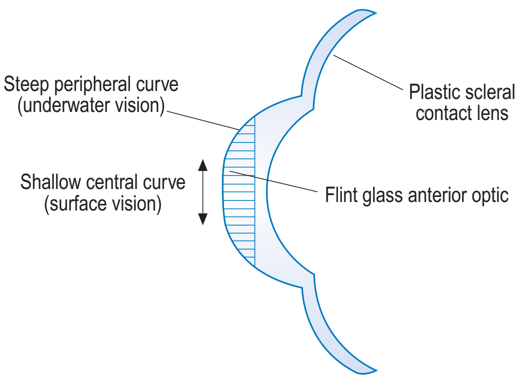

Figure 9.3 The bifocal underwater contact lens

low levels of light under water allow the large pupil to utilize the peripheral area of the flint glass button which is bonded to the scleral lens with an epoxy resin. The higher light levels on the surface produce pupil constriction which allows the central section of the button to dominate the image formation. This design is free of the disadvantages listed for the air cell lens. However, the problem of how the pupillary region shares the image formation encountered with conventional bifocals still applies. The surface vision may be improved by using a larger diameter surface section but this will induce a deterioration in the quality of the underwater vision. The opposite is true for a reduced diameter surface section.

The front surface radii of the flint glass button are easily calculated following the methods described in Section 8.3.1. The design might be further improved by moving the shallow curve zone into the lower half of the flint button, since divers tend to look up towards their foreheads as they swim in

a prone attitude (face down) under water and yet on the surface they tread water in a vertical position with their heads well back which results in them looking down their noses in order to scan the horizon.

9.2 The low vision aid telescope

The Galilean telescope is used as a spectacle-mounted magnification device for the visually disabled patient requiring help for distance, intermediate or near work. The distance system is illustrated in **Figure 9.4** where a distant object h, standing on the optical axis, subtends a visual angle ω at the telescope objective lens O which is separated from the eyepiece lens E by distance d. The diagram illustrates that the incident pencil of light rays coming from the top of the object emerges from the eyepiece still as a parallel pencil but one which subtends a visual angle of ω'. **Figure 9.4** also illustrates that, for a telescope set for distance viewing, the principal foci F_1' (objective), and F_2 (eyepiece) coincide. The light rays from the distant point source (the upper point of the object) would be focused in the plane of F_1', if the eyepiece was removed. Thus the light incident on the eyepiece is converging on to its principal focus F_2 and this ensures that the light emerges as a parallel pencil.

The magnification can be defined as the ratio of the visual angle of the object ω to the visual angle of the image seen through the telescope ω'. Therefore

$$\text{magnification} = \frac{\omega'}{\omega}$$

$$\tan\omega = \frac{h'}{f_1'} \qquad \tan\omega' = \frac{h'}{f_2}$$

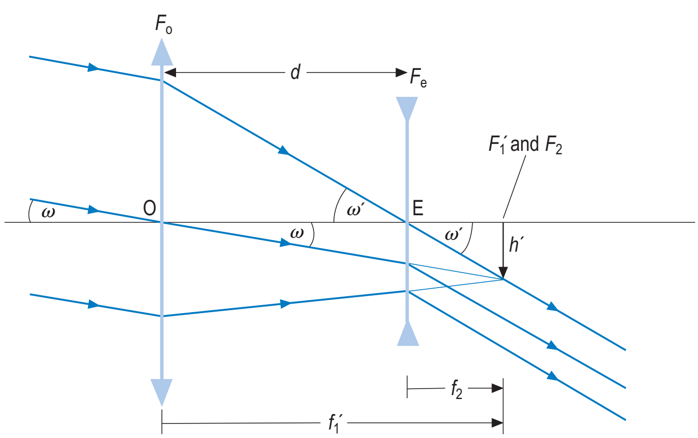

Figure 9.4 The Galilean telescope

Therefore

$$\text{magnification} = \frac{h'/f_2}{h'/f_1'} = \frac{f_1'}{f_2} = \frac{F_e}{F_o}$$

where F_e is the dioptric power of the eyepiece (ignoring its sign) and F_o is the dioptric power of the objective when f_1' and f_2 are measured in metres. Therefore

$$\text{magnification} = \frac{\text{power of eyepiece}}{\text{power of objective}} \tag{9.1}$$

Also

$$f_2 = f_1' - d$$

Therefore

$$\text{magnification} = \frac{f_1'}{f_2} = \frac{f_1'}{f_1' - d}$$

$$\text{magnification} = \frac{1}{1 - d/f_1'} = \frac{1}{1 - dF_o} \tag{9.2}$$

where F_o is again the dioptric power of the objective and d is the separation between the objective and eyepiece in metres.

The cosmetic appearance of a conventional telescope can be dramatically improved if it is made up by using a positive spectacle lens as the objective and a negative contact lens as the eyepiece. This initially attractive idea, however, rapidly becomes less attractive when consideration is given to the fact that the lens separation d will be the vertex distance of the spectacles and will of necessity be small. This means that the lens powers must be large, and even then the magnification achieved will be modest. Let us take a typical example.

A +25.00 D spectacle lens is to be glazed into a spectacle frame of vertex distance 14 mm. What power is required for a contact lens to act as the eyepiece for a Galilean telescope, assuming the eye is emmetropic and requires to view distant objects? What magnification will be produced by this device?

In Figure 9.5

power of the objective = +25.00 D

Therefore

$$f_1' = \frac{1000}{25} = 40 \text{ mm}$$

$$d = 14 \text{ mm}$$

Therefore

$$f_2 = f_1' - d = 26 \text{ mm}$$

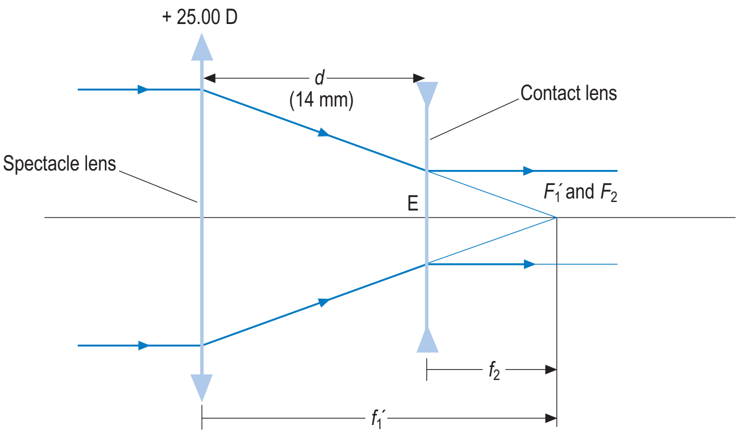

Figure 9.5 The contact lens/spectacle lens telescope

Therefore

$$\text{power of the eyepiece} = \frac{1000}{26} = 38.46\ \text{D}$$

$$\text{Magnification} = \frac{F_e}{F_o} = \frac{38.46}{25} = 1.54\times$$

The problems become immediately obvious. The high plus spectacle lens will not be particularly good from a cosmetic point of view. The high minus contact lens will be difficult to fit and may never be particularly comfortable. Any movement of the contact lens will produce apparent movement of the visual field since the contact lens is now the eyepiece of a telescope. Finally, as with all Galilean telescopes, the field of view will be significantly restricted.

9.2.1 *Field of view*

Strictly speaking, we must be most concerned with the field of fixation. To find this we simply need to trace a limiting light ray from the edge of the spectacle lens to the centre of rotation of the eye and determine the angle.

The light ray in **Figure 9.6** is a limiting light ray that goes to R, the centre of rotation of the eye. Let us assume that the distance from R to the contact lens is 13 mm and we also assume that we are dealing with the telescopic system of the previous example, where $d = 14$ mm. Let us also assume that the objective diameter is 32 mm, i.e. $y = 16$ mm. Now

$$\tan\theta = \frac{16}{13 + 14} = 0.5926$$

$$\theta = 30°\ 39'$$

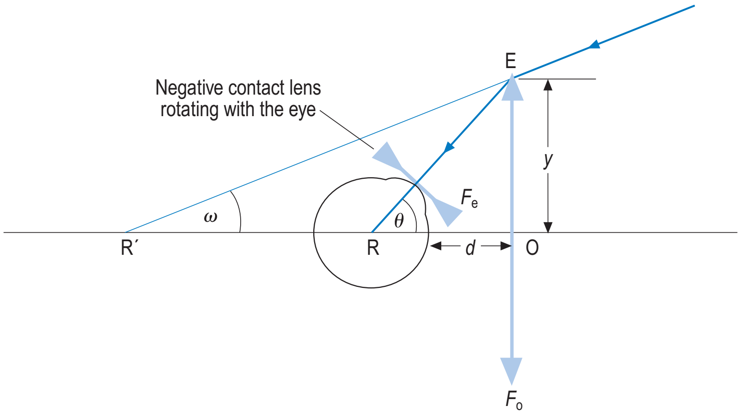

Figure 9.6 The field of fixation for a contact lens telescope. The contact lens is assumed to maintain a well-centred position on the cornea. R is the centre of rotation of the eye, 2ω is the true field of fixation

The patient's apparent field of fixation is given by the angular subtense of the lens at the centre of curvature of the eye.

The eye needs to rotate $2\theta = 61°\ 18'$ in order to look from edge to edge of the apparent field of fixation.

If we now assume that R is an object point for the +25.00 D spectacle lens, the divergent incident light rays will be refracted by the lens to give an emergent vergence that is less negative. Thus the image of R will be formed at R′.

$$\text{Object distance RO} = -27\text{ mm}$$

$$\text{Giving an object vergence of } \frac{1000}{-27} = -37.04\text{ D}$$

$$\text{Objective power } F_o = \underline{+25.00\text{ D}}$$

$$\text{Therefore image vergence} = -12.04\text{ D}$$

$$\text{Therefore distance R'O} = \frac{1000}{-12.04} = -83.06\text{ mm}$$

$$\tan\omega = \frac{16}{83.06} = 0.1926$$

$$\omega = 10.9°$$

$$\text{True field of fixation } 2\omega = 21°\ 48'$$

9.2.2 The Telecon system

Figure 9.7 illustrates the Telecon system devised by Filderman. The contact lens has a central circular zone of diameter 2.5 mm which is sighted with a BVP of –50.00 D. The objective is mounted at a vertex distance of 20 mm, and consists of a plano-meniscus carrier with a +25.00 D lenticular aperture cemented onto the carrier. This provides a system which allows for image

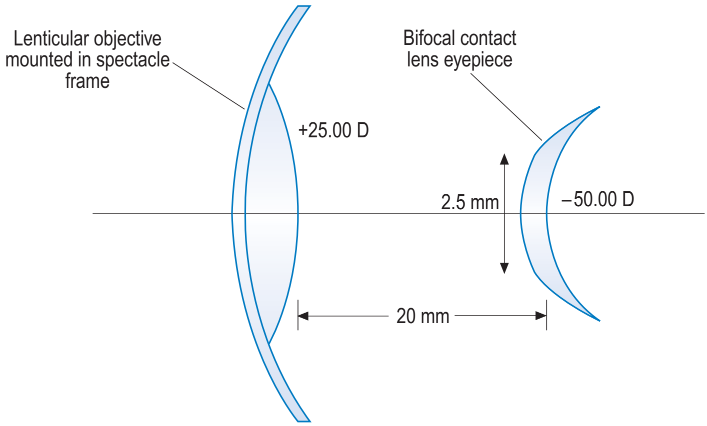

Figure 9.7 The Telecon system

magnification using the central zone of the contact lens and the spectacle lenticular, with the advantage of also observing an unmagnified periphery via the contact lens peripheral zone (which can carry the distance correction) and the plano-periphery of the spectacle lenticular. The magnification achieved with this system is 2×.

Once again the practical problems encountered in fitting and adapting to this system are considerable. Bennett (1985) has described the effects of wearing a contact lens telescope which produced nausea after a short time. Chromatic aberration, pincushion distortion and apparent movement of the visual field all produced noticeable effects, with the magnification making estimation of distances very difficult.

9.2.3 Near vision

A distance telescope is most conveniently converted to a near telescope by using a positive-powered lens cap placed in front of the objective. A +4.00 D lens, for example, would convert the divergent light rays from an object 250 mm away to a parallel pencil, which the distance telescope is designed to deal with. In the case of the Telecon system, a second reduced aperture lens bonded to the lower section of the carrier portion, with a power which is 4.00 D more positive than the central reduced aperture lens provides magnification for near if required.

9.3 The correction of the aphakic or high hyperopic eye

The correction of the aphakic or high hyperopic eye is fully covered by Douthwaite (1993) with a more comprehensive discussion than the one which follows here. Aphakic eyes usually require a substantial positive power (often

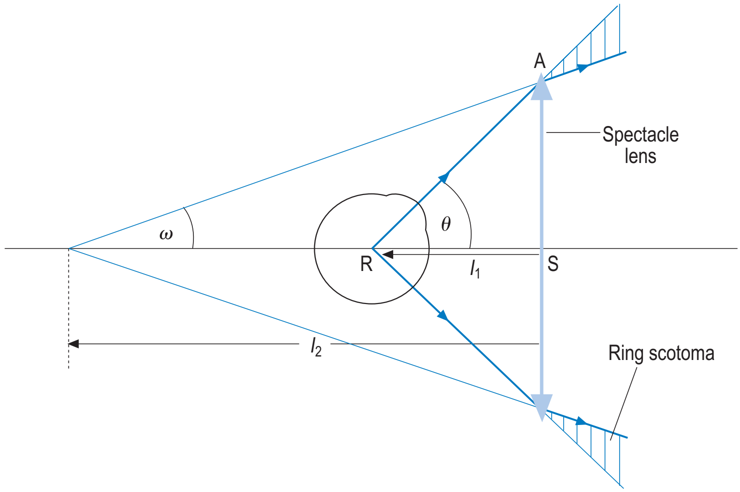

Figure 9.8 The motor field or field of fixation. R is the centre of rotation of the eye

around +12.00 D) to effect a correction. This means that a measurement of vertex distance is essential during the refraction and a contact lens made to correct an individual eye will be more powerful than the spectacle correction. The spectacle correction, however, will suffer from a number of disadvantages.

9.3.1 The motor field of view or field of fixation

In **Figure 9.8** we see an eye which has rotated to look through the upper extremity (point A) of a spectacle lens whose optical centre is at S. This illustrates the extent of the field constriction. Also any object positioned in the shaded areas of **Figure 9.8** would not be seen at all since this type of correction produces a ring scotoma arising out of the prismatic deviation at the lens periphery. This appears to cause most distress to the wearer when observing at intermediate distances around 1 to 3 m.

The deviation of the limiting light ray at A can be calculated by regarding the centre of rotation of the eye as a point object. Let us assume that the lens diameter is 50 mm and the distance RS is –25 mm.

$$\text{Lens semi-diameter} = 25\ \text{mm}$$

Therefore

$$\tan\theta = \frac{25}{25}$$

$$\theta = 45°$$

Therefore

$$\text{field of fixation without the spectacle lens} = 90°$$
$$\text{field of fixation with the spectacle lens in place} = 2\omega$$

In order to deduce ω we must first know l_2. Let us suppose that the lens has a power of +13.00 D.

$$\text{Object vergence } L_1 = \frac{1000}{-25} = -40.00\text{ D}$$

$$\text{Lens power} = \underline{+13.00\text{ D}}$$

Therefore

$$\text{image vergence} = -27.00\text{ D}$$

Therefore

$$l_2 = \frac{1000}{-277} = -37.04\text{ mm}$$

$$\tan\omega = \frac{25}{37.04} = 0.675$$

$$\omega = 34^\circ\ 1'$$

Therefore

$$\text{true field of fixation for the corrected aphake} = 2\omega = 68^\circ\ 2'$$

It is as well to remember that the spectacle lens will suffer from oblique aberrations and the visual acuity may show some deterioration when the visual axis passes through the spectacle lens periphery. Also the spectacle lens may cause distortion of the retinal image, which may cause patient adaptation difficulties. The spectacle lens may need constant adjustment to maintain the desired vertex distance. Also the 'Jack in the box' effect may become an annoying distraction.

9.3.2 The Jack in the box effect

In Figure 9.9(a), an aphakic eye, corrected by a spectacle lens, is in the primary position, where it can see object A which lies just within the superior field. In this case the limiting light ray passes through C which is the centre

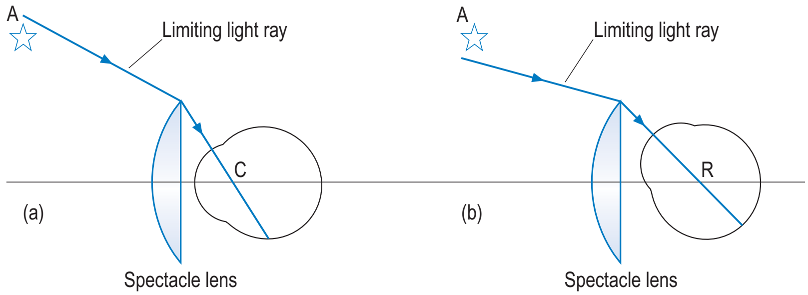

Figure 9.9 (a) The limiting light ray passes through the centre of curvature of the refracting surface C. Thus the object A is within the field of vision. (b) The limiting ray now passes through the centre of rotation of the eye R and the ring scotoma moves in to cover object A

of curvature of the single refracting surface in **Figure 9.9(a)**. If the eye elevates to look at this object then the pupil will be displaced superiorly during the rotation, and this displacement may be enough for the object to disappear into the ring scotoma as in **Figure 9.9(b)** where the limiting light ray will now pass through R, the centre of rotation of the eye. As a result of this phenomenon, the wearer will see peripheral objects disappearing when s/he attempts to fixate the object of interest, only to find the object reappearing when the eye returns to the primary position. The patient must become accustomed to rotating the head instead of rotating the eyes in order to deal with this problem.

9.3.3 Retinal image size

The topic of retinal image size, spectacle magnification and relative spectacle magnification is discussed in Chapter 1. The summary here is derived from the relationships described in that chapter (Sections l.5 and 1.6).

The high plus spectacle lens will produce a magnification of the retinal image which increases with increased vertex distance. The retinal image size will typically be around 30% larger in a corrected aphake than in an emmetrope. A contact lens correction results in the correcting lens being very near the entrance pupil of the eye and the spectacle magnification is much reduced; however, it is not unity and it is increased by an increase in the shape factor (see Section 1.5.3.1) of the correcting lens. An examination of typical contact lens shape factors reveals that scleral lenses have larger shape factors than corneal or soft lenses. Therefore a scleral lens produces a larger retinal image because of its larger shape factor, which is due, in the main, to a larger central thickness than that which occurs in the RGP or soft lens.

The significant magnification produced by a spectacle correction in the case of a monocular aphake will make it impossible to fuse the two retinal images. A contact lens correction will minimize the retinal image magnification. However, even so, the lens will produce a small magnification of the corrected retinal image and, although the monocular aphake may well be able to fuse the images formed in the two eyes, some aniseikonic adaptive problems are to be expected. Many aphakic contact lens wearers require auxiliary spectacles and this is quite likely in a monocular aphake who needs a prescription for the other eye. In this case the practitioner can over-correct the positive power in the contact lens and the eye is then corrected for distance by a negative spectacle lens. This negative distance spectacle lens will reduce the retinal image size. Let us take a typical example. It is worth pointing out that the aphakic eye is classified as a refractive (not axial) defect and therefore the relative spectacle magnification will equal the spectacle magnification. This means that calculation of the spectacle magnification will indicate the increase in the retinal image size compared to that of the standard emmetropic eye.

A numerical example follows.

An aphakic eye will be corrected by using a soft contact lens. The contact lens BOZR on the eye is assumed to be 7.80 mm (the anterior corneal surface radius) because the fluid lens power is deduced to be plano. This eye requires a contact lens BVP of +12.00 D to correct the eye for distance vision. It is decided to over-correct the eye by +5.00 D. The contact lens thus has a BVP of +17.00 D and the centre thickness is 0.4 mm. We will assume that the entrance pupil of the eye is 3 mm behind the anterior cornea and the contact lens refractive index is 1.444. What will the spectacle magnification be for the contact lens? If the spectacle lens which neutralizes the over-correction is placed 15 mm from the cornea, what will the spectacle minification be for this spectacle lens?

The first move must be to calculate the contact lens front surface power (F_{a0}) as in Section 2.3. This can be done by the step along method, which gives a value of +72.44 D.

The spectacle magnification due to the contact lens is given by

$$\text{spectacle magnification} = \text{power factor} \times \text{shape factor (Section 1.5.5)}$$

$$\text{Power factor} = \frac{1}{1 - xF_v'} \qquad (1.1)$$

where x is the distance from the back vertex of the lens to the entrance pupil of the eye (in metres) and F_v' is the BVP (in dioptres).

$$\text{Shape factor} = \frac{1}{1 - (t/n)\,F_1} \qquad (1.5)$$

where t is the lens centre thickness (in metres), n is the lens refractive index and F_1 is the lens front surface power (in dioptres).

If we now substitute the values of our example into these two equations we get

$$\text{Power factor} = \frac{1}{1 - (0.003\ .\ 17)} = 1.0537$$

$$\text{Shape factor} = \frac{1}{1 - (0.000277\ .\ 72.44)} = 1.0205$$

$$\text{Spectacle magnification} = 1.0537\ .\ 1.0205 = 1.0753$$

This means that the retinal image is enlarged by 7.53% by the contact lens.

When a negative spectacle lens is used to neutralize the extra 5.00 D incorporated into the contact lens and is placed at a vertex distance of 15 mm, then the step along method indicates that the spectacle lens BVP will need to be –5.41 D. We will therefore use a –5.50 D lens. The best form for this lens (assuming it is made in spectacle crown glass) will produce a front surface power of +3.50 D and a centre thickness of 1.5 mm. The shape factor of this lens is 1.0035 which indicates that the retinal image size will be increased by 0.35%. Thus the shape factor of the spectacle lens has a very limited influence on the retinal image size.

$$\text{Power factor} = \frac{1}{1 - (0.018 \,.\, -5.5)} = 0.91$$

So the spectacle lens will reduce the image to 91% of its former size.

The spectacle magnification of the spectacle lens $= 0.91 \,.\, 1.0035 = 0.913$

The total spectacle magnification for the contact lens and spectacle lens $= 1.0753 \,.\, 0.91 .\, 1.0035 = 0.982$

Thus when wearing this system (contact lens and spectacle lens) the retinal image size is 98% of the size of that of the emmetropic eye.

A method is required to determine what spectacle lens power is needed for a given vertex distance in any individual case. The problem can be tackled by considering the spectacle lens and contact lens as the objective and the eyepiece of a reversed Galilean telescope. We have already seen in Section 9.2 that magnification (m) for a Galilean telescope is

$$m = \frac{1}{1 - dF_o} \tag{9.2}$$

where d is the separation (in metres) between the objective and the eyepiece and F_o is the power of the objective (in dioptres). Therefore

$$1 - dF_o = \frac{1}{m}$$

$$dF_o = 1 - \frac{1}{m}$$

$$F_o = \frac{1 - (1/m)}{d} \tag{9.3}$$

Equation (9.3) allows us to calculate the power of the spectacle lens when we know the magnification required and the vertex distance of the spectacle lens. The magnification required will be determined by the magnification produced by the contact lens, because the spectacle lens must minify by the same amount. The initial move is to determine the spectacle magnification of the contact lens (with no over-correction) and a reversed Galilean telescope is used to minify the image by the same amount. This is illustrated in **Figure 9.10**. The application of equation (9.3) is shown in the following example.

An aphakic eye is corrected by a soft lens of BOZR 7.80 mm on the eye, central thickness 0.5 mm, BVP +12.00 D and has a refractive index of 1.444. What spectacle lens is required to eliminate the magnification induced by the contact lens, when the spectacle lens is placed

(a) at vertex distance 15 mm?
(b) at vertex distance 12 mm?

The power of the front surface of the contact lens on the eye (F_{a0}) can be calculated using the step along method. This produces a value of +67.32 D for F_{a0}.

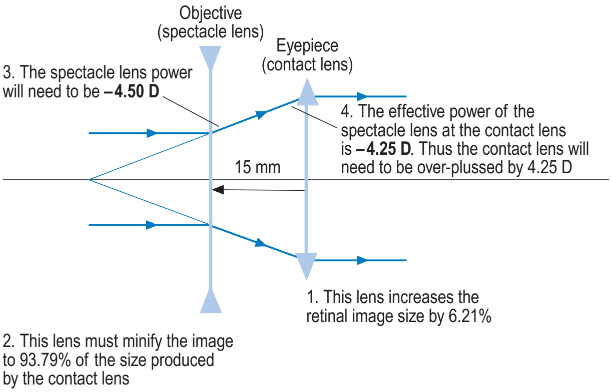

Figure 9.10 The spectacle lens (at vertex distance 15 mm) required to eliminate the magnification of the retinal image produced by the contact lens

$$\text{Power factor} = \frac{1}{1 - (0.003\ .\ 12)} = 1.0373 \qquad (1.1)$$

$$\text{Shape factor} = \frac{1}{1 - (0.0003463\ .\ 67.32)} = 1.0239 \qquad (1.5)$$

$$\text{Spectacle magnification} = 1.0373\ .\ 1.0239 = 1.0621$$

So we see that the contact lens is increasing the retinal image size by 6.21%. We need to use a reversed Galilean telescope to restore the image to the size when uncorrected. Therefore the telescope must provide a magnification of 0.9379 times (the image is reduced to 93.79% of its contact lens corrected size). It must be noted that this argument is approximate in the sense that the two percentages above apply to the uncorrected and corrected images respectively. This will produce a small error.

We can now substitute our values into equation (9.3).

(a) vertex distance 15 mm

$$F_o = \frac{1 - (1/0.9379)}{0.015} = -4.41\ \text{D}$$

Thus we need a spectacle lens of BVP –4.41 D at vertex distance 15 mm and the step along method can be used to deduce that the contact lens must be over-corrected by an amount equal to the power of the eyepiece, which is +4.14. In fact we would use a –4.50 D spectacle lens and the contact lens power would be over-corrected by +4.25 D. This means that the contact lens power would need to be +12.00 +4.25 = +16.25 D. The increase of 4.25 D of positive power to the contact lens will increase both its power factor and shape factor which means that the spectacle lens will not return the retinal image exactly to its unmagnified state. The residual error, however, will be small.

(b) Vertex distance = 12 mm

$$F_o = \frac{1 - (1/0.9379)}{0.012} = -5.52\ \text{D}$$

so that we would probably use a –5.50 D spectacle lens at vertex distance 12 mm with the contact lens over-plussed by 5.25 D giving it a power of +17.25 D.

9.3.4 *Near vision*

The aphakic eye is incapable of accommodating in order to focus near work. Some practitioners have suggested adding extra positive power to the distance correction in order to help the aphake with intermediate and near work. However, there appear to be some reservations, and caution on the part of the practitioner is advised. Stone and Francis (1980) have noted that the positive spherical aberration in a high plus contact lens produces a correction which may well be around 2.00 D more positive in the lens periphery than at its centre. Thus an aphake whose contact lenses ride high when the eyes are depressed for close work may well be able to manage without a supplementary reading correction. If, on the other hand, some extra positive power is needed for close work this may most conveniently be prescribed as a pair of reading spectacles, which simply contain the reading addition, to be used in conjunction with the distance contact lenses.

9.4 Orthokeratology

The application of orthokeratology started with the Chinese who attached bags of sand to their eyes over-night in an attempt to alter their refractive status. The possibility of altering refractive error using contact lenses was apparent when changes in refractive error induced by hard PMMA contact lens wear were noted by a number of investigators. Some patients who stopped wearing contact lenses reported improved unaided vision that lasted for days or even weeks. Monitoring these individuals has demonstrated large changes in refractive error induced by rigid contact lens wear and these changes may take a few months to completely disappear. The evidence suggests, however, that the refractive error ultimately returns to its former value. Orthokeratology (OK) developed as the practice of fitting progressively flatter rigid contact lenses to flatten the cornea sufficiently to reduce myopia. In the early days it was associated with refractive changes that were small, variable and temporary.

There is some evidence that the wearing of rigid lenses discourages myopia development but this has been disputed (Kerns, 1976–78). This is a form of myopia control and is not to be confused with OK.

There is an undoubted ability to induce corneal curvature changes by wearing rigid lenses. The questions that arise are:

- how much modification of corneal curvature can be safely induced?
- how permanent are the results?
- at what age might the therapy best begin?

In 1974 a special project team for the American Optometric Association was set up to investigate the practice and feasibility of OK. They defined OK as the reduction, modification or elimination of refractive anomalies by the programmed application of contact lenses or other related procedures.

The factors that influence corneal curvature changes may include:

- lens–cornea bearing relationship.
- lens diameter.
- centre thickness.
- mass.
- position.
- duration of wear.
- individual tissue response.
- the role of blinking.
- IOP.
- tensile strength of the cornea.
- ocular rigidity.
- lid tension.

Kerns (1976–78) found, that by fitting progressively flatter rigid corneal lenses, the corneal curvature flattened, the refractive error of the myopic patient was reduced, the unaided vision was improved and the corneal topography was less aspheric. He deduced that the direction of change was quite predictable but the magnitude of change was not. Thus there was a question about how precise the technique was for correcting refractive error.

He noted that control of the vertical corneal meridian was difficult. The lenses may well flatten the horizontal (flatter) corneal meridian but often left the vertical meridian unchanged. This inevitably means an increase in uncorrected astigmatism.

The variability of response comparing one individual with another raises concerns for control of the extent of the refractive error. This may be associated with corneal or ocular rigidity. It was not possible to conclude if age, refractive error or corneal curvature had any influence on the results. The lens total diameter (TD), BOZR, BOZD and centre thickness might all play a part in changing corneal curvature.

Kerns concluded that the extent of corneal flattening in the horizontal meridian was poorly correlated to the degree of cornea-to-lens flattening and the response variability increased when the degree of flattening was greater than 0.1 mm. The corneal response seemed relatively independent of the cornea-to-lens fitting relationship. Thus there was no direct relationship between the amount of flattening of the BOZR and the flattening of the corneal curvature.

Corneal flattening in the vertical corneal meridian occurred when the cornea-to-lens relationship was less than 0.1 mm (lens flatter). Over this amount produced greater variability and often produced corneal steepening.

The flat fit inevitably resulted in a mobile lens that took up a decentred position, often superior and temporal. Decentred lenses were often observed where corneal toricity was induced. Was this due to the original fit that encouraged more corneal toricity or was the existing corneal toricity inducing the lens decentration? A centred lens produced more symmetrical corneal curvature changes.

Corneal curvature changes developed in a uniform manner over the first 200 days. Curvature changes appeared to decrease corneal asphericity and stop when the cornea had lost its asphericity. Variability increased with a very flat lens (0.3 mm flatter than K). Variability increased with lens wearing time. Variability was greatest from lens removal to about five days after removal. The best unaided vision could occur any time from lens removal to just before lens insertion on the next day.

Curvature changes varied from subject to subject and were possibly associated with corneal rigidity. The lid tension may also have had some influence on the induced changes. The vertical (steeper) corneal meridian could not be moulded to the shape of a contact lens no matter whether the fit was flat or steep because the flat meridian separated the cornea from the lens in the steep meridian. Lens centration may have had an important influence on the OK results.

The problem with OK was its lack of predictability, working well with some subjects and not at all with others. Also there appeared to be no way to control the degree of change with any precision.

Binder *et al.* (1980) attempted OK in 23 patients and concluded that the responses to OK are unpredictable and uncontrollable. When lenses are removed, the corneal parameters return to pre-fit levels. There was no change in corneal thickness, anterior chamber depth or axial length throughout the study (three years). Patients who obtained good unaided vision (nine patients) did so within the first nine months of the study – average dioptric change was around 1.00 to 2.00 D with an increase in with-the-rule astigmatism. The OK eyes appeared to suffer no significant trauma or any other undesirable change but the unaided vision was poorer than the corrected visual acuity. Again it appeared that corneal curvature changes were more likely if the cornea was aspheric at the outset whereas spherical corneas ($p = 1$, $e = 0$) did not respond. The best patient would be a low myope with against-the-rule astigmatism and an aspheric cornea. Having said that, they concluded that it was impossible to predict who would benefit from OK and who would not. For refractive errors over 4.00 D, little improvement in unaided vision was to be expected. There was no direct relationship between refractive error change and improvement in unaided vision suggesting that there were changes not measured by conventional clinical techniques. There was no direct relationship between change in refractive error and change in corneal curvature. Only four patients reached the retainer lens stage in this study.

Polse *et al.* (1983a) concluded that OK produces modest reductions in myopia but the effect will not persist without continued lens wear. They used

34 treatment and 32 control subjects with a masked examination. Lenses were worn for 12 months after full adaptation. The average reduction in refractive error was about 1.00 D. The curvature changes were inconsistent with the refractive error changes in some subjects. They posed the question – Does a large induced change in refractive error induce a more permanent change in refractive error when lens wear ceases? Their results provided no convincing relationship. They concluded that the cornea has a strong tendency to resume its original shape.

In the Berkeley Orthokeratology Study, Polse *et al.* (1983b–d) performed a treatment/control group double blind study and concluded that OK treatment was safe for the patients that participated in their 1.5 year study. They used PMMA or Polycon lenses. They checked corneal thickness, curvature, refractive astigmatism, spectacle blur, corneal oedema and staining and finally endothelial cell density. They did not observe an increase in corneal astigmatism. This may be due to attempting to maintain good lens centration. However their selection criteria included limiting astigmatism to less than 0.75 D. They also noted that fluctuations in refractive error after lens wear ceased were greater in subjects who had shown the greatest refractive change. Also the vision during periods of non-lens wear was unstable making it difficult to predict what the quality of vision would be under a retainer lens wear program. There was an average reduction in myopia of around 1.00 D compared to 0.50 D in the control group. Four out of 30 subjects had a reduction over 2.00 D. The improvement was not permanent. The visual instability after lens wear was accompanied by curvature fluctuations. There were discrepancies between refractive change and curvature change. They concluded that OK was a safe procedure but it lacked predictability.

Coon (1984) used a double blind study to investigate the effects of wearing a PMMA lens with an aspheric periphery using the Tabb method. The Tabb method was an OK procedure where the BOZR was 0.05 to 0.15 steeper than K. The TD was equal to K plus 1 mm. The BOZD was equal to K. The method involves adjusting the tear reservoir. This was defined as the percentage of total posterior lens surface occupied by the intermediate and peripheral curve area. This was achieved by changing the intermediate curves, the peripheral curves and the BOZD. A number of biometric measurements were taken including anterior chamber depth and corneal thickness. The parameters that appeared most involved with refractive and acuity changes were corneal thickness and corneal asphericity.

Mountford (1997) reviewed the topic of OK. He defined OK as the reduction, modification or elimination of a visual defect by the programmed application of contact lenses. He stated that this is limited to myopic errors. He described two techniques for OK:

1. *The May–Grant technique*. Initial lens about 0.1 mm flatter than K with the TD 1.30 mm larger than K and the BOZD equal to K. The central thickness is greater than normal to discourage high riding lenses. Progressively

flatter lenses are used until the cornea resists further change. Grant reported on a study of 100 patients and concluded that the limits achievable were up to 4.00 D of myopia with up to 2.00 D of astigmatism. The lower the initial refractive error, the better the chances of success.

2. *The Tabb technique*. The peripheral curves of this four curve lens are each 1.00 mm flatter than the preceding curve, 0.20 mm wide and well blended. The BOZD is calculated to give the back optic zone 70% of the total lens area. The peripheral curves make up the remaining 30% and this is called the tear reservoir. OK is achieved by progressively increasing the 30% tear reservoir in 2.5% steps up to 45%, by reducing the BOZD.

Mountford notes that past investigations in the 1970s and 1980s indicated a steepening of the corneal periphery as the central cornea flattens. There was no correlation between the change in refractive error and the improvement in unaided vision. The keratometry changes accounted for around 50% of the refractive changes. The central corneal thickness decreased in OK patients with the peripheral thickness increasing. The average decrease in myopia was around 1.00 D and the maximum effect was gained after six months of treatment but there may be some remission in succeeding months. All studies agreed that the myopia reverted to its pre-fitting levels when contact lens wear was terminated. Maintenance of the refractive change required lens wear for at least eight hours per day. OK was concluded to be a safe procedure capable of inducing a modest reduction in myopia that was, unfortunately, unpredictable. There appeared to be a relationship between the degree of corneal asphericity and the amount of refractive change. Also the lower the initial myopia, the better the chance of success.

Thus the limitations of the early attempts at OK can be summarized as:

- Only a small reduction in refractive error can be achieved.
- Astigmatism will produce complications and may increase with OK.
- The unaided vision will fluctuate during no lens wear.
- A lack of precision in the elimination of refractive error.
- The refractive error will regress if lens wear ceases.
- Lack of association between corneal and refractive changes.
- Patient inconvenience during treatment and the need to wear retainer lenses for a considerable part of the day.

These findings resulted in a reduction of OK activity until novel approaches were suggested.

9.4.1 *Modern OK techniques*

9.4.1.1 Accelerated orthokeratology by reversed geometry lenses

In this approach (Wlodyga and Bryla, 1989), the contact lens is designed with a periphery steeper than the back optic zone. This reverse geometry was suggested as a feature to encourage peripheral corneal steepening with

central corneal flattening in a lens that was likely to centre well on the cornea. It was found that lenses centred well even up to 0.4 mm of central flat fitting and produced rapid (days rather than months), stable corneal changes. They used the OK series of lenses in fluoro-silicone acrylate material (Dk 88 for daily wear, 120 for extended wear). BOZDs of 6.00 to 8.00 mm in 0.50 mm steps with a BPD1 giving a bandwidth from 0.50 to 1.80 mm and BPR1 0.20 to 1.80 mm steeper than the BOZR. The TD was 10.60 to 11.20 mm. The edge curve bandwidth was 0.50 mm and had a flat aspheric curve approximately 0.60 mm flatter than the BOZR. They also came up with an aspheric series where all the peripheral curves were aspheric. These lenses gave less flare and less tendency to bind but induced slower changes. The asphericities have not been disclosed. All OK standard design lenses are constant axial edge lift lenses (AEL = 0.12 mm) although this can be altered in 0.02 mm steps. A standard design OK603 lens has a BOZD of 6.00 mm and a BPR1 3.00 D (approx 0.6 mm) steeper than the BOZR. This is usually the lens of first choice. An OK754 has BOZD 7.50 mm and BPR1 steeper by 4.00 D. A prescription order might look like

8.25/10.60/OK704/XL

the XL is requesting an extra low edge lift.

Their routine:

$$\text{Projected myopia reduction } \Delta\text{Rx} = 2(K_c - K_t) + 1.00\text{ D}$$

Here, K_c is central keratometry (D). K_t is temporal keratometry (D) where the patient fixes on the nasal plus sign of a Bausch and Lomb keratometer mire.

The extra 1.00 D is added to allow for axial length reduction produced by corneal flattening, although previous studies have failed to demonstrate a reduction in axial length induced by OK

$$\text{if} \quad K_c - K_t = 2.00\text{ D} \quad \text{then} \quad \Delta\text{Rx} = 5.00\text{ D}$$

Thus the initial corneal asphericity was believed to be an important predictor of the degree of myopia reduction. If corneal astigmatism is present then steeper lenses are fitted and the corneal astigmatism is reduced first before attempting myopia reduction.

Patient selection

You require a well-motivated patient with a normal healthy eye. Myopia should be less than 4.00 D and astigmatism less than 1.50 D. Patients may have to accept an unaided vision of 6/12 or worse. Large pupil diameters will cause problems due to the nature of the modified cornea with its flat centre and steep periphery.

Fitting routine

The first fitting session will be fitting then wearing for six hours followed by re-assessment. If there is no change in refractive error/unaided vision after six hours, then the likelihood of a good outcome is low. The first lens is typically 0.30 to 0.60 mm flatter than K and will display:

- A 3.00 mm diameter region of central touch with a wide and deep tear reservoir.
- 1 to 2 mm of movement and good centration of the area of touch over the pupil zone.
- Active tear exchange without the presence of bubbles in the tear reservoir.

Other fitting guidelines are:

- The larger the BOZD the looser the fit.
- The steeper the tear reservoir (BPR1) the tighter the fit.
- The larger the TD the tighter the fit.
- Use base down prism as an aid to centration if lenses locate superiorly.

The cornea will change from 1.00 to 3.00 D after six hours of wear. Assess movement before introducing fluorescein. When fluorescein is instilled there should now be less apical touch and a shallower tear reservoir. Once the lens is removed retinoscopy can be used to check for corneal distortion. A video-keratoscope (VK) can be used with picture subtraction to reveal the changes from before to after lens wear. The next lens to be fitted will be 0.2 mm flatter than the first lens. Centration is important. If it is poor then try a deeper tear reservoir (steeper BPR1), 1–3Δ base down prism or a larger BOZD. Correct centration and good movement are essential. Subsequent lenses will flatten in 0.1 mm stages. The first pair of lenses may require changing after one to three days. The second pair will take one to two weeks to induce a full change. Subsequent pairs may need more time to complete the corneal change. Note that the lenses to be provided on the sixth visit will be checked for fit and ordered on the fifth visit. When the corneal topography/refractive error/ unaided vision do not change over two visits (each a monthly visit) then it is unlikely that further change will take place and the retainer phase starts.

Retainer lenses

Ideally the unaided vision will be at least 6/6 with slight hyperopia. The last pair of lenses could be the retainer lenses that the patient wears overnight. Alternatively the lens specification may require alteration. Once the retainer lens design is decided, the lens is worn all day for one week and then assessed. If all is well then extended wear (seven days/ six nights) should commence for a few weeks. If the corneal state is stabilized then the patient switches to overnight wear only, to remove the lenses one hour after waking and to note the length of time that vision remains clear. When blurring

reaches an unacceptable level, the lenses are re-inserted. This continues until the lenses can be removed for an entire day. Eventually the lenses may need to be worn for only every third night but this depends on the individual patient. The use of very high Dk materials is advised remembering that the lens thickness will be greater than a conventional lens. The retainer lens design should give good centration, an evenly distributed pressure over the pupil area, a large optic zone and high oxygen transmissibility. The cornea will now no longer be prolate in form. It will be either spherical or oblate. The retainer must not bind to the cornea and must not trap debris. It might be worth considering three or four fenestrations to increase lens movement, aid in oxygen transmission and to help flush out debris.

One common problem is the lens decentering during sleep. If it rides high then the superior cornea will flatten and the central cornea will steepen producing poor unaided vision. A larger BOZD and larger TD may help prevent this. These OK lenses tend to encourage an increase in against-the-rule astigmatism and so are not recommended where against-the-rule astigmatism is present at the outset. These OK lenses reduce with-the-rule astigmatism.

Carkeet *et al.* (1995) attempted to identify the factors leading to individual variability in the success of OK. The OK group of subjects used the OK-3 lens design. They noted from past work that there may be a correlation between change in IOP and change in subjective refraction. Also that the faster the eye responds to OK, the faster the regression when lens wear is terminated. The OK-3 lens was fitted 0.3 mm flatter than (flattest) K but had a secondary curve that was usually 0.6 mm steeper than its base curve with standard aspheric peripheral curves that were based on the secondary curve. The lens had a 6.00 mm BOZD and a secondary curve width of 1.1 mm. The material was a fluoro-silicone acrylate (Dk 88 $cm^2 ml\ O_2/s\ ml\ mmHg$). Refractive error, central corneal thickness, a corneal thickness profile, axial length, anterior chamber depth, lens thickness, vitreous chamber depth, corneal diameter, corneal topography, corneal epithelial fragility threshold (CFT) and ocular rigidity were all measured. None of these biometrical measurements were predictive for OK changes. The only predictive feature was the starting refractive error. The poor responders had the highest initial levels of myopia. It must be noted that the number of subjects used in this study was small (only nine subjects in the OK group).

Joe *et al.* (1993) considered IOP and corneal asphericity as possible predictors of OK success. The maximum myopia reduction using the OK-3 lens was noted, by other workers, to be around 5.00 to 6.00 D. Joe *et al.* used 11 subjects. They believed that refractive changes around 2.00 D were likely to occur with the OK-3 lens. They concluded that corneal asphericity and IOPs may be predictors of OK change. However, the subject numbers were small and their results were not convincing in that their conclusions were based on linear relationships in their scatterplots where the correlation coefficients were very low.

The reverse geometry approach appears to offer improvement in the results of OK. However, the fitting procedure appears protracted and each patient must be prepared to comply throughout the lengthy process. The patient will also need to be supplied with a number of lenses.

9.4.1.2 Modern OK

The use of reverse geometry lenses for OK stimulated renewed interest. OK is a non-surgical treatment for the reduction of myopia that is reversible. It is seen by many patients as an attractive alternative to refractive surgery. The renewed activity resulted in a number of OK lens designs. These included tetracurve lenses that claimed to be able to eliminate greater amounts of myopia, using only one pair of lenses with a more rapid myopia reduction.

The function of each of the four curves is as follows:

1. A central flat curve for myopia control.
2. A reverse curve that allows contact between the cornea and curve three.
3. Curve three is a curve that aligns to the cornea to help ensure good movement and centration.
4. The final curve is a flat curve that generates an appropriate edge clearance.

In all cases the BOZR is made flatter than the cornea but it does not always follow that extra flattening of the BOZR will produce a greater decrease in myopia. If we consider a four curve back surface design then the second (reverse curve) and third back surface curves will determine centration and movement of the lens. The usual rules apply in that curves that are too flat will encourage movement and decentration with steep curves reducing movement to the point that the fixed central position of the lens will produce very little tear exchange. The peripheral curve is there to provide adequate corneal clearance to encourage some movement and tear circulation.

The modern orthokeratology pioneers include Mountford, Tabb, Rinehart, Reeves, El Hage, Legerton, Edwards, and Breece. Mountford (1997) found a convincing relationship between corneal asphericity and change in corneal apical power based on the notion that the cornea will change until the asphericity is eliminated ($p = 1$, $e = 0$). He approximates the relationship as follows

$$\text{change in eccentricity } (\Delta e) = 0.21 \times \text{change in refractive error}$$

So if a cornea has an eccentricity of 0.5, we might expect a change down to $e = 0.0$. From the above the maximum change in refractive error is

$$\frac{0.5}{0.21} = 2.40 \text{ D}$$

It may be that a suitable lens design may allow corneal moulding into an oblate form, in which case the asphericity will not be a limiting factor.

It must be noted here that the eccentricity *e* cannot be used to quantify the asphericity of an oblate cornea. The asphericity of an oblate conic section can be quantified by using the p-value.

Mountford also noted that a number of workers claimed that the keratometric changes were approximately half the subjective refraction changes but this approaches a one-to-one relationship when comparing refractive change to corneal power change measured by a VK. He has advocated calculation of the tear layer profile for the initial and subsequent OK lenses to be used to eliminate refractive error. He stated that the ideal fitting OK lens must exhibit:

- Good centration.
- 3 to 3.5 mm 'apparent central touch' (TLT must be 0.02 mm deep before fluorescein becomes apparent).
- A wide deep tear reservoir that tapers from the area of central touch to the peripheral contact band.
- 1 to 2 mm movement depending on diameter.
- Active tear exchange without the presence of bubbles in the tear reservoir.

Dave and Ruston (1998) reviewed current trends in OK. They give the Contex OK series lenses specification as follows:

- BOZR 7.5 to 9.00 in 0.05 mm steps.
- BOZD 6.00 to 8.00 mm.
- Edge lift XL, L (low), H (high), XH.
- Tear reservoir (TR) 1 D to 5 D.
- TD 10.60 or 11.20 mm.

The specification recorded for the most commonly used lens is as follows

OK 70 4 C Y F \ 10.6

(OK)OK series (70)BOZD 7.00 mm (4) secondary curves (C) aspheric periphery (Y) extended wear material (F) fenestrated (10.6) total diameter.

The peripheral curve width is 1.10 mm. The edge lift varies from the standard 0.12 mm in 0.02 mm steps.

Lenses are fitted by ensuring

sag of lens = sag of cornea +10 μm over a specified chord length

The chord length is the lens TD minus 2 × the peripheral curve band width.

The optimum level of myopia decrease with these lenses appears to be 2.50 D. Thus low myopes are over-corrected and higher myopes are under-corrected. There were individual variations so that errors up to 3.50 D were eliminated.

Patient selection should be based on:

- Good Rx and corneal asphericity.
- With-the-rule astigmatism less than 1.75 D.
- No keratoconus, corneal dystrophy or active eye disease.

• No deep set eyes or loose lids.
• No long term history of rigid lens wear.
• A good response to a six hour or overnight trial (myopia reduction around 0.75 to 1.50 D).

They suggest that lens fitting should be assessed only after at least 10 minutes of wear. An ideal fit shows a central area of touch of 3 to 4 mm diameter, a tapering tear reservoir, good centration, movement of 1 to 2 mm and peripheral clearance (broad band of fluorescein). The appearance of steep, optimum and flat fits are summarized in Table 9.1.

Summary of effect of increasing lens parameters on the lens fit

BOZR increase
Flattens lens fit. Not a good idea because it changes the corneal/lens relationship too much.

BOZD increase
Increasing the BOZD does not affect the TR depth but increases the TR width and the area of central touch. It appears to flatten the lens fit.

Table 9.1 The behaviour of OK lenses according to the fit

	Flat fit	Ideal fit	Steep fit
Centration	Superior or inferior	Well-centred	Well-centred or inferior
Central touch	> 3–4 mm	3–4 mm	< 3–4 mm
Tear reservoir	Wide TR	Wide tapering TR 50 μm deep	Deep TR bubbles Abrupt TR
Peripheral touch	Reduced or absent	0.75 mm area of touch	Wide and abrupt
Lens movement	1.5 mm	1 to 1.5 mm	<1.0 mm
Topography	Difference maps show displaced zone of corneal flattening. Inferior steepening may be observed – Smiley pattern	Difference maps show central flattening with circular area of steepening	Central islands superior steepening with peripheral compression

Tear reservoir depth increase
Improves the centration. Depths greater than 5.00 D will reduce the central TLT to significant touch and are not advised.

TD increase
Increasing the TD effectively steepens the fit. This produces only subtle corneal changes because there is only a slight increase in the TR depth.

Mountford (1998) considered the problem of refractive regression during periods of no lens wear. He suggested an initial over-correction by OK of 0.50 D in order to minimize the effects of regression.

9.4.2 Pacific University Oregon recommendations

Patrick Caroline of Pacific University Oregon describes reverse geometry lenses as follows.

Spherical optic zone defined by the BOZD is typically 6.00 mm. This is followed by the reverse curve that is around 1 mm wide. There are then one or two curves that align with the cornea each of width 0.5 mm. The second peripheral alignment curve could be flattened to give a suitable edge clearance.

All this totals up to a diameter of 10.00 mm. Add to this another 0.3 mm for the edge bevel and we have a lens total diameter of 10.60 mm. The lens design is illustrated in **Figure 9.11**. The tear layer profile illustrated in **Figure 9.12** has been produced by using the orthokeratology program described in Chapter 11.

In the program, the BOZR should give a TLT of 0.005 mm. The axial edge clearance should be 0.08 mm. The best way of achieving the appropriate fitting relationship is to use a program like the one described in Chapter 11 and illustrated in **Figure 9.12**. The practitioner needs to know the apical radius and asphericity of the two corneal principal meridians and must decide on the amount of myopia reduction. The program will then calculate the back surface specification. The apical radii and asphericities can be derived from a captured VK image. Note that this will be required for each of the two principal corneal meridians.

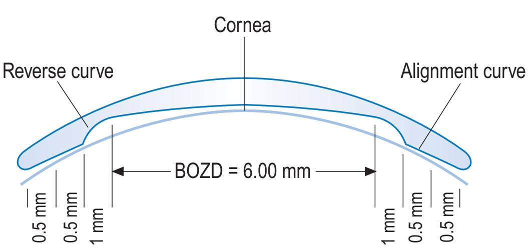

Figure 9.11 A reverse geometry design

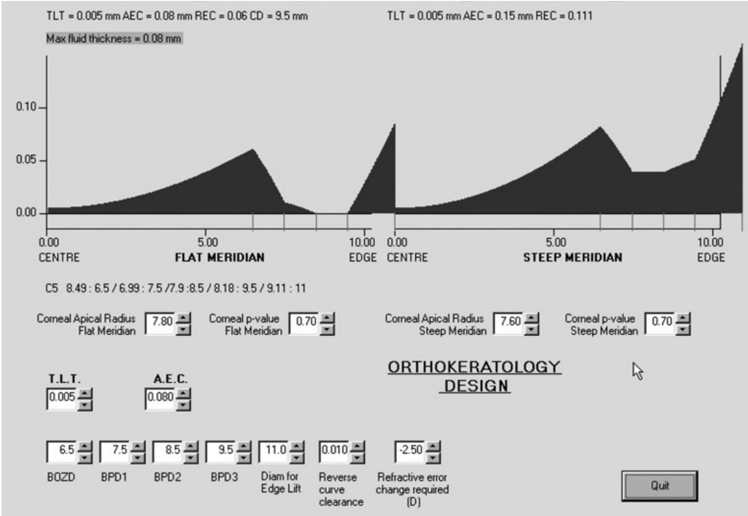

Figure 9.12 The tear layer profile for a reverse geometry pentacurve design. The program calculates the lens back surface specification and then illustrates the theoretical fluid lens profile (Plate 17)

Caroline suggests taking six VK images and calculating the apical radius and asphericity, dropping the highest and lowest values and then taking the mean of the four remaining values. Suitable patients should have:

- Myopia from 0.50 to 4.50 D.
- With-the-rule astigmatism from 0 to 1.50.D
- Career requirements for improved vision.
- An acceptance of realistic goals.

Problems arise with:

- Any against-the-rule astigmatism.
- Residual (lenticular) astigmatism greater than 0.50 D.
- Active or recurrent ocular disease.
- Keratoconus or any other corneal thinning disease.

In the program in this book, the BOZR is fitted flatter than the apical radius of the flat meridian by an amount equal to the target Rx plus –1.00 D. For example

Rx –4.00/–0.50 × 180. Target Rx –4.00 D. Add –1.00D = –5.00 D
The flatter corneal apical radius is 7.80 mm (43.27D)
The BOZR required is 337.5/38.27 = 8.82 mm.

Lens BVP will always be +1.00 DS because the refractive error is corrected by the fluid lens power which has been set to over-correct the myopia by 1.00 D.

The program adopts most of Caroline's recommendations as default values, which suggest:

- Centre thickness of 0.18 mm.
- Lens TD will usually be 10.6 mm.
- The reverse curve width default value is 0.5 mm.
- The alignment curve width default value is 0.5 mm. It is suggested that there is a second alignment curve of width 0.5 mm.
- The peripheral curve width default value is 0.80 mm.
- The reverse curve radius is calculated to give a TLT of 0.005 mm.
- The first alignment curve is radius adjusted until there is 10 µm (0.01 mm) clearance at the start of the curve. The second alignment curve is calculated to give exact alignment.
- The peripheral radius is calculated to give an edge clearance of 0.08 mm at the lens edge.

According to Caroline, when the lens is placed on the eye it should:

- Sit in a centred position.
- Show a 1 to 4 mm area of central bearing.
- Show an obvious fluorescein pool in the reverse curve area – the tear reservoir.
- Show 1 to 1.5 mm of mid peripheral bearing in the area of the alignment curve.
- Show 360° of edge clearance.
- Show 0.5 to 1.00 mm of blink induced movement.

The lens material used must be a high permeability (Dk > 100) material because the lenses will be worn overnight. It has been suggested that patients may start overnight wear without prior adaptation experience. The RGP lens would need to have high oxygen transmission for this to be acceptable.

It is suggested that follow-up visits include new VK maps to observe the corneal changes.

9.4.3 Overnight OK

The modern OK lens designs are intended to be worn overnight. The lenses are removed on waking and the patient will not return to lens wear until retiring to bed at the end of the day. This has obvious advantages particularly if the overnight wear of retainer lenses starts from the first time any lens is worn. Thus a patient collects her/his first pair of lenses and inserts them at bedtime for wearing overnight. Patients are advised to remove their lenses around 30 minutes after waking and to use wetting or rewetting drops last thing at night and first thing in the morning. These steps are necessary to deal with any lens binding that may occur during overnight wear. The RGP material used must be of very high permeability. The patient would normally be examined in the morning after the first night of lens wear.

The results of overnight OK studies have revealed:

- a more rapid reduction in myopia than daily wear studies.
- the OK lens used to reduce the myopia is often also the retainer lens.
- the myopia reduction is often within 0.50 to 1.00 D of the desired result.
- the refractive change achieved is, on average, around 2.00 D.
- a reduction of with-the-rule astigmatism of around 50%.

It may therefore be reasonable to consider myopia up to 4.50 D and with-the-rule astigmatism up to 1.50 D as the limits for OK treatment. A large pupil diameter (greater than 5 mm in normal room illumination) is a contra-indication.

9.4.4 The value of a corneal topographer

Any practitioner considering OK should be equipped with a VK or some other type of instrument for measuring the corneal topography. Some workers claim that the degree of corneal asphericity will help determine how much refractive correction can be achieved by OK. They suggest that refractive change ceases when the corneal asphericity is lost and $p = 1$ ($e = 0$). If this is the case then measurement of corneal asphericity will be a necessary part of the initial examination.

Corneal topography difference maps will be invaluable in demonstrating the regions of the cornea being affected by the OK lens and will thus be useful in helping to determine what changes are required to a lens specification in order to make it more effective.

From the foregoing discussion of OK lenses, it is obvious that a successful outcome relies on establishing the appropriate relationship between the cornea and the contact lens in the various vital regions. The use of a corneal topographer to measure the anterior corneal surface is the most obvious and precise way of ensuring that the OK lens is:

- flatter than the apical radius by the required value.
- steep enough in the reverse curve region.
- aligned in the mid periphery.
- provided with a periphery that produces an appropriate edge clearance.

A topographer allows the corneal topography to be described in mathematical terms which in turn allows calculation of the contact lens back surface curves required to achieve a desired result.

The computer program in Chapter 11 allows the deduction of the lens back surface specification tailor made to the cornea under examination. The user is asked to enter the apical radius and the p-value of the two corneal principal meridians. This information can be derived from the radius and perpendicular distance of each VK ring as described in Section 5.13.1. The user is also asked to indicate the amount of myopia that requires correction. The program then calculates the back surface specification required to

produce the optimal clearance values for this OK lens design. Any of the default diameters and clearances can be altered to allow practitioners to achieve whatever fitting relationship they desire. The program calculates the radii of curvature required for the five curve back surface design.

The program also includes an OK lens based on a polynomial back surface. This is a somewhat hypothetical and untried design. There are not many contact lens laboratories that would be prepared to attempt to produce a polynomial back surface design. However, the program tear layer profile indicates how well suited the polynomial surface is to OK.

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CHAPTER 10

Checking the lens specification

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The topic of lens checking is well covered from a practical point of view in many contact lens textbooks. This chapter will concentrate on the optical principles of the various techniques.

10.1 The optical microspherometer

In the UK the optical microspherometer is usually called a radiuscope. This name was originally the trade name of the American Optical Company instrument. This instrument may be used to measure both the back and front optic radii, the lens thickness and the refractive index of the lens material for any lens. The fundamental working principle for radius measurement is derived from Drysdale's method and is illustrated in **Figure 10.1**.

In **Figure 10.1** the microscope is fitted with a self-luminous target T which is introduced into the main optical system by the semi-silvered mirror M. The microscope objective produces an image of T at T′. If this image is focused on the surface under examination as in **Figure 10.1(a)**, then the central ray is reflected back along its own path with the left-hand incident ray reflected back along the right-hand ray, and vice versa. These reflected rays will form an image at T. However, since the mirror is semi-silvered, 50% of the light will pass through the mirror to form an image at T″ and this can be observed via the microscope eyepiece. The image T″ is in the first focal plane of the eyepiece which is at the same distance from the mirror centre as

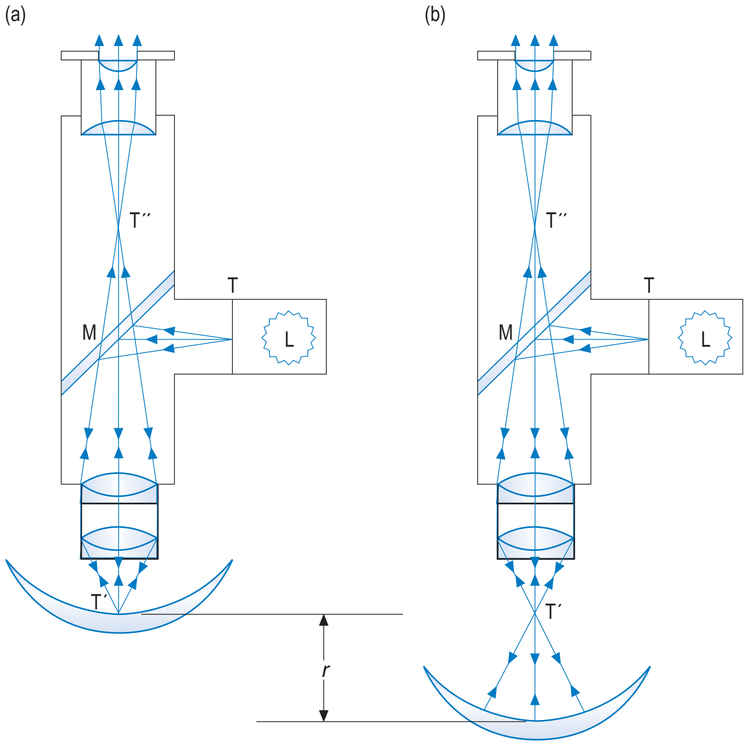

Figure 10.1 The optical microspherometer. (a) Focused on the lens posterior surface. (b) Focused on the centre of curvature of the surface

the target T. **Figure 10.1(b)** illustrates that an image of the target will also be formed in the first focal plane of the eyepiece when image T′ is at the centre of curvature of the surface under investigation. In these circumstances all light rays will strike the surface at normal incidence and will therefore be reflected back up their own paths. The distance between the two positions of the microscope is equal to the radius of curvature of the surface.

If the concave surface of a contact lens is being examined, the reflections from the convex surface must be eliminated by floating the contact lens on a fluid of a refractive index equal to that of the contact lens. From a practical point of view, water will reduce the unwanted reflection to an acceptable level but it must be pointed out that some RGP lenses may be distorted by surface tension forces and in these cases paraffin, with its lower surface tension, may be a more suitable substitute.

In **Figure 10.1(a)** the image T′ is formed on the surface of the lens and, therefore, this surface is seen magnified by courtesy of the microscope. Thus the instrument can be used to assess the quality of the surface in this position. Even superficial surface scratches will be obvious. The image T″ will not be displaced from the centre of the field of view as the contact lens is

horizontally displaced on the microscope table. However, in position (b) the image T′ must coincide with the centre of curvature of the contact lens surface. Any horizontal displacement of the lens will result in the image formed by reflection being displaced in the same direction as the lens movement. It is therefore essential to set the instrument up in position (b) and ensure that the image T″ is in the centre of the field of view which indicates that the centre of curvature of the back optic zone lies on the optical axis of the microscope. When this has been achieved then a radius measurement can be attempted.

Figure 10.1 illustrates the instrument being used on a concave surface, however, it is easy to see that it could be used to measure convex surfaces although an objective with a longer focal length may be required.

The optical microspherometer scale divisions are 0.01 mm. An objective lens magnification not less than ×10 and a numerical aperture not less than 0.25 are recommended. The total magnification should be not less than ×100.

10.1.1 Toric surfaces

In the case of a toric surface, position (a) will produce the target image T″ in exactly the same way as a spherical surface since Figure 10.1(a) illustrates that the image is theoretically produced by reflection at a single point on the surface. Thus the form of the surface (concave, convex, toric or aspheric) is of no significance in this position. However, in position (b) the toric surface will produce two principal centres of curvature and the target image T″ will look very similar to a focimeter target seen when examining a sphere/cyl ophthalmic lens. The operator will see line images from point objects and must endeavour to focus the long edge of the line as accurately as possible. This will give the radius value for one of the two principal meridians. Further movement of the microscope will result in the focal lines reforming at 90° to their previous orientation and the radius of curvature of the remaining principal meridian can be measured.

10.1.2 Aspheric surfaces and peripheral curves

In the case of an aspheric surface we require the apical radius. Inspection of Figure 10.1(b) reveals that a finite central area of the contact lens is used to produce the image T″. The diameter of this area can be reduced by stopping down the aperture of the illuminator. Most instruments are provided with reduced apertures for this purpose. This precaution is also necessary for measuring the radius of peripheral zones where the width of the surface of interest is small. In this latter case the lens is tilted on the microscope table in order to ensure normal incidence of the light rays, in position (b), striking the peripheral region of the lens back surface. The peripheral curve bandwidth will need to be at least 1 mm for there to be any possibility of acquiring a measurement. The presence of any blending of the transitions may make

measurement difficult. Narrow bands will be impossible to measure. Many of the current multicurve designs possess narrow peripheral bands with blended transitions.

10.1.3 *Edge lift*

The importance of the peripheral curves in determining the edge lift and edge clearance has been described earlier in this book. It is unfortunate that the peripheral curves cannot be directly measured in modern lens designs. A check on the peripheral curves is as important as a check on the BOZR. The optical microspherometer can be adapted to measure axial edge lift indirectly. The underlying principle is discussed fully in Section 3.2.2. Essentially we need to measure only two parameters: the BOZR, and the overall sag of the back surface at a diameter equal to the diameter for the stated or calculated edge lift.

The axial edge lift l_a is determined by

$$l_a = s_0 - x \tag{3.3}$$

where x is the overall sag measured by the optical spherometer and s_0 is the sag of the central curve (radius is the BOZR) out to semi-diameter y.

The radial edge lift l_r can be determined using the equation

$$l_r = \sqrt{[(r_0 - x)^2 + y^2]} - r_0 \tag{3.2}$$

where r_0 is the BOZR and x is the overall sag of the contact lens over a semi-diameter y.

Let us take an example to illustrate the indirect assessment of the peripheral curves.

A tetracurve back surface RGP lens has the following specification

C4 7.60:7.50/8.10:8.00/8.60:8.50/9.10:9.50

This lens back surface specification requires checking. It is possible to calculate the axial and radial edge lift of this lens as shown in Section 3.2. Alternatively you could use the appropriate program described in Chapter 11. For a measurement of edge lift it is necessary to avoid errors arising from the presence of any edge taper or edge polish. Therefore in the above example we could calculate the edge lift at diameter 9.00 mm. This will be found to be

Axial edge lift 0.076 mm Radial edge lift = 0.061 mm

All we now need to do is to determine one of these values by calculating the sag of the BOZR for diameter 9.00 mm and measuring the overall sag for diameter 9.00 mm using the optical spherometer.

Figure 10.2 illustrates that the sag of the BOZR for diameter 9.00 mm is

$$s_0 = 7.60 - \sqrt{(7.60^2 - 4.5^2)} = 1.476 \text{ mm}$$

Let us assume that the overall sag (OS) is measured as 1.40 mm.

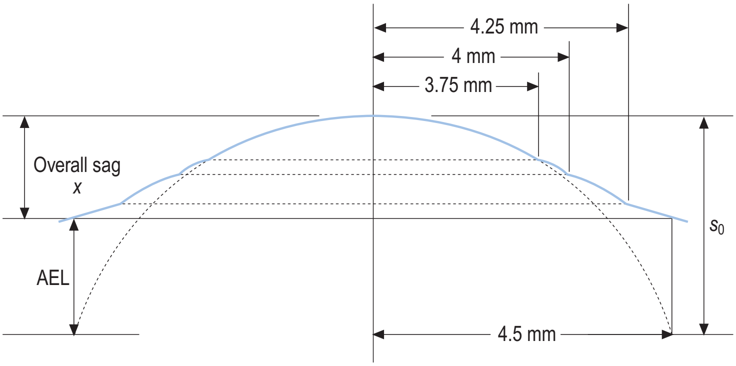

Figure 10.2 The AEL is the sag of the central curve s_0 minus the back surface overall sag x

$$\begin{aligned}\text{Axial edge lift } &= s_0 - x\\ &= 1.476 - 1.40\\ &= 0.076 \text{ mm}\end{aligned}$$

$$\begin{aligned}\text{Radial edge lift } &= \sqrt{[(r-x)^2 + y^2]} - r\\ &= \sqrt{[(7.6-1.4)^2 + 4.5^2]} - 7.6\\ &= 0.061 \text{ mm}\end{aligned}$$

Douthwaite and Hurst (1998a, b) described a clinical technique that deals with the practical problems encountered. The main problem is that a small error in the assumed diameter for the measurement induces a large error in the calculated edge lift. The second problem that induces inaccuracy is the need to ensure accurate centration of the contact lens when the overall sag is measured. They proposed using a pillar of known diameter with an attached collar of internal diameter 0.2 mm larger than the lens TD. The arrangement for our lens example is illustrated in **Figure 10.3**.

The measurement technique is as follows:

1. The combined collar and pillar are placed on the optical spherometer stage and the microscope is focused on the horizontal face of the pillar.
2. The pillar is centred on the optical axis of the microscope and the position on the microscope movement scale is noted.
3. The contact lens is placed on the pillar as illustrated in **Figure 10.3** and the microscope is focused on the lens front vertex. The position on the microscope movement scale is noted for a second time.
4. The difference between the two movement positions gives a distance equal to the overall sag plus the lens centre thickness. The contact lens centre thickness is then measured using a thickness gauge.

The overall sag has thus been measured and this allows calculation of the edge lift. Douthwaite and Hurst (1998b) found that the precision of the

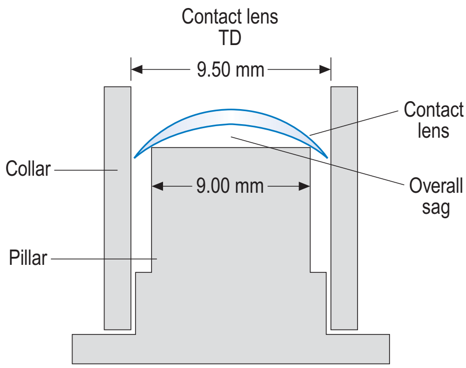

Figure 10.3 The pillar and collar support used in conjunction with the optical spherometer to measure the overall sag of the contact lens for diameter 9.00 mm

method was adequate to meet the ISO standards for measurement of BPR and the old BSI standard for axial edge lift.

It must be noted that the comparison between the measurement and calculation of edge lift is determined by the measurement and calculation of the overall sag. Thus it is not, in fact, necessary to consider the edge lift as such and it is therefore not necessary to calculate the sag s_0. The assessment of the lens periphery can be made by simply comparing the measured overall sag with the calculated overall sag. The difference between these two values will indicate the error and this value will be the same as the error in AEL.

Dietze *et al.* (2003) used this technique on aspheric (conicoidal) surface contact lenses and concluded that this was an acceptable technique for the assessment of surface asphericity.

The p-value of a conic section is

$$p = \frac{2r_o x - y^2}{x^2} \qquad (4.8)$$

Thus measurement of the apical radius r_o using the optical spherometer with a reduced aperture and measurement of the overall sag x on a pillar of diameter y will allow calculation of the p-value.

10.1.4 Thickness measurement

If the contact lens is placed on the microscope table as for a radius measurement but without the use of fluid, reflections from both the upper and lower surfaces will be seen. When the microscope is set at the position shown in **Figure 10.1(a)**, then the two images will be seen simultaneously. If the image formed by the upper surface is seen clearly then the other slightly blurred image, which is formed by reflection from the lower surface, can be made clear by a small movement of the microscope in a downward direction. This movement is equal to the apparent central thickness of the contact lens. The real thickness is acquired by multiplying the apparent thickness by the contact lens material refractive index. The actual point being investigated can

be observed if the instrument is used in a darkened room, since the image T′ is formed on the surface of the lens which acts like a projection screen. Pearson (1980) has suggested making an apparent thickness measurement as described above, then making a real thickness measurement by focusing on the contact lens upper (concave) surface, at the lens back vertex, and then removing the lens and focusing on the supporting surface. The ratio of the real and apparent thicknesses provides us with a convenient way of determining the refractive index of the material since

$$\text{refractive index} = \frac{\text{real thickness}}{\text{apparent thickness}}$$

Axial edge apparent thickness measurement can be made with the optical microspherometer at any specified point. However, radial edge thickness cannot be measured with any great precision since there is no way of knowing how much lens tilt is required on the microscope table to ensure a truly radial measurement.

10.1.5 Diameter measurement

Some optical microspherometers come equipped with a secondary eyepiece which contains an eyepiece graticule marked off in 0.1 mm divisions. The instrument is then used like a conventional microscope with external illumination. It must be emphasized that the lens image and graticule must coincide; this can be checked by the observer moving the head from side to side, when no parallax between the image and the graticule should be observed. Alternatively, the observer must be convinced that the lens surface and graticule are simultaneously clear.

10.1.6 Optical spherometer measurement of soft lenses in a wet cell

The flexibility of a soft lens means that the surfaces are likely to warp in air under the influence of gravity. Also the soft lens will be dehydrating during the measurement, which may well alter the surface specification. The lens temperature will be influenced by the air temperature in the checking room and this may not be particularly well controlled. Also the dehydrating surfaces will produce a poor reflected image. It is thus preferable to check the lens in a wet cell where the fluid supports the lens and maintains full hydration at a steady temperature. The flat surface of the wet cell will refract the emerging light rays and this must be taken into account.

In **Figure 10.4** the optical microspherometer target T′ (of **Figure 10.1**) will need to be positioned at A′ and C′. The microscope movement from position *(a)* to position *(b)* will be the distance A′C′.

When focused on the lens surface

the real depth of saline $\quad AH = n.A'H$

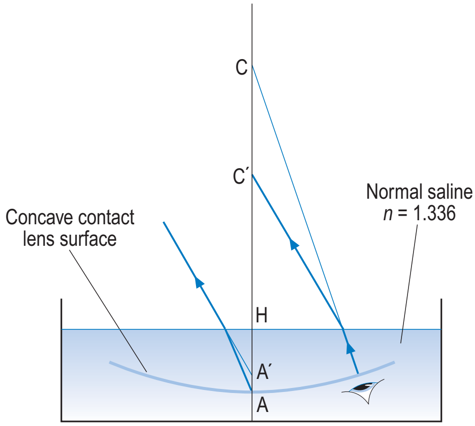

Figure 10.4 The effect of refraction in a wet cell for a light ray directed to the centre of curvature of the surface C and a light ray from the vertex A. AC is the radius of curvature of the concave contact lens surface: A′C′ is the apparent radius of curvature measured by the optical microspherometer

where n is the refractive index of the saline and A′H is the apparent depth.

When focused on the centre of curvature, if we reversed the path of the light ray so that we assumed that it is coming from point C′ in air to be viewed in the normal saline apparently coming from C, then we have the relationship

$$\frac{1}{n} = \frac{\text{real depth}}{\text{apparent depth}} = \frac{HC'}{HC}$$

Therefore

$$HC = n.HC'$$

Now from **Figure 10.4**, the radius of curvature of the surface is

$$\begin{aligned} AC &= AH + HC \\ AH + HC &= n.A'H + n.HC' = n(A'H + HC') \\ &= n.A'C' \end{aligned}$$

A′C′ is the apparent radius of curvature measured by the optical microspherometer. Therefore, when using an optical microspherometer on a soft contact lens in a saline wet cell, we must multiply the measured radius by the refractive index of the normal saline. The precision of the measurement is lowered a little by the shorter radius measurement.

An optical microspherometer has been produced with an objective which can be immersed in a deep saline cell and this avoids the effects of refraction at the plane surface of the wet cell. The movement of this instrument gives a direct reading of the radius of curvature.

The light reflected from the contact lens surface in the wet cell is reduced due to the similarity of the refractive indices of the lens and the normal saline.

Fresnel's law for light of normal incidence is

$$R = \left(\frac{n_L - n_S}{n_L + n_S}\right)^2 \times 100$$

where R is the percentage of reflected light, n_L is the refractive index of the lens and n_S is the refractive index of the saline solution. If n_L is taken to be 1.43 and n_S is 1.336, then R in air is 3.13% and R in the wet cell is 0.115%. Therefore, if an optical microspherometer is to be used, the light source will need to have a high light output. Even with this precaution, however, we are faced with two images in close approximation from the two surfaces of the contact lens, and careful considered observation will be needed.

The soft lens can be measured in air like an RGP corneal lens, however, a suitable holder is needed which allows the lens to be supported by floating it on a saline solution without capillary attraction forces distorting the lens. The surface of interest will require blotting dry but the dry surface may preclude a clear image.

10.2 The keratometer

The keratometer can be used to measure concave surfaces. However, since the mire images are formed by reflection at regions of the surface outside the paraxial one, an allowance must be made during calibration for the non-paraxial nature of the image formation. Bennett (1966) has shown that the difference in the aberrations of convex and concave surfaces can account for the corrections which have to be applied when a keratometer is used to measure the BOZR of a contact lens. Some instrument manufacturers provide conversion tables for concave surfaces. These tables illustrate that the concave surface has a radius of curvature which is longer by around 0.02 mm for steep BOZRs (6.5 mm), and by up to 0.04 mm for flat BOZRs (9.50 mm). Therefore an addition of 0.03 mm to the radius indicated by the keratometer will give a realistic result for typical contact lens BOZRs.

It will be recalled from Section 5.4 that an annulus of approximate diameter 3 mm is used for the generation of the mire images. The keratometer will therefore be unsuitable for measuring intermediate and peripheral radii on a multicurve contact lens and the reading obtained from an aspheric surface will obviously not indicate the apical radius. On the other hand, a keratometer is likely to give a better indication of the exact orientation of the principal meridians of a toric lens surface. As with the optical microspherometer the surface not under investigation is neutralized by floating the lens on a suitable fluid. A 45° mirror or reflecting prism can be used to allow convenient observation of the contact lens by the keratometer.

10.2.1 Keratometric measurement of scleral lenses

The optic portion of a scleral lens produces no new problems when using a keratometer to check the radius of curvature of the surface. However, the radius of curvature of the scleral portion is likely to be so large as to be outside the normal keratometer scale range. It will be recalled that in Section

5.7 we observed that a negative auxiliary lens placed in front of the keratometer telescope objective results in the instrument reading steep. This therefore means the keratometer can now be used for measuring flat radii. Usually a –1.00 D or –2.00 DS trial case lens will be used, with the practitioner constructing a conversion table from measurements made on steel balls of accurately known diameter. It goes without saying that a positive auxiliary lens of similar power can be used for steep radii, most commonly encountered in keratoconic cases and when measuring soft lenses in a wet cell.

10.2.2 *Keratometric measurement of soft lenses*

If the soft lens is suitably prepared and suitably supported, then a keratometer like the optical microspherometer could possibly be used to measure the BOZR of the soft lens in air. However, as with the microspherometer, the measurement is more likely to be successful if the lens remains fully supported, fully hydrated and at a controlled temperature in a saline wet cell, where the radius indicated by the instrument will require multiplying by the refractive index of the saline solution.

If, for the sake of simplicity, we assume that the keratometer is a telecentric instrument, then in **Figure 10.5(a)** the mire image h_1' produced by reflection in the contact lens surface will be in the focal plane F. Since

$$\tan i = \frac{h_1'}{f}$$

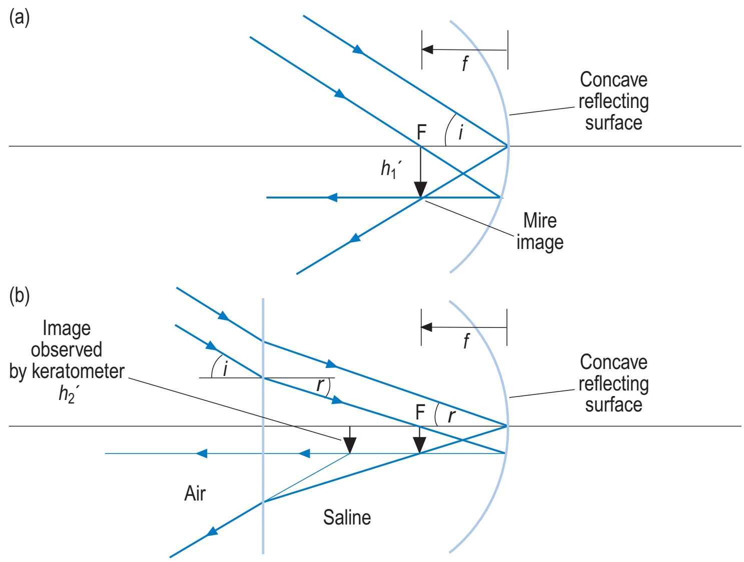

Figure 10.5 Keratometer image formation (a) in air and (b) in a wet cell.

the image size is

$$h_1' = f \tan i \tag{10.1}$$

If the same lens is placed in a wet cell, as in **Figure 10.5(b)**, then the angle of incidence i is reduced by refraction at the air/saline boundary to the angle of refraction r. The pencil of light rays within the saline solution is still parallel and so the point image will form as before in the focal plane F. However, the reduction of angle i to angle r ensures that the image size h_2' is smaller than that formed when the lens was in air. **Figure 10.5(b)** also shows that the size h_2' is the size of the image which is observed via the keratometer telescope. The keratometer must be moved away from the lens a little to maintain a correct focus at the eyepiece

$$\tan r = \frac{h_2'}{f}$$

Therefore the image size in the wet cell is

$$h_2' = f \tan r \tag{10.2}$$

From (10.1) and (10.2)

$$\frac{h_2'}{h_1'} = \frac{f \tan r}{f \tan i}$$

Therefore

$$h_2' = \frac{\tan r}{\tan i} h_1' \tag{10.3}$$

for small angles

$$\frac{\tan r}{\tan i} = \frac{\sin r}{\sin i} = \frac{1}{n}$$

where n is the refractive index of the saline solution. So, from equation (10.3) the image size in the wet cell is

$$h_2' = \frac{h_1'}{n}$$

The radius of curvature measured by a keratometer is

$$r = -2 \frac{h_1'}{h_1} d \tag{5.2}$$

The wet cell reduces h_1' to h_1'/n. It therefore follows that the radius of curvature indicated by the instrument will be reduced to r/n. Thus the radius of curvature given by the instrument when a wet cell is employed must be multiplied by the refractive index of the saline solution. All the above has neglected to include the correction factor of +0.03 mm required for concave surfaces. Since the wet cell results in steep radius measurements it may be more appropriate to add 0.02 mm to the indicated radius, so that if the

keratometer reads 6.22 mm we must apply the correction factor of +0.02 mm, giving 6.24 mm, and this must be multiplied by the refractive index of the saline solution. Therefore

$$\text{BOZR} = 6.24 \times 1.336 = 8.34 \text{ mm}$$

It must be noted that many keratometers will not measure radii as steep as those encountered when measuring a soft lens in a wet cell. An auxiliary lens, of power around +1.00 D, placed on the keratometer telescope objective will extend the keratometer scale to cover steeper radii after re-calibration.

As with the optical microspherometer, the surface reflections are severely reduced and so the mire luminosity must be increased considerably and even then the practitioner must cope with an image formed by each of the two contact lens surfaces. The image formed by the surface which is nearest to the mires will be the brightest and the smaller of the two images will be produced by the steeper of the two surfaces.

It must be noted that the angles i and r will not be particularly small and this means that the sine and tangent will not be equal. If we consider an angle of incidence of 20°, then the angle of refraction will be 14.833° for a fluid of refractive index 1.336. If the tangents of these angles are used to determine the refractive index, a result of 1.3744 is produced. The accuracy of the result obtained by this approach is suspect.

10.3 The microspherometer used to measure soft lenses

The problems described above make the use of the optical microspherometer or keratometer a less than attractive proposition. One of the most popular approaches to the problem of measuring the BOZR of a soft contact lens is to use a microspherometer. This is simply an instrument that measures the sag over a given chord and from this the radius of curvature of the surface can be deduced. The instrument utilizes the fundamental working principle of the lens measure.

The microspherometer instrument can be either a wet or dry cell device. In either case, the instrument working principle is illustrated in **Figure 10.6**. The moving micrometer probe is raised until it just touches the back surface of the contact lens, measuring the sag which is then converted to radius of curvature. In **Figure 10.7**, from Pythagoras' theorem

$$r^2 = y^2 + (r - s)^2$$

Therefore

$$\begin{aligned} r^2 &= y^2 + r^2 - 2rs + s^2 \\ 2rs &= y^2 + s^2 \\ r &= \frac{y^2 + s^2}{2s} \end{aligned} \qquad (10.4)$$

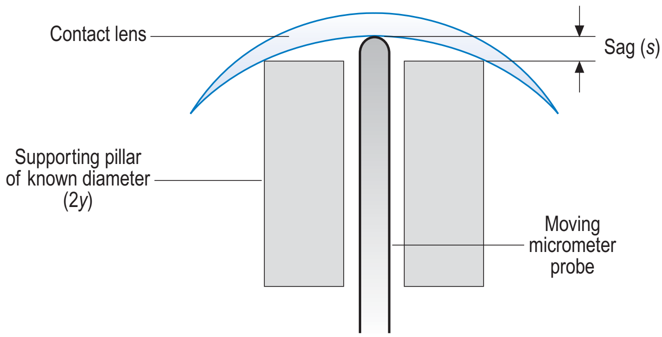

Figure 10.6 The microspherometer. The moving probe measures the sag of the surface (s) over the pillar diameter ($2y$)

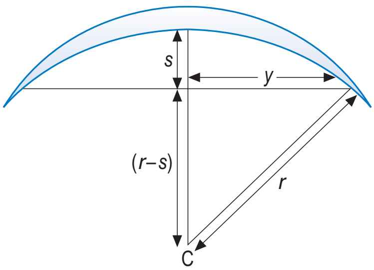

Figure 10.7 The principle of the microspherometer. C is the centre of curvature of the back surface of the contact lens of radius r and sag s over a semi-diameter y

The point of contact between the microspherometer probe and the lens surface can be observed by viewing, as in **Figure 10.6**, a magnified sagittal projection of the lens as it sits on the supporting pillar. When contact has been made, any further raising of the probe will induce movement of the contact lens. The micrometer adjustment screw uses a scale that is marked in 0.1 mm radius divisions.

As already stated, this approach can be used with a wet cell or dry cell technique. The problems associated with dry cells have already been listed. As far as spherometry is concerned there is an extra problem which is associated with the lack of physical support for the lens if the measurement is made in air. This results in a positive-powered lens, with its greatest thickness at the lens centre, tending to warp into a flatter form under the influence of gravity. The negative lens, on the other hand, with its thick edge hanging off the sides of the supporting pillar, will tend to warp into a steeper form.

For all lenses, whether wet or dry cell, it is important to centralize the lens on the supporting pillar. Some instruments incorporate devices to help with this centration. As an alternative to observing the point of contact between the probe and the lens surface, some instruments utilize the electrical conductivity of the soft lens with the point of contact completing an electrical

circuit, which results in the activation of a light emitting diode or the digital display of the surface radius. Alternatively an ultrasonic system can be used to measure the sag.

Some of the wet cell devices allow for measurement of lens central thickness when observing the sagittal projection. Also a conventional projection system is incorporated to allow measurement of the lens total diameter and inspection for surface engravings or defects.

10.4 The focimeter

The focimeter provides the optometrist with an accurate and convenient method of determining the back vertex power (BVP) of any ophthalmic lens. The lens under examination must rest against the focimeter stop. If the back surface of the lens is in contact with the stop then the end point of the measurement will give the BVP. The assumption is made that the back vertex of the lens coincides with the plane of the stop. A typical contact lens possesses surfaces which are very steeply curved and so the above assumption may introduce a significant position error when a standard focimeter stop is used as in **Figure 10.8(a)**, which illustrates that the contact lens back vertex is *x* mm to the right of the focimeter stop. In this case the focimeter reading at the end point will be the effective power of the lens *x* mm from the back surface.

For a positive contact lens the focimeter will give a power which is greater than the BVP, with the power being less than the BVP for a negative lens. The discrepancy will become significant as the contact lens power is increased. The problem is minimized as shown in **Figure 10.8(b)** by fitting a reduced

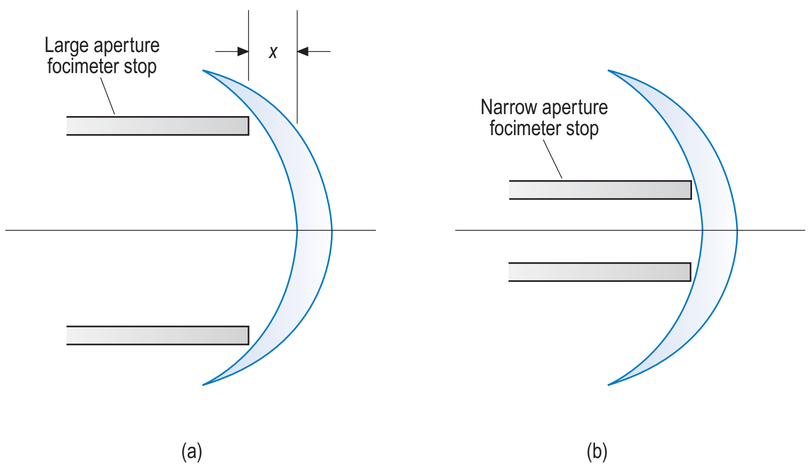

Figure 10.8 The contact lens resting on the focimeter stop. (a) The standard focimeter stop diameter results in the contact lens back vertex being positioned *x* mm to the right of the stop. (b) The reduced focimeter stop diameter for contact lens work

diameter stop to the focimeter. Many focimeters are provided with a reduced diameter stop as a standard accessory to be used when measuring the BVP of a contact lens. If the stop aperture diameter is too small then the pinhole effect will induce a less precise end point focus. The central aperture should be between 4 mm and 5 mm in diameter with the central aperture projecting 0.55 ± 0.02 mm less than the spectacle lens support which it replaces (BS ISO 9337–1:1999). This provides an accurate power reading on lenses with a BOZR of 8.00 mm.

10.4.1 Power measurement of soft lenses

The soft lens BVP can be measured in air provided certain precautions are taken. A focimeter with a vertical optical axis should be used with the power scale set at the expected reading. The lens surfaces should be carefully and *thoroughly* dried and the reading taken as soon as possible although Mandell (1974) suggested that shrinkage on dehydration does not substantially affect the readings for about 4 min. A reliability around ± 0.25 D is claimed. However, the lens is subjected to the environmental influences of air temperature and humidity. Undoubtedly a soft lens will be in a more stable state if examined in a wet cell. A focimeter with a vertical optical axis is essential for a wet cell measurement. The lens/wet cell system will be as shown in **Figure 10.9**. The power of the lens is dramatically reduced by the saline cell as illustrated below.

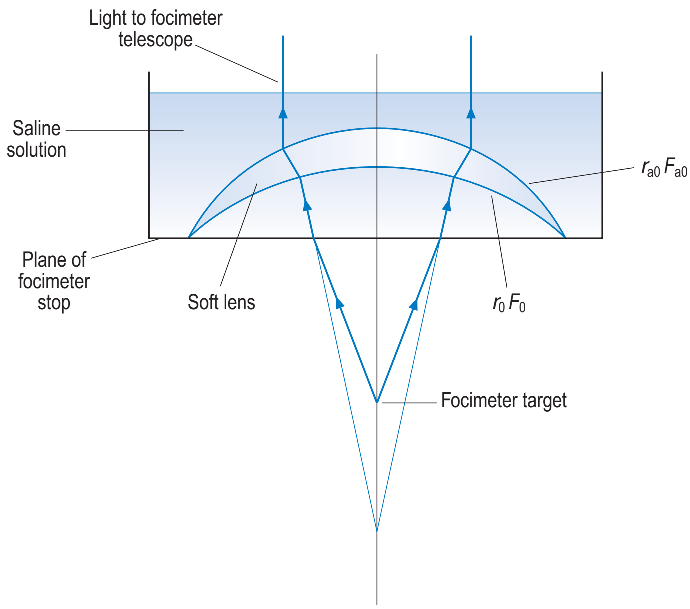

Figure 10.9 The soft lens in a wet cell. The surface powers are much reduced due to being in contact with the saline solution

Back surface

In air

$$F_0 = \frac{n_L - 1}{r_0}$$

where n_L is the refractive index of the lens.

In the saline cell

$$F_0 = \frac{n_L - n_s}{r_0}$$

where n_S is the refractive index of the saline solution.

$$\frac{F_0\text{ (air)}}{F_0\text{ (saline)}} = \frac{n_L - 1}{r_0} \cdot \frac{r_0}{n_L - n_s} = \frac{n_L - 1}{n_L - n_s}$$

Therefore

$$F_0\text{ (air)} = \frac{n_L - 1}{n_L - n_s} F_0\text{ (saline)}$$

If we take n_L as 1.444 and n_S as 1.336, then

surface power in air = surface power in saline × 4.111

We can therefore state that the surface power in the saline cell will be approximately one-quarter of the power in air.

Front surface

The above relationship will hold for the front surface power F_{a0}, and if we consider the contact lens to be a thin lens then

$$\text{BVP} = F_{a0} + F_0$$

Therefore

BVP in air = BVP in saline × 4.111

However, this does not hold for a thick lens system.

Suppose the soft lens has a BOZR of 8.30 mm, a central thickness t_C of 0.3 mm, a front surface radius r_{a0} of 7.10 mm and a refractive index n of 1.444.

BVP in air

In Figure 10.10

$$F_{a0} = \frac{444}{7.1} = +62.54\text{ D}$$

$$F_0 = \frac{-444}{8.3} = -53.49\text{ D}$$

$$\frac{t}{n} = 0.21\text{ mm}$$

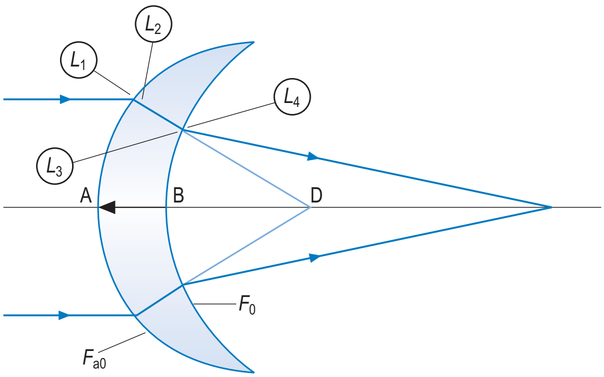

Figure 10.10 The path of light rays through the lens

Vergence (D) | **Distance (mm)**

$$
\begin{array}{llcccl}
L_1 = & 00.00 & & & & \\
F_{a0} = & \underline{+\,62.54} & & & & \\
L_2 = & +\,62.54 & \longrightarrow & \dfrac{1000}{62.54} & \longrightarrow & 15.99 = \text{AD} \\
 & & & & & \underline{-0.21} = \dfrac{t}{n} \\
L_3 = & +\,63.37 & \longleftarrow & \dfrac{1000}{15.87} & \longleftarrow & 15.78 = \text{BD} \\
F_0 = & \underline{-53.49} & & & & \\
L_4 = & +\,9.88 & = \text{BVP in air} & & &
\end{array}
$$

BVP in saline solution

In Figure 10.10

$$F_{a0} = \frac{108}{7.1} = +15.21 \text{ D}$$

$$F_0 = \frac{-108}{8.3} = -13.01 \text{ D}$$

$$\frac{t}{n} = 0.21 \text{ mm}$$

Vergence (D) | **Distance (mm)**

$$
\begin{array}{llcccl}
L_1 = & 00.00 & & & & \\
F_{a0} = & \underline{+15.21} & & & & \\
L_2 = & +15.21 & \longrightarrow & \dfrac{1000}{15.21} & \longrightarrow & 65.75 = \text{AD} \\
 & & & & & \underline{-0.21} = \dfrac{t}{n} \\
L_3 = & +15.26 & \longleftarrow & \dfrac{1000}{65.54} & \longleftarrow & 65.54 = \text{BD}
\end{array}
$$

$$F_0 = \underline{-13.01}$$
$$L_4 = +2.25 = \text{BVP in saline}$$

If

$$\text{BVP in air} = \text{BVP in saline} \times 4.111$$
$$\text{BVP in air} = +2.25 \times 4.111$$
$$= +9.25\ \text{D}$$

Clearly this produces an unacceptable error since the BVP in air is +9.88 D.

If we return to **Figure 10.9** we can see that one further problem remains and that is that light will be refracted at the plane saline/air boundary that rests on the focimeter stop which is a considerable distance from the contact lens back vertex. Let us continue with the example above. We have calculated that the BVP in saline is +2.25 D. In order to reach the plane of the focimeter stop, the light will need to travel from the back vertex of the lens to the saline/air boundary, a distance equal to the sag of the lens. The refraction at the saline/air boundary will reduce this distance to the apparent sag.

Let us assume for the sake of simplicity that the lens has a monocurve back surface of BOZR 8.3 mm and diameter 13 mm.

$$\text{Sag} = r_0 - \sqrt{(r_0^2 - y^2)}$$
$$= 8.3 - \sqrt{(8.3^2 - 6.5^2)} = 3.14\ \text{mm}$$

$$\text{Reduced sag } \frac{s}{n} = \frac{3.14}{1.336} = 2.35\ \text{mm}$$

Vergence (D) **Distance (mm)**

$$\text{BVP} = +2.25 \longrightarrow \frac{1000}{2.25} \longrightarrow 444.44$$

$$\underline{-\ 2.35}\ \text{reduced sag}$$

$$+2.26 \longleftarrow \frac{1000}{442.09} \longleftarrow 442.09$$

Therefore the emergent vergence in the plane of the focimeter stop is +2.26 D. Thus the dramatic reduction of the lens power in the saline means that there is a negligible change in vergence between the lens back vertex and the focimeter stop and so we can assume that we are measuring the lens BVP in the saline solution without the need to consider any effective power error due to the lens sag.

By far the greatest disadvantage of this approach is that the focimeter is working at a reduced precision. In our example above, the lens in air will read +9.88 D on the focimeter power scale, but in the saline cell the same lens reads +2.26 D, which indicates that the focimeter scale divisions of 0.25 D represent approximately 1.00 D when the wet cell is used. Thus the resulting measurement is likely to lack precision. The partial neutralization of the surface

powers by the saline will be accompanied by a similar partial neutralization of surface irregularities or unwanted cylinders or prisms.

It must be noted that BS EN ISO 9337-2:2004 recommends that the measurement of lenses immersed in saline be made by using either a Moire deflectometer or the Hartmann microlens instrument. These instruments, as mentioned in Chapter 8, are unlikely to be available to the practitioner. In these circumstances, the best clinical method available to us is to use the focimeter and to measure the soft lens in air.

10.4.2 The radius checking device

The radius checking (RC) device was an attachment to a focimeter that allowed the measurement of the BOZR of PMMA corneal contact lenses. The RC device developed by Sarver and Kerr (1964) was made from PMMA with a refractive index of 1.490 and is as illustrated in **Figure 10.11**. The convex surface has a radius r of 8.87 mm which gives it a power of

$$\frac{490}{8.87} = +55.24 \text{ D}$$

This surface is allowed to rest against the focimeter stop. The focimeter optical axis must be vertical. The contact lens under examination is placed concave surface upward onto the upper concave dish of the device. The contact lens floats on a *small* quantity of fluid of refractive index 1.490. If the RC device has a thickness t_1 and the contact lens has a central thickness t_2 with a BOZR of r_0 then we have in the focimeter, a thick lens with surface radii r and r_0 and central thickness $t_1 + t_2$. The focimeter measures the FVP (if we consider that the convex surface is the front surface). We know the power of one surface and the system central thickness, it is therefore an easy matter to deduce the power and the radius of the other surface when we have measured the FVP.

Let us take an example.

Suppose the RC device thickness t_1 is 1.5 mm and the lens under examination has a central thickness of 0.2 mm with a BOZR (r_0) which is unknown. If the focimeter reads +2.75 D when the RC device is used, what is the BOZR of the contact lens?

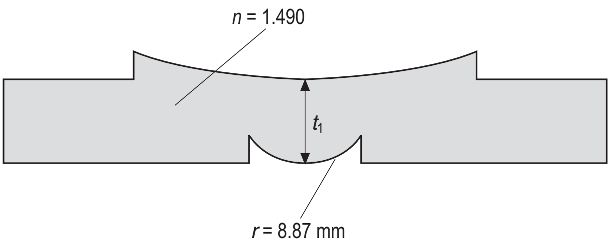

Figure 10.11 The RC device in cross-section

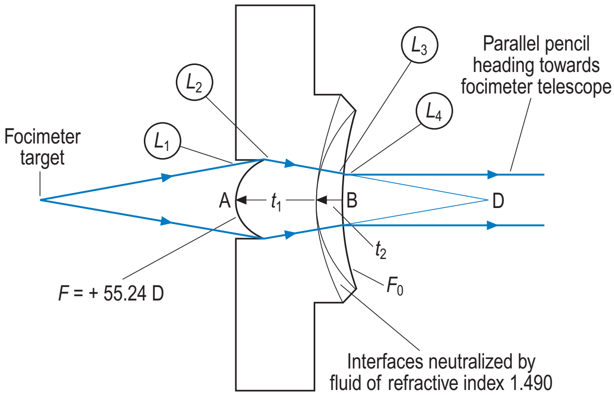

Figure 10.12 The RC device and its effect on the incident pencil of light rays from the focimeter target

In **Figure 10.12** the focimeter reads + 2.75 D. Therefore

Vergence $L_1 = -2.75$ D $\qquad t = t_1 + t_2$

$$\frac{t}{n} = \frac{1.5 + 0.2}{1.490} = 1.14 \text{ mm}$$

Vergence (D) **Distance (mm)**

$$
\begin{array}{llcll}
L_1 = & -2.75 & & & \\
F = & \underline{+55.24} & & & \\
L_2 = & +52.49 \longrightarrow & \dfrac{1000}{52.49} & \longrightarrow 19.05 & = \text{AD} \\
 & & & \underline{-1.14} & = t/n \\
L_3 = & +55.83 \longleftarrow & \dfrac{1000}{17.91} & \longleftarrow 17.91 & = \text{BD} \\
L_4 = & \underline{00.00} & & & \\
F_0 = & -55.83 & & &
\end{array}
$$

Power of the back surface of the contact lens $F_0 = -55.83$ D

Therefore

$$\text{BOZR} = \frac{1-n}{F_0} = \frac{-490}{-55.83} = 8.78 \text{ mm}$$

The calculation can be made more convenient by using the equation which reflects the relationship between front and back surfaces derived in Section 2.4. It will be recalled that we start off by assuming that the above system is thin and we calculate the BOZR for a thin lens. We then apply the correction factor $\{(n-1)/n\}t$ where t is the central thickness of the system, and since we are concerned with the radius of the back surface we subtract the correction factor from the thin lens radius.

In Figure 10.12

$$
\begin{aligned}
L_1 &= -\ \ 2.75\text{ D}\\
F &= \underline{+55.24\text{ D}}\\
L_2 &= +52.49\text{ D} = L_3 \text{ for a thin lens}\\
L_4 &= \underline{\ 00.00}\\
F_0 &= -52.49\text{ D}\\
r_0 &= \frac{1-n}{F_0} = \frac{-490}{-52.49} = 9.34\text{ mm}
\end{aligned}
$$

For a thick lens

$$
\begin{aligned}
\text{BOZR} &= r_0 - \frac{n-1}{n}\cdot t \qquad \text{from (2.1)}\\
&= 9.34 - 0.33t = \quad 9.34 - 0.56\\
&= 8.78\text{ mm}
\end{aligned}
$$

If the above exercise is repeated for a focimeter reading of +3.00 D then the BOZR works out to be 8.82 mm, i.e. a radius change of 0.04 mm. This gives some idea of the sensitivity of the instrument since focimeters are usually calibrated in 0.25 D steps.

10.5 The pachometer used for edge thickness measurement

This instrument has already been described in Section 5.17, where its major application was seen to be for measuring corneal thickness and monitoring corneal thickness changes. It could also be used to measure contact lens thickness. The pachometer holds promise as a means of measuring axial edge thickness. It is possible to specify with this instrument (suitably adapted) the axial thickness measurement at an exact location from the lens edge. Axial thickness measurements are not possible with any precision using a dial gauge, which measures the edge thickness in a more or less radial direction. The question is, however, *Radial to what?* Is the edge thickness radial to the back surface or the front surface, or is it somewhere between the two? The dial gauge is therefore of limited value to the practitioner for measuring edge thickness.

Guillon et al. (1987) modified a slit lamp to register the exact location of the pachometer measurement from the lens edge. It is therefore possible to specify with some precision the axial edge thickness and its location. If this approach were used at, say, three or four agreed distances from the edge of the lens then we would have a means of quantifying and checking the form of the lens edge.

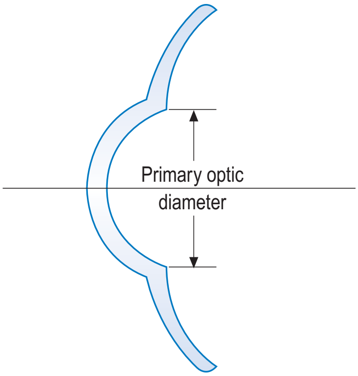

Figure 10.13 The primary optic diameter of a scleral lens

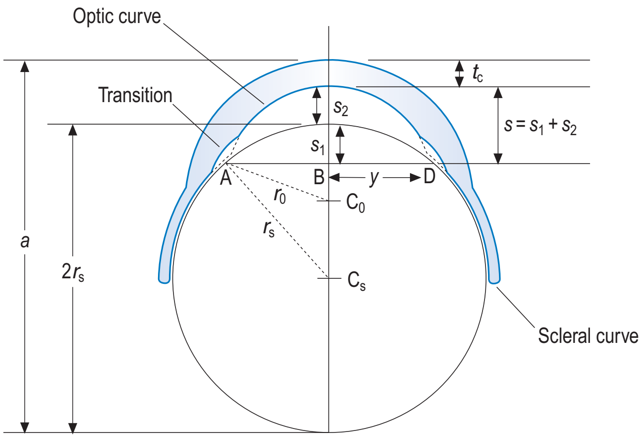

Figure 10.14 The scleral lens mounted on a sphere of radius of curvature equal to that of the scleral curve

10.6 The primary optic diameter of a scleral lens

The primary optic diameter (POD) may be stated when ordering a scleral contact lens. The dimension is illustrated in **Figure 10.13** where it can be seen that the POD is the diameter of the junction between the optic and scleral portions of the lens. This, however, is not usually available for direct inspection on a finished lens because of a transition curve being worked on the lens between the optic and scleral curves.

It can be measured indirectly as follows.

In **Figure 10.14**, AD = $2y$ is the primary optic diameter.

Using Pythagoras' theorem

In triangle ABC_0

$$\begin{aligned} r_0^2 &= y^2 + (r_0 - s)^2 \\ r_0^2 &= y^2 + r_0^2 - 2r_0 s + s^2 \\ y^2 &= 2r_0 s - s^2 \end{aligned} \tag{10.5}$$

In triangle ABC_s

$$y^2 = 2r_s s_1 - s_1^2 \tag{10.6}$$

$$s = s_1 + s_2 \tag{10.7}$$

Substituting (10.7) into (10.5)

$$y^2 = 2r_0(s_1 + s_2) - (s_1 + s_2)^2 \tag{10.8}$$

From equations (10.6) and (10.8)

$$\begin{aligned}
2r_s s_1 - s_1^2 &= 2r_0(s_1 + s_2) - (s_1 + s_2)^2 \\
2r_s s_1 - s_1^2 &= 2r_0 s_1 + 2r_0 s_2 - s_1^2 - 2s_2 s_1 - s_2^2 \\
2r_s s_1 &= 2r_0 s_1 + 2r_0 s_2 - 2s_2 s_1 - s_2^2 \\
2r_s s_1 - 2r_0 s_1 + 2s_2 s_1 &= 2r_0 s_2 - s_2^2 \\
2(r_s - r_0 + s_2)s_1 &= 2r_0 s_2 - s_2^2
\end{aligned}$$

$$s_1 = \frac{2r_0 s_2 - s_2^2}{2(r_s - r_0 + s_2)} \tag{10.9}$$

Equation (10.9) allows us to find s_1. We can measure r_0 and r_s using an optical spherometer or keratometer. We can then find s_2 from

$$s_2 = a - 2r_s - t_c$$

after measuring a and t_c.

We now have all we need to calculate s_1 using equation (10.9).

We can now find y by substituting into equation (10.6)

$$y^2 = 2r_s s_1 - s_1^2$$

A numerical example follows.

Measure a, r_s, r_0 and t_c.

$$s_2 = a - 2r_s - t_c$$

s_2 is found to be 1.3 mm for a lens with a BOZR r_0 of 8.00 mm and a scleral radius r_s of 13.00 mm. From equation (10.9)

$$s_1 = \frac{2(8)(1.3) - (1.3)^2}{2(13 - 8 + 1.3)}$$

$$s_1 = \frac{19.11}{12.6} = 1.5167$$

$$y^2 = 2r_s s_1 - s_1^2 \tag{10.6}$$

$$\begin{aligned}
y^2 &= 2(13)(1.5167) - (1.5167)^2 \\
y^2 &= 37.1338 \\
y &= 6.0938 \\
\text{Primary optic diameter} &= 2 \times 6.0938 = 12.1875 \text{ mm} \\
&= 12.19 \text{ mm (to 2 decimal places)}
\end{aligned}$$

10.7 Tolerances

The object of specifying dimensional tolerances should be to ensure that the product is sufficiently accurate to fulfil its function. There is no point in working to greater accuracy, even if it is technically possible, since this inevitably increases costs to no advantage. A tolerance of 0.05 mm on the diameter of a hydrated soft lens, for example, is very difficult (if not impossible) to achieve and practitioners would no doubt feel that the diameter of a hydrated soft lens does not require that degree of precision in order to ensure an acceptable performance and reproducibility on re-ordering.

10.7.1 Rigid corneal lenses

The dimensional tolerances for rigid corneal contact lenses are given in British Standard 7208: Part 1: 1992 'Contact lenses – Part 1: Specification for rigid corneal and scleral contact lenses', which is identical to the International Organisation for Standardisation publication ISO 8321 – 1:1991 'Optics and optical instruments – Contact lenses – Part 1: Specification for rigid corneal and scleral contact lenses'.

Measurements should be taken at a temperature of 20° ± 5° C.

The optical microspherometer is calibrated by measuring surfaces of known radius. Each surface is measured by taking 10 independent readings (independent meaning remove the surface and remount it between measurements). The mean and standard error of the mean for each set of 10 readings can be calculated and the standard error must be no greater than 0.02 mm. The mean values of each surface can be used to plot a calibration graph. This is recommended to be done by using a least squares best fit (linear regression).

The lens parameters to which tolerances have been recommended follow below. The common tolerances are summarized in **Table 10.1**.

It is recommended that the recorded value for any measurement is the mean of three independent readings.

BOZR – back optic zone radius

The tolerance is ± 0.025 mm for PMMA lenses
and ± 0.05 mm for RGP lenses
using a previously calibrated optical microspherometer, taking three independent measurements and calculating the mean.

The tolerances are increased for toroidal surfaces (see **Table 10.1**)

BOZD – back optic zone diameter

The tolerance is ±0.20 mm for both lens types (PMMA and RGP).

The recommended method of measurement, for BOZR and the other diameters listed below, is by use of a calibrated projection system producing

Table 10.1 Dimensional tolerances of rigid corneal lenses. All dimensions are expressed in mm

Back optic zone radius	±0.025 (PMMA)	±0.05 (RGP)
Toroidal surfaces where the radius difference between the two principal meridians is		
Less than or equal to 0.2	± 0.025 (PMMA)	± 0.05 (RGP)
Greater than 0.2 but less than or equal to 0.4	± 0.035 (PMMA)	± 0.06 (RGP)
Greater than 0.4 but less than or equal to 0.6	± 0.055 (PMMA)	± 0.07 (RGP)
Greater than 0.6	± 0.075 (PMMA)	± 0.09 (RGP)
Back optic zone diameter	± 0.20	
Back or front peripheral radius	± 0.10	
Back peripheral diameter	± 0.20	
Total diameter	± 0.10	
Front optic zone diameter	± 0.20	
Bifocal segment height	– 0.10 to + 0.20	
Centre thickness	± 0.02	

an overall magnification of not less than ×15, taking three independent measurements and calculating the mean. Blended transitions will add to the difficulty of making the measurement.

BPR or FPR – back peripheral radius or front peripheral radius

The tolerance is ±0.10 mm for both lens types.

The standard includes the words 'where measurable' which implies that there may be problems encountered which make the measurement impossible. The problems arise when narrow peripheral bands require measuring and this is exacerbated by blended transitions.

The method described in this chapter for calculating the edge lift by measuring the overall sag of the lens represents an indirect method of assessing the accuracy of the peripheral regions of the back surface of any RGP lens design.

BPD – back peripheral diameter

The tolerance is ±0.20 mm for both lens types.

It is worth noting that this is the same tolerance as the BOZD. Again the presence of transition blending will make the measurement more difficult.

TD – total diameter

The tolerance is ±0.10 mm for both lens types.

FOZD – front optic zone diameter

The tolerance is ±0.20 mm for both lens types.

A transition on the front surface of a lens is observed in a lenticulated lens. The tolerance is the same as that of the BOZD and the BPD.

Bifocal segment height

The tolerance is –0.10 to +0.20 mm for both lens types.

The method of measurement is as described for the measurement of diameters, i.e. utilization of a ×15 projection system.

Centre thickness

The tolerance is ± 0.02 mm for both lens types.

It is suggested that the measurement is made with a calibrated dial gauge that complies with BS 907 and that the gauge scale interval is not more than 0.01 mm. This recommendation is supported by the concept that an instrument must be able to measure to an accuracy equal to or better than twice the tolerance. Three independent readings of thickness should be taken and the average recorded.

BVP – back vertex power

The tolerance is the same for PMMA and RGP lenses. In general the tolerance is wider for higher lens powers and this is shown in **Table 10.2**.

10.7.2 Scleral lenses

The dimensional tolerances for scleral lenses should be measured using the same equipment and technique adopted for the equivalent tolerance in the rigid corneal lens. All that remains to be described, therefore, are the tolerances themselves and these are shown in **Table 10.3**.

The optical tolerances for scleral lenses are identical to those for the corneal rigid lenses (see **Table 10.2**).

10.7.3 Soft lenses

Soft lens specification is currently based on BS EN ISO 8321–2:2000 (BS7208–24:2000) 'Ophthalmic optics – Specifications for materials, optical and dimensional properties of contact lenses' Part 2: Single vision hydrogel contact lenses.

Table 10.2 Optical tolerances of rigid corneal and scleral lenses

Back vertex power (in the weaker meridian)	
For powers in the range	
0 to ± 5.00 D	± 0.12 D
over ± 5.00 to ± 10.00 D	± 0.18 D
over ± 10.00 to ± 15.00 D	± 0.25 D
over ± 15.00 to ± 20.00 D	± 0.37 D
over ± 20.00 D	± 0.50 D
Prismatic error (measured at the geometric centre of the optic)	
Back vertex power 0 to 6.00 D	± 1/4 Δ
Back vertex power over 6.00 D	± 1/2 Δ
Prescribed prism	± 1/4 Δ
Cylinder power	
up to 2.00 D	± 0.25 D
over 2.00 to 4.00 D	± 0.37 D
over 4.00 D	± 0.50 D
Cylinder axis	± 5°

Table 10.3 Dimensional tolerances for scleral lenses (BS 7208 : Part 1: 1992). All dimensions are in mm

Back optic zone radius	± 0.10
Toroidal surfaces where the radius difference between the two principal meridians is	
Less than or equal to 0.2	± 0.12
Greater than 0.2 but less than or equal to 0.4	± 0.13
Greater than 0.4 but less than or equal to 0.6	± 0.15
Greater than 0.6	± 0.17
Back optic zone diameter	± 0.20
Back scleral radius – preformed lens	± 0.10
Basic or primary optic diameter	± 0.20
Back or front peripheral radius	± 0.10
Back peripheral diameter – preformed lens	± 0.20
Total diameter	± 0.25
Front optic zone diameter	± 0.20
Bifocal segment height	– 0.10 to + 0.20
Centre thickness	± 0.10
Vertex clearance from cast – impression lens	± 0.02

Table 10.4 Dimensional and optical tolerances of soft lenses

	Tolerances in mm	
Back optic zone radius	± 0.20	
Total diameter	± 0.20	
Central optic zone diameter	± 0.20	
Sag at specified diameter	± 0.05	
Centre thickness		
Up to 0.1 mm	± 0.01	+ 10%
Over 0.1 mm	± 0.015	+ 5%
Back vertex power	Tolerances in dioptres	
In weaker meridian		
Plano to ± 10.00 D	± 0.25	
± 10.00 D to ± 20.00 D	± 0.50	
Over ± 20.00 D	± 1.00	
Cylinder power		
Plano to 2.00 D	± 0.25	
2.25 to 4.00 D	± 0.37	
Over 4.00 D	± 0.50	
Cylinder axis	Tolerances in degrees ± 5	
Refractive index	± 0.005	
Water content	± 2%	
Oxygen permeability	± 20% (of nominal Dk)	

The suggestion is made that when measuring the power of a lens which incorporates a cylindrical component, then the prescription should be transposed into the cross-cylinder form and the tolerance for cylindrical power applied to each meridian.

10.7.4 *Internal defects and surface imperfections*

The standards also describe the need to check that the lens is free of inclusions, surface imperfections and is free of strain within the material. The edge of the lens will require close examination. These matters are outside the scope

of this book but it is worth noting that the British standard suggests using a ×6 magnification for examining internal defects, a ×2 magnification for surface imperfections and a ×10 magnification for inspecting the edge profile.

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CHAPTER 11

Computer programs

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Books dealing with the optics of contact lenses often are accompanied by pages of tables that speed up the process of calculation. One such example would be a set of tables converting radius to power and vice versa for a given refractive index. This book replaces tables with suitable computer programs written in Visual Basic. A simple program which performs, for example, the effective power calculation can replace a number of pages of tables and using such a program is more convenient than referring to tables.

A number of the programs provide the practitioner with information which allows a more precise assessment of the inter-relationships encountered in fitting. Without this type of information the practitioner relies on her or his intuitive deductions and, after only a short time using programs like the ones included in this chapter, it becomes obvious that our intuitive conclusions often result in misleading deductions which lead to deteriorating rather than improving fitting relationships between the contact lens and the cornea.

Previous textbooks have provided students with questions and answers in order to test the students' abilities. The computer programs provided here

achieve the same end in that the student can set her/himself a question and the programs will provide the answer to each and every question. The student can then attempt the question and check her/his answer with that provided by the program. The big advantage of using the programs in this way is that there is no end to the number of questions that may be asked. The programs deal with the following:

1. *EPower* – This program converts spectacle refraction to ocular refraction and vice versa.
2. *Heine* – This program calculates surface power given surface radius or surface radius given surface power for any material refractive index in air.
3. *Poweradius* – This program calculates surface radius or surface power of an interface between two media of given refractive indices.
4. *Frontradius* – This program calculates the front surface radius and the front vertex power of any contact lens of given BOZR, BVP, centre thickness and refractive index.
5. *AEL AET* – This program calculates the axial edge lift, radial edge lift, axial edge thickness and radial edge thickness for any contact lens given the back surface specification (radii and diameters), BVP, centre thickness and refractive index. It also calculates the overall sagitta of the lens. It can deal with multicurves (C2 to C5), offset and conicoidal lenses.
6. *Periphradius* – This program calculates the back surface radii required to give a specified axial edge lift given the lens diameters. It can deal with multicurves (C2 to C5), offset and conicoidal lenses.
7. *Radthick* – This program displays the radial thickness profile for multicurves (C1 to C5), offset and conicoidal lenses. It also calculates the arithmetic mean thickness and the harmonic mean radial thickness of the lens.
8. *Axthick* – This program displays the axial thickness profile for multicurves (C1 to C5), offset and conicoidal lenses with or without lenticulation. It also calculates the arithmetic mean thickness and the harmonic mean axial thickness of the lens.
9. *TLT AEC* – This program calculates the tear layer thickness and axial edge clearance for RGP multicurve lenses (C2 to C5), offset and conicoidal lenses given the corneal apical radius and p-value and the lens back surface specification (radii and diameters). It also displays a fluid lens profile for both principal meridians.
10. *Design* – This program calculates the RGP back surface radii to achieve a specified tear layer thickness and axial edge clearance given the corneal apical radius and p-value and the lens diameters. It also displays the fluid lens profile for both principal meridians. It can deal with multicurves (C2 to C5), offset, conicoidal and polynomial lenses.
11. *Orthok* – This program calculates the RGP lens back surface specification for orthokeratology fitting of myopic eyes given the corneal apical radius and p-value and the refractive error for a C5 and a polynomial design.

12. *Pandro by kerat* – This program will calculate the apical radius and p-value from keratometric measurements using the Zeiss telecentric keratometer.
13. *Torics* – This program will calculate the theoretical BVP to order when fitting either toric soft or toric RGP lenses. In the case of soft lenses, the calculation provides the effective power of the spectacle correction at the eye for the two principal meridians and assumes that the fluid lens power is plano. The calculation does exactly the same thing for RGP lenses but provides the contact lens power that also neutralizes the power of the fluid lens in both meridians.
14. *Rotation* – This program calculates either the theoretical cylinder induced by an inappropriate orientation of a toric contact lens *or* it will calculate the amount of unwanted rotation given the over-refraction measurement of the resultant unwanted cylinder power and cylinder axis orientation *or* it will calculate the BVP to order when a soft toric lens of inappropriate power and/or orientation is used for the over-refraction.

For all programs, there are windows that require values to be entered. This can be done in one of two ways:

1. Each window includes vertical scroll bars that allow you to increase (more positive) or decrease (more negative) a value by pressing the upper or lower button with the mouse.
2. The value you wish to use can be typed into the window by a left click on the mouse button with the cursor positioned within the window and then typing the value that you wish to use.

The instructions that follow, with the exception of the first example in Section 11.1, will use the second option in describing how the program should be used.

11.1 Effective power

The effective power of a spectacle correction at the cornea is always required when, during an over-refraction, the lens in the refractor head or trial frame has a power greater than 4.00 D. The program called *EPower* will perform this task.

In order to check the program try the following.

Example 1

- To use this program, first ensure that the radio button is activated for *Ocular refraction required*
- Use the vertical scroll bar arrows on the *vertex distance (mm)* window to increase the distance to **14**

• Use the vertical scroll bar arrows on the *Power of spectacle lens (D)* window to decrease the power down to **–12** or left click the mouse on the window and type **–12**
• Left click the mouse on the *Calculate ocular refraction* command button

The answer

The effective power at the eye is –10.27 DS

should appear at the bottom of the screen. The screen display is shown in Figure 11.1.

• The *Clear* command button will change the values in the windows to their default settings so that you can enter new values if you wish. Do this now and try Example 2.

Example 2

• Left click the mouse on the *Spectacle lens power required* radio button
• Use the vertical scroll bar arrows on the *vertex distance (mm)* window to increase the distance to **14**
• Left click the mouse on the *Ocular refraction (D)* window and type **8**
• Left click the mouse on the *Calculate spectacle refraction* command button

The answer

The spectacle lens power required is 7.19 DS

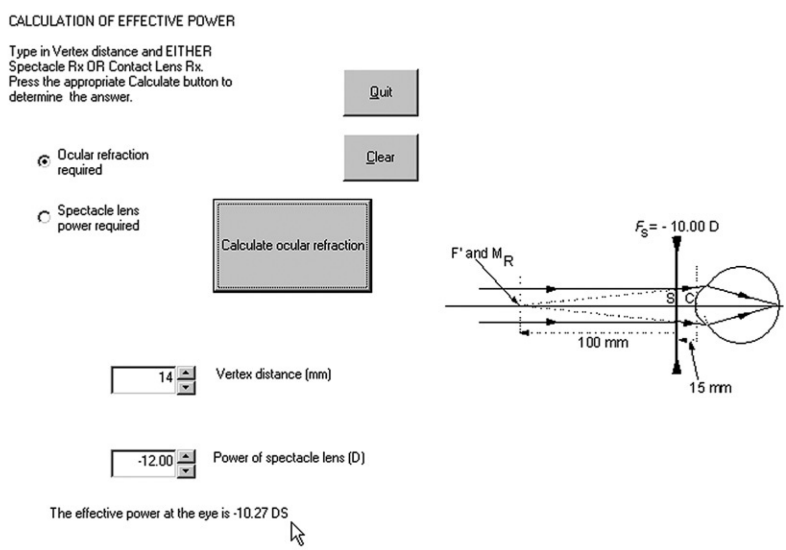

Figure 11.1 The screen display for calculation of the ocular refraction from the spectacle lens power at a given vertex distance

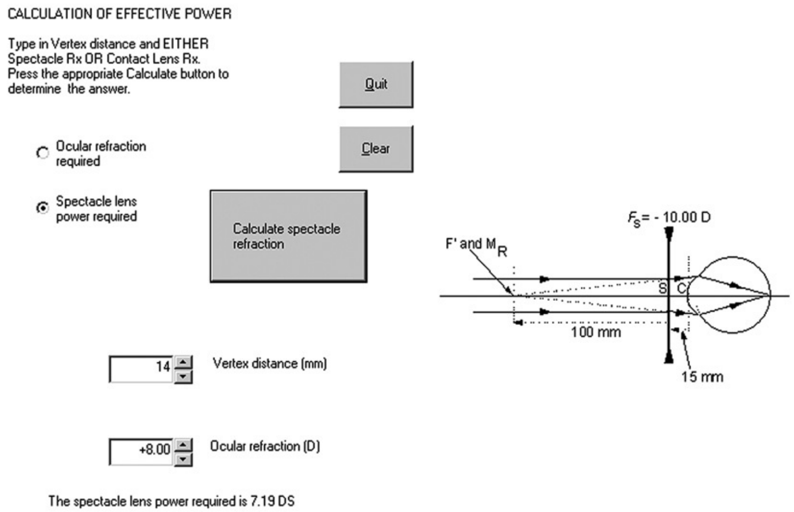

Figure 11.2 The screen display for calculation of the spectacle lens given the ocular refraction

should appear at the bottom of the screen. The screen display is shown in Figure 11.2.

- The *Quit* command button is used when you wish to leave the program

11.2 Conversion of radius to power and vice versa

11.2.1 Radius to power change in air

The conversion of the radius of curvature to surface power or vice versa is a common requirement which, when dealt with in tabular form, is restricted to a material of given refractive index in air for each set of tables. The computer program has the advantage that it will deal with any refractive index. If the user enters a refractive index for the material of 1.336 then the program becomes a computerized version of Heine's scale converting radius to power and vice versa for a medium of refractive index 1.336 in air. Thus this program is called *Heine*.

In order to check the program try the following.

Example 1

- Ensure that the *Calculate Power* radio button is active
- Left click the mouse on the *Refractive Index* window and type **1.490**
- Left click the mouse on the *Radius of Curvature (mm)* window and type **7.4**
- Left click the mouse on the *Calculate Power* command button

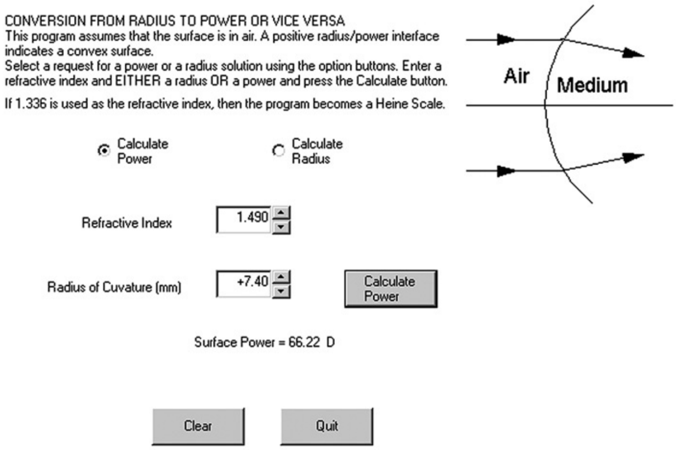

Figure 11.3 The screen display for calculating the surface power in air given the refractive index of the lens and the radius of curvature of the surface

The answer

Surface Power = 66.22 D

should appear on the screen display that is shown in **Figure 11.3**.

- Left click the mouse on the *Clear* command button to prepare for Example 2

Example 2

- Left click the mouse on the *Calculate Radius* radio button
- Left click the mouse on the *Refractive Index* window and type **1.336**
- Left click the mouse on the *Surface Power (D)* window and type **43.64**
- Left click the mouse on the *Calculate Radius* command button

The answer

Surface Radius = 7.7 mm

should appear on the screen display which is shown in **Figure 11.4**.

- Left click the mouse on the *Quit* command button to leave this program

11.2.2 Radius to power conversion with any combination of refractive index

The Section 11.2.1 program will only deal with a surface in air. It is sometimes necessary to consider the radius to power and power to radius conversion for two media neither of which is air. The program called *Power radius* is provided to deal with this type of problem. It is assumed that light is

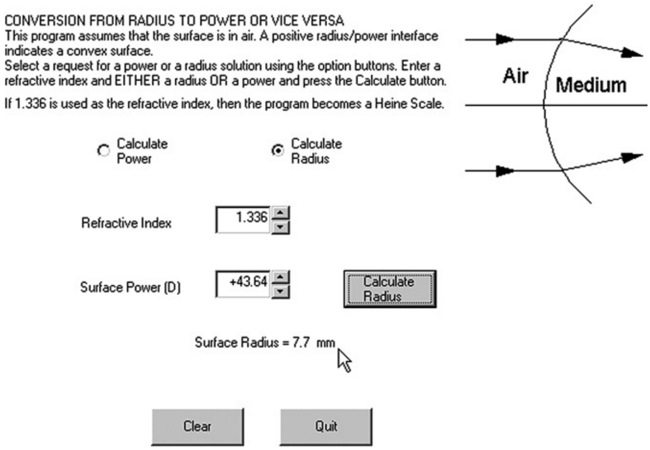

Figure 11.4 The screen display for calculating the surface radius of curvature given the refractive index of the lens and the surface power

travelling from left to right, with medium 1 to the left of medium 2 and the sign convention for the Cartesian system is upheld. Thus any surface with a radius of curvature measured from the surface to the centre of curvature will have a positive radius where this is measured in the same direction as the incident light. This will be true for both the front and back surface of a contact lens.

In order to check the program try the following.

Example 1

- Ensure that the *Calculate Radius* radio button is activated
- Left click the mouse on the *Surface Power (D)* window and type **20**
- Left click the mouse on the *Refractive index of Medium 1* window and type **1.336**
- Left click the mouse on the *Refractive index of Medium 2* window and type **1.49**
- Left click the mouse on the *Calculate Radius* command button

The answer

SURFACE RADIUS is 7.7 mm

should appear in the screen display which is illustrated in **Figure 11.5**.

- Left click the mouse on the *Clear* command button in order to prepare for Example 2.

Example 2

- Left click the mouse on the *Calculate Power* radio button
- Left click the mouse on the *Radius of Curvature (mm)* window and type **7.8**

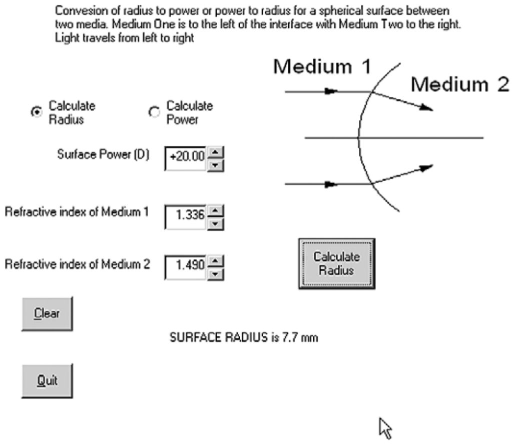

Figure 11.5 The screen display for calculating the surface radius of curvature when the surface power is known for two media

- Left click the mouse on the *Refractive index of Medium 1* window and type **1.49**
- Left click the mouse on the *Refractive index of Medium 2* window and type **1.336**
- Left click the mouse on the *Calculate Power* command button

The answer

SURFACE POWER is –19.74 D

should appear in the screen display which is illustrated in **Figure 11.6**.

- Left click the mouse on the *Quit* command button to leave this program

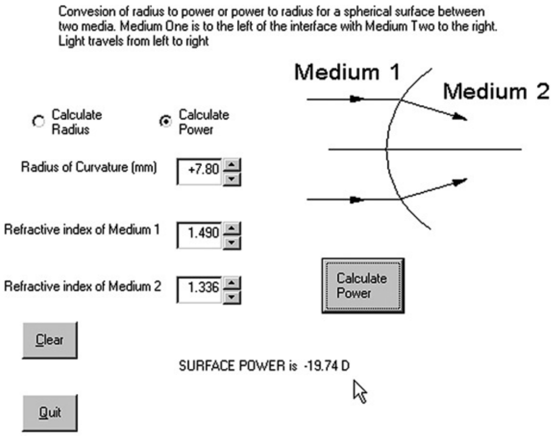

Figure 11.6 The screen display for calculating the surface power between two media when the surface radius of curvature is known

11.3 Front surface radius, FVP and spectacle magnification

This program called *Frontradius* calculates the front surface radius and front vertex power (FVP) of a contact lens, given the BVP, central thickness, BOZR and refractive index of the lens material. This is of course what the manufacturer needs to know when producing a lens with a BVP equal to that requested by the optometrist. It may be useful to the practitioner in that it gives an indication of the relationship between the front and back surfaces of the contact lens. The program also calculates the spectacle magnification from the product of the power factor and shape factor.

In order to check the program try the following.

Example 1

- Left click the mouse on the *BOZR (mm)* window and type **7.9**
- Left click the mouse on the *BVP (D)* window and type **5.5**
- Left click the mouse on the *Centre thickness (mm)* window and type **.6**
- Left click the mouse on the *Refractive index* window and type **1.49**
- Leave the *Distance from cornea to entrance pupil (mm)* at the default value of **3.0**
- Left click the mouse on the *Calculate* command button

The answers

Front surface radius is 7.45 mm
Front Vertex Power is 5.22 D
Spectacle magnification = 1.017X

will appear in the screen display which is illustrated in **Figure 11.7**.

- Left click the mouse on the *Quit* command button to leave this program

11.4 Peripheral radii required to give a specific axial edge lift

The program called *Periphradius* calculates the BPRs required for any C2, C3, C4, C5 offset or conicoidal lens to give a stipulated axial edge lift. In the case of the C2 lens the single peripheral curve produces all the edge lift. The two peripheral curves in the C3 share the edge lift, with the mid-curve producing two-thirds and the outer curve one-third of the total edge lift at the lens edge. In the case of the C4, the first peripheral curve is responsible for generating half of the total edge lift, with the second peripheral curve generating one-third and the final outer curve one-sixth of the total edge lift. In the C5, the share of the edge lift is 0.4 for the first peripheral curve, 0.3 for the second, 0.2 for the third and 0.1 for the final peripheral curve at the requested diameter. The sharing of the edge lift can be altered by changing the default values in the appropriate windows. Note that the individual shares of the edge lift must add up to unity.

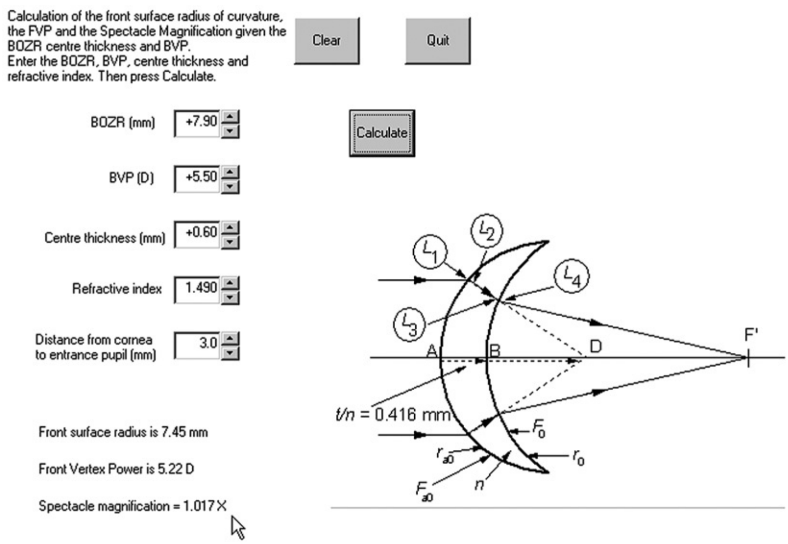

Figure 11.7 The screen display for calculating the front surface radius, the front vertex power and the spectacle magnification for any given contact lens

It must be pointed out here that the share for any intermediate curve is the share at the actual lens edge. In other words if we consider the C3, the mid curve radius would produce two-thirds of the requested edge lift when the mid curve is extended to the lens edge. The alternative approach, which is not used here, would be to consider the edge clearance of the mid curve where it stops, i.e. at the transition between the second and the third curves.

The edge lift sharing described above produces lenses similar to those described and used by Rabbetts (1976).

In the case of the conicoidal lens, the program calculates the p-value required to produce the requested axial edge lift. It also gives the overall sag which can be checked using a radiuscope.

The first page of the computer display is illustrated in **Figure 11.8**. The user is required to select one of the lens options. This is achieved by double clicking on the required option. The second screen will then be displayed which allows the selected option to be assessed.

In order to check the program try the following.

Example 1 – the bicurve back surface

- Left double click the mouse on the *C2* option on the first screen and the bicurve option will then be displayed
- Left click the mouse on each of the four windows and type the following in each window:

BOZR type **7.8**
BOZD type **6.5**

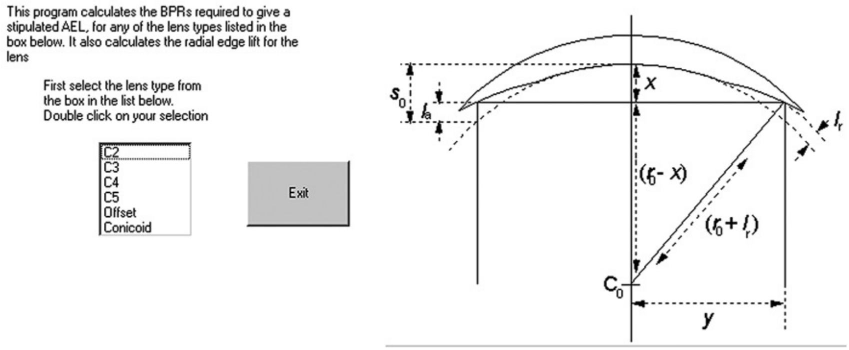

Figure 11.8 The first screen display for selecting the type of lens where the BPRs will be calculated for a given AEL

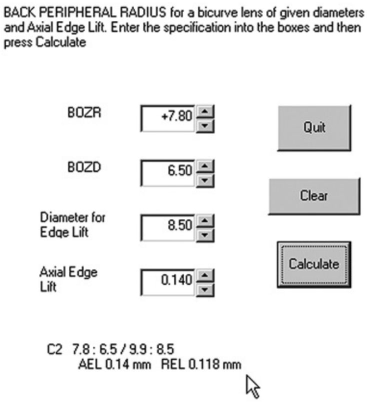

Figure 11.9 The second screen display for calculating the BPR of a bicurve lens given the AEL

Diameter for Edge Lift type **8.5**
Axial Edge Lift type **0.14**

- Left click the mouse on the *Calculate* command button

The answer

C2 7.8:6.5/9.9:8.5

AEL 0.14 mm REL 0.118 mm

appears on the screen display which is illustrated in **Figure 11.9**. Thus the program has calculated the single peripheral radius required to give the requested axial edge lift and has also calculated the radial edge lift.

- Left click the mouse on the *Quit* command button

Example 2 – the tricurve back surface

- Left double click the mouse on the *C3* option on the first screen and the tricurve option will then be displayed

Note that there are two windows that indicate the default share of the axial edge lift between the two peripheral curves.

- Left click the mouse on each of the remaining five windows and type the following in each window:

BOZR	type **7**
BOZD	type **7**
BPD1	type **7.8**
Diameter for Edge Lift	type **8.6**
Axial Edge Lift	type **0.12**

- Left click the mouse on the *Calculate* command button

  The answer

  C3 7:7/7.86:7.8/8.99:8.6

  AEL 0.12 mm REL 0.095 mm

  appears on the screen display which is illustrated in **Figure 11.10**. Thus the program has calculated the two peripheral radii required to give the requested axial edge lift and has also calculated the radial edge lift.
- Left click the mouse on the *Quit* command button

Example 3 – the tetracurve back surface

- Left double click the mouse on the *C4* option on the first screen and the tetracurve option will then be displayed.

Note that there are three windows that indicate the default share of the axial edge lift between the three peripheral curves.

- Left click the mouse on each of the six windows and type the following in each window:

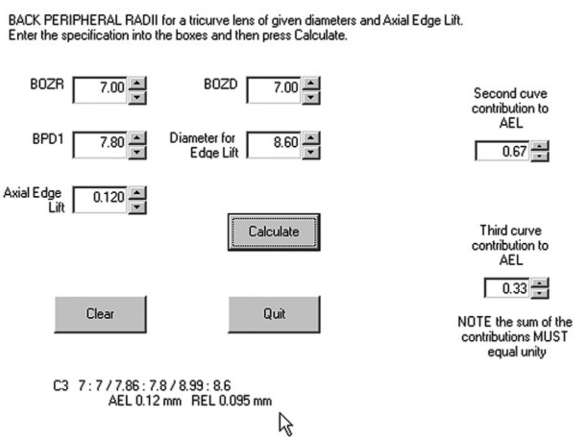

Figure 11.10 The second screen display for calculating the BPRs of a tricurve lens given the AEL. Note the default values for the share of the contribution to the AEL. These can be changed if you wish to change them

BOZR type **8.6**
BOZD type **7**
BPD1 type **7.8**
BPD2 type **8.6**
Diameter for Edge Lift type **9.2**
Axial Edge Lift type **0.15**

- Left click the mouse on the *Calculate* command button
  The answer

  C4 8.6:7/9.58:7.8/10.88:8.6/13.03:9.2
  AEL 0.15 mm REL 0.127 mm

  appears on the screen display which is illustrated in **Figure 11.11**. Thus the program has calculated the three peripheral radii required to give the requested axial edge lift and has also calculated the radial edge lift.
- Left click the mouse on the *Quit* command button

Example 4 – the pentacurve back surface

- Left double click the mouse on the *C5* option on the first screen and the pentacurve option will then be displayed

Note that there are four windows that indicate the default share of the axial edge lift between the four peripheral curves.

- Left click the mouse on each of the seven windows and type the following in each window:

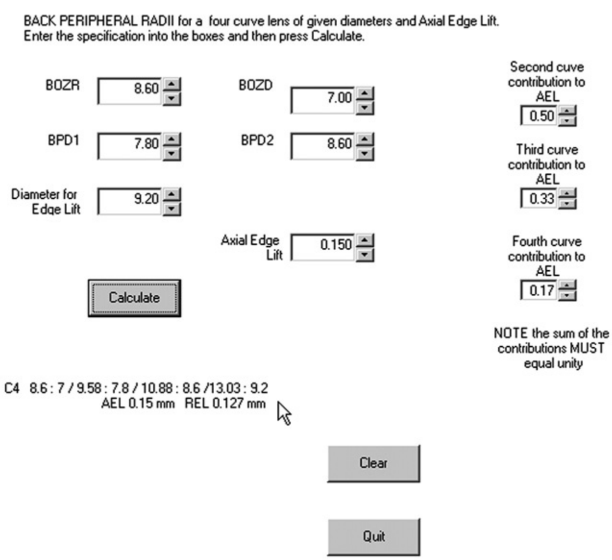

Figure 11.11 The second screen display for calculating the BPRs of a tetracurve lens given the AEL

BOZR type **7.8**
BOZD type **7**
BPD1 type **8**
BPD2 type **8.5**
BPD3 type **9**
Diameter for Edge Lift type **9.5**
Axial Edge Lift type **0.15**

- Left click the mouse on the *Calculate* command button

  The answer

  C5 7.8:7/8.26:8/8.9:8.5/9.66:9/10.59:9.5
  AEL 0.15 mm REL 0.119 mm

  appears on the screen display which is illustrated in **Figure 11.12**. Thus the program has calculated the four peripheral radii required to give the requested axial edge lift and has also calculated the radial edge lift.
- Left click the mouse on the *Quit* command button

Example 5 – the offset bicurve back surface

- Left double click the mouse on the *Offset* option on the first screen and the offset bicurve option will then be displayed
- Left click the mouse on each of the four windows and type the following in each window:

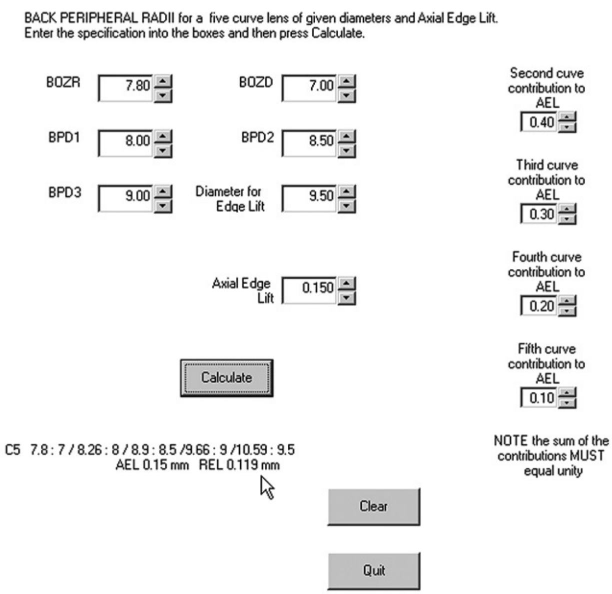

Figure 11.12 The second screen display for calculating the BPRs of a pentacurve lens given the AEL

BOZR type **7.2**
BOZD type **6.5**
Diameter for Edge Lift type **9.5**
Axial Edge Lift type **0.12**

- Left click the mouse on the *Calculate* command button

The answer

OFFSET 7.2:6.5/12.65:9.5
AEL 0.12 mm REL 0.091 mm

appears on the screen display which is illustrated in **Figure 11.13**. Thus the program has calculated the single peripheral radius required to give the requested axial edge lift and has also calculated the radial edge lift.

- Left click the mouse on the *Quit* command button

Example 6 – the conicoidal back surface

- Left double click the mouse on the *Conicoid* option on the first screen and the conicoid option will then be displayed
- Left click the mouse on each of the three windows and type the following in each window:

Apical Radius type **7.85**
Diameter for Edge Lift type **9.5**
Axial Edge Lift type **0.12**

- Left click the mouse on the *Calculate* command button

The answer

The p-value of this surface is 0.309
The overall sagitta = 1.48 mm at diameter 9.5 mm

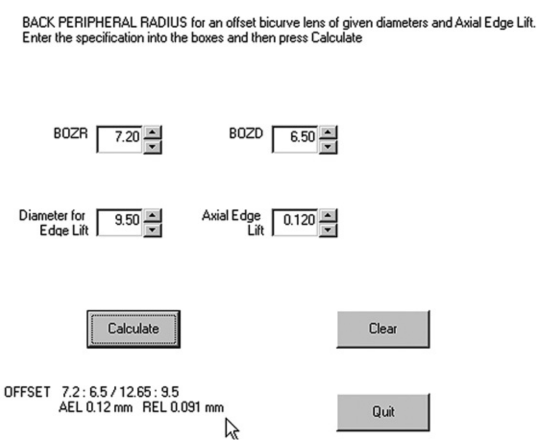

Figure 11.13 The second screen display for calculating the BPR of an offset bicurve lens given the AEL

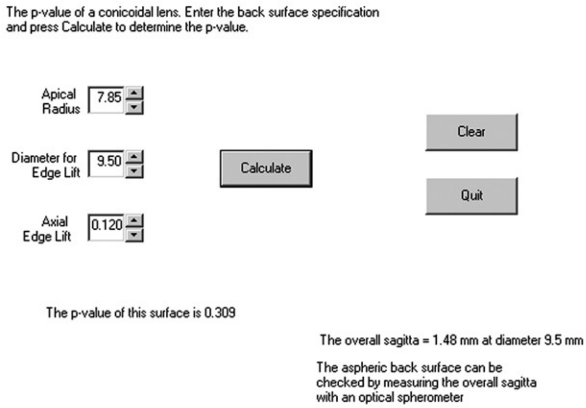

Figure 11.14 The second screen display for calculating the p-value of a conicoidal lens given the AEL

appears on the screen display which is illustrated in **Figure 11.14**. Thus the program has calculated the surface asphericity and the overall sag.

- Left click the mouse on the *Quit* command button
- Left click the mouse on the *Exit* command button to leave this program

11.5 Edge thickness and edge lift

This program called *AEL AET* represents an alternative and more flexible approach to the problem of edge lift. The user simply enters the lens specification and the computer calculates the edge lift. The previous program (Section 11.4) can be used to determine some suitable radii from which to start. These radii can then be altered and the effect of the alterations on the axial edge lift will be immediately seen. This program also gives the radial edge lift (measured normal to the back optic zone surface), and can calculate the axial and radial edge thickness where the lens power, central thickness and refractive index are known. The program is designed to work for C2 to C5, offset and conicoid lenses although in the latter two types the program requests the axial edge lift and simply calculates the radial edge lift with, once again, the option for calculating the edge thickness. In all cases the program calculates the overall sag of the lens which can be used to check the peripheral radii and diameters for accuracy by measuring an actual lens overall sag and checking this against the calculated result.

In order to check the program try the following.

The first screen is similar to that illustrated in **Figure 11.8** asking the user to select one of the lens options. Again this is achieved by double clicking on the required option. The second screen will then be displayed which allows this option to be assessed.

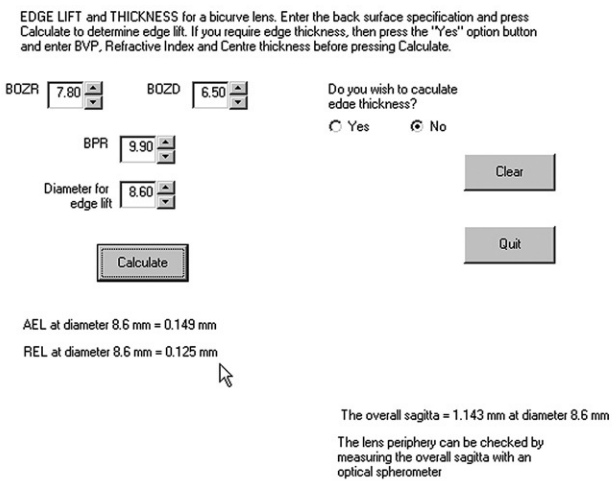

Figure 11.15 The second screen display for calculating the edge lift of a bicurve lens

Example 1 – the bicurve back surface

- Left double click the mouse on the *C2* option on the first screen and the bicurve option will then be displayed
- Make sure that the *Do you wish to calculate edge thickness?* radio button is activated on *No*
- Left click the mouse on each of the four windows and type the following in each window:

BOZR type **7.8**
BOZD type **6.5**
BPR type **9.9**
Diameter for edge lift type **8.6**

- Left click the mouse on the *Calculate* command button

  The answer

  AEL at diameter 8.6 mm = 0.149 mm
  REL at diameter 8.6 mm = 0.125 mm
  The overall sagitta = 1.143 mm at diameter 8.6 mm

  appears on the screen display as is illustrated in **Figure 11.15**.
- Left click the mouse on the *Quit* command button

Example 2 – the tricurve back surface

- Left double click the mouse on the *C3* option on the first screen and the tricurve option will then be displayed
- Make sure that the *Do you wish to calculate edge thickness?* radio button is activated on *No*

- Left click the mouse on each of the six windows and type the following in each window:

BOZR	type **7**	*BOZD*	type **7**
BPR1	type **7.85**	*BPD1*	type **7.8**
BPR2	type **9**	*Diameter for edge lift*	type **8.6**

- Left click the mouse on the *Calculate* command button

The answer

AEL at diameter 8.6 mm = 0.12 mm
REL at diameter 8.6 mm = 0.095 mm
The overall sagitta = 1.356 mm at diameter 8.6 mm

appears on the screen display.

- Left click the mouse on the *Quit* command button

Example 3 – the tetracurve back surface

- Left double click the mouse on the *C4* option on the first screen and the tetracurve option will then be displayed
- Left click the *Do you wish to calculate edge thickness?* radio button on the *Yes* option
- Left click the mouse on each of the windows and type the following in each window:

BOZR	type **7.8**	*BOZD*	type **7**
BPR1	type **8.3**	*BPD1*	type **7.8**
BPR2	type **8.8**	*BPD2*	type **8.6**
BPR3	type **11**	*Diameter for edge lift*	type **9**
BVP	type **–6**		
Refractive Index	type **1.49**		
Centre thickness	type **.3**		

- Left click the mouse on the *Calculate* command button

The answer

AEL at diameter 9 mm = 0.102 mm
REL at diameter 9 mm = 0.084 mm
AET at diameter 9 mm = 0.376 mm
RET at diameter 9 mm = 0.32 mm
The overall sagitta = 1.327 mm at diameter 9 mm

appears on the screen display which is illustrated in **Figure 11.16**. Note that any negative values for the axial edge thickness *AET* or radial edge thickness *RET* indicate that the lens centre thickness is inadequate and will need to be increased. A message box will come up on the screen whenever the radial thickness is less than 0.03 mm.

- Left click the mouse on the *Quit* command button

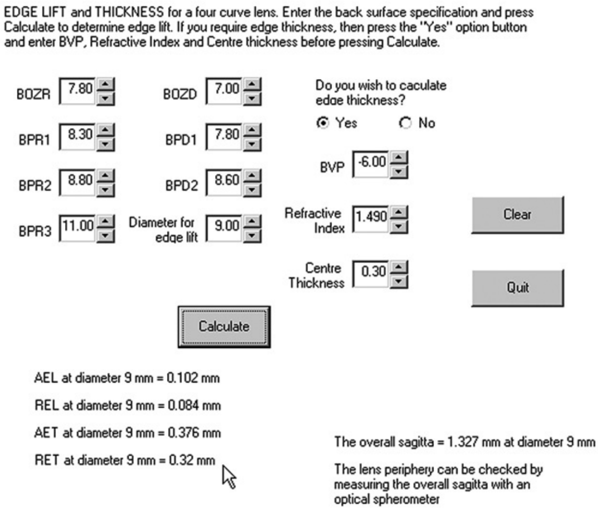

Figure 11.16 The second screen display for calculating the edge lift and edge thickness of a tetracurve lens

Example 4 – the pentacurve back surface

- Left double click the mouse on the *C5* option on the first screen and the pentacurve option will then be displayed
- Make sure that the *Do you wish to calculate edge thickness?* radio button is activated on *No*
- Left click the mouse on each of the windows and type the following in each window:

BOZR	type **7.2**	*BOZD*	type **7**
BPR1	type **7.7**	*BPD1*	type **7.8**
BPR2	type **8.2**	*BPD2*	type **8.6**
BPR3	type **10.5**	*BPD3*	type **9**
BPR4	type **11.5**	*Diameter for edge lift*	type **9.5**

- Left click the mouse on the *Calculate* command button

The answer

AEL at diameter 9.5 mm = 0.229 mm
REL at diameter 9.5 mm = 0.173 mm
The overall sagitta = 1.56 mm at diameter 9.5 mm

appears on the screen display.

- Left click the mouse on the *Quit* command button

Example 5 – the offset bicurve back surface

- Left double click the mouse on the *Offset* option on the first screen and the offset bicurve option will then be displayed

- Make sure that the *Do you wish to calculate edge thickness?* radio button is activated on *No*
- Left click the mouse on each of the windows and type the following in each window:

BOZR	type **7.2**	*BOZD*	type **6.5**
BPR	type **12.65**	*Diameter for edge lift*	type **9.5**

- Left click the mouse on the *Calculate* command button

The answer

AEL at diameter 9.5 mm = 0.12 mm
REL at diameter 9.5 mm = 0.091 mm
The overall sagitta = 1.669 mm at diameter 9.5 mm

appears on the screen display.

- Left click the mouse on the *Quit* command button

Example 6 – the conicoidal back surface

- Left double click the mouse on the *Conicoid* option on the first screen and the conicoid option will then be displayed
- Left click the *Do you wish to calculate edge thickness?* radio button on the *Yes* option
- Left click the mouse on each of the windows and type the following in each window:

Apical radius	type **7.2**
p-value	type **.53**
Diameter for edge lift	type **9.5**
BVP	type **–6**
Refractive Index	type **1.49**
Centre Thickness	type **.3**

- Left click the mouse on the *Calculate* command button

The answer

AEL at diameter 9.5 mm = 0.12 mm
REL at diameter 9.5 mm = 0.09 mm
AET at dimeter 9.5 mm = 0.405 mm
RET at diameter 9.5 mm = 0.322 mm
The overall sagitta = 1.669 mm at diameter 9.5 mm

appears on the screen display

Note that any negative values for the axial edge thickness *AET* or radial edge thickness *RET* indicate that the lens centre thickness is inadequate and will need to be increased. A message box will come up on the screen whenever the radial thickness is less than 0.03 mm.

- Left click the mouse on the *Quit* command button
- Left click the mouse on the *Exit* command button to leave this program

11.6 Radial thickness

The calculation of edge thickness is described in Chapter 3. This involves calculation of the sags with their inter-relationships that can be used in a computer algorithm starting with calculations performed for a very small lens diameter and then repeating the calculations for small diameter increments. All the calculated thicknesses for all the diameters are held in the memory and this allows a thickness profile of the contact lens to be graphically displayed. Also the stored thickness values can be averaged to give an arithmetic mean thickness for the displayed section and the harmonic mean thickness of the section can also be calculated. The discussion in Chapter 3 indicates that the compensated arithmetic mean thickness and the compensated harmonic mean thickness can be calculated and these apply to the entire lens, not just the section. The algorithm uses axial theory to derive the compensated average thickness and the compensated harmonic mean values. These are, therefore, approximations. They are nevertheless good approximations.

11.6.1 Radial thickness profiles

The program *Radthick* allows calculation of lens radial thickness for hard, RGP and soft contact lenses with monocurve, bicurve, tricurve, four curve, five curve, offset or conicoidal back surfaces. The program calculates radial thickness (normal to the front surface of the lens), displays a radial thickness profile of the lens which includes markers for the transition positions and when this has been completed the following are specified to three decimal places:

1. radial edge thickness
2. maximum radial thickness
3. approximate compensated arithmetic mean radial thickness
4. approximate compensated harmonic mean radial thickness
5. front surface radius

1. is self-explanatory
2. is the maximum radial thickness encountered during the calculations.
3. is the average thickness of the lens which is compensated to take into consideration the extra weighting required for the more peripheral parts of the lens. This compensated arithmetic mean thickness is a useful value for giving the practitioner more idea of the physical dimensions of the lens, than simply noting the centre or edge thickness.

4. is the compensated harmonic mean thickness of the lens which is compensated to take into consideration the extra weighting required for the more peripheral parts of the lens. This compensated harmonic mean thickness is the value required when considering the gas flow through the lens, since this is proportional to $1/t$ where t is the thickness. If we take the average of the $1/t$ values acquired and then take the reciprocal of this mean, we have the harmonic mean thickness. This then requires compensation for the fact that a thickness measurement near the edge of the lens will have a larger circumference than a thickness measurement near the lens centre.

The program calculates the radial thickness from the centre of the lens to the edge at points which are spaced at 0.001 mm semi-diameter intervals across the lens. If the radial thickness decreases down to zero, then this will be obvious from the tear layer profile drawn on the screen. A label appears suggesting that the lens is too thin and a thicker lens should be considered.

Note that the calculation indicates lens thickness before any edge profile has been worked or edge polishing has been performed on the lens.

In order to check the program try the following soft lens example.

Example 1 – the monocurve lens

- Left double click the mouse on the *Monocurve* option on the first screen and the monocurve option will then be displayed
- Left click the mouse on each of the five windows and type the following in each window:

BVP	type **–6**
Refractive index	type **1.444**
Centre thickness	type **.05**
BOZR	type **8.7**
Total diameter	type **14**

- Left click the mouse on the *Calculate* command button

The screen will display a radial thickness profile from the lens centre to the lens periphery. In this example the following values are displayed on screen:

Edge thickness = 0.463 mm
Max thickness = 0.463 mm at diameter = 14 mm
Compensated arithmetic mean thickness = 0.238 mm
Compensated harmonic mean thickness = 0.169 mm
The front surface radius is 9.87 mm

All of this is illustrated in **Figure 11.17**.

- Left click the mouse on the *Quit* command button

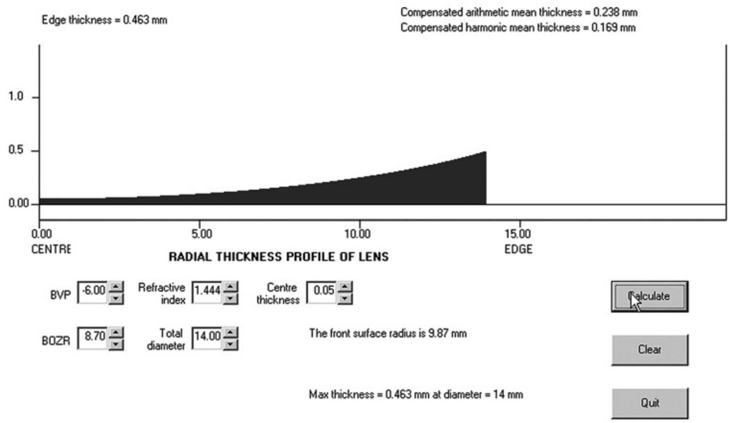

Figure 11.17 The second screen display for calculating the radial thickness profile of a monocurve soft contact lens. The program also indicates the edge thickness, maximum thickness, compensated arithmetic mean and compensated harmonic mean thicknesses. The contact lens front surface radius is also displayed

Example 2 – the bicurve lens

- Left double click the mouse on the *C2* option on the first screen and the bicurve option will then be displayed
- Left click the mouse on each of the windows and type the following in each window:

BVP	type **–6.5**		
Refractive index	type **1.444**		
Centre thickness	type **.17**		
BOZR	type **8.5**	*BOZD*	type **11**
BPR	type **9.7**	*Total diameter*	type **14.5**

- Left click the mouse on the *Calculate* command button

The screen will display a radial thickness profile from the lens centre to the lens periphery. In this example the following values are displayed on screen:

Edge thickness = 0.333 mm
Max thickness = 0.402 mm at diameter = 11 mm
Compensated arithmetic mean thickness = 0.318 mm
Compensated harmonic mean thickness = 0.3 mm
The front surface radius is 9.76 mm

Note that the single vertical short red line marks the position of the transition which is where the maximum thickness occurs in this example.

- Left click the mouse on the *Quit* command button

Example 3 – the tricurve lens

- Left double click the mouse on the *C3* option on the first screen and the tricurve option will then be displayed
- Left click the mouse on each of the windows and type the following in each window:

BVP	type **–2.5**		
Refractive index	type **1.49**		
Centre thickness	type **.2**		
BOZR	type **7**	*BOZD*	type **7**
BPR1	type **7.85**	*BPD1*	type **7.8**
BPR2	type **9**	*Total diameter*	type **8.6**

- Left click the mouse on the *Calculate* command button

The screen will display a radial thickness profile from the lens centre to the lens periphery. In this example the following values are displayed on screen:

Edge thickness = 0.13 mm
Max thickness = 0.216 mm at diameter = 7 mm
Compensated arithmetic mean thickness = 0.199 mm
Compensated harmonic mean thickness = 0.196 mm
The front surface radius is 7.33 mm

There are now two vertical short red lines marking the position of the two transitions.

- Left click the mouse on the *Quit* command button

Example 4 – the tetracurve lens

- Left double click the mouse on the *C4* option on the first screen and the tetracurve option will then be displayed
- Left click the mouse on each of the windows and type the following in each window:

BVP	type **2**		
Refractive index	type **1.49**		
Centre thickness	type **.35**		
BOZR	type **7.8**	*BOZD*	type **7**
BPR1	type **8.8**	*BPD1*	type **8**
BPR2	type **11**	*BPD2*	type **8.6**
BPR3	type **12.25**	*Total diameter*	type **9.5**

- Left click the mouse on the *Calculate* command button

The screen will display a radial thickness profile from the lens centre to the lens periphery. In this example the following values are displayed on screen:

Edge thickness = 0.054 mm
Max thickness = 0.35 mm at diameter = 0 mm
Compensated arithmetic mean thickness = 0.266 mm
Compensated harmonic mean thickness = 0.221 mm
The front surface radius is 7.67 mm

The edge thickness in this example is low.

- Left click the mouse on the *Quit* command button

Example 5 – the pentacurve lens

- Left double click the mouse on the *C5* option on the first screen and the pentacurve option will then be displayed
- Left click the mouse on each of the windows and type the following in each window:

BVP	type **3**		
Refractive index	type **1.49**		
Centre thickness	type **.4**		
BOZR	type **7.2**	*BOZD*	type **7**
BPR1	type **7.7**	*BPD1*	type **7.8**
BPR2	type **8.2**	*BPD2*	type **8.6**
BPR3	type **10.5**	*BPD3*	type **9**
BPR4	type **11.5**	*Total diameter*	type **9.5**

- Left click the mouse on the *Calculate* command button

The screen will display a radial thickness profile from the lens centre to the lens periphery. In this example the following values are displayed on screen:

Edge thickness = 0.082 mm
Max thickness = 0.4 mm at diameter = 0 mm
Compensated arithmetic mean thickness = 0.302 mm
Compensated harmonic mean thickness = 0.266 mm
The front surface radius is 7.03 mm

The screen display is illustrated in **Figure 11.18** where it can be seen that the edge thickness is reduced by the presence of the four peripheral curves. If the edge thickness calculation produces a negative value then the message *TOO THIN – Try again with thicker lens* is displayed.

- Left click the mouse on the *Quit* command button

Example 6 – the offset bicurve lens

- Left double click the mouse on the *Offset* option on the first screen and the offset bicurve option will then be displayed
- Left click the mouse on each of the windows and type the following in each window:

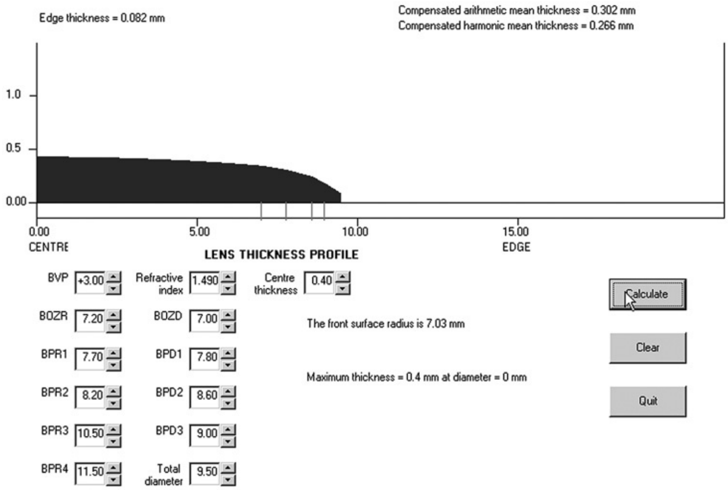

Figure 11.18 The second screen display for calculating the radial thickness profile of a pentacurve RGP contact lens. The short vertical lines indicate the position of the transitions

BVP	type **–8**		
Refractive index	type **1.49**		
Centre thickness	type **.2**		
BOZR	type **7.2**	*BOZD*	type **6.5**
BPR	type **12.65**	*Total diameter*	type **9.5**

• Left click the mouse on the *Calculate* command button

The screen will display a radial thickness profile from the lens centre to the lens periphery. In this example the following values are displayed on screen:

Edge thickness = 0.29 mm
The Axial Edge Lift is 0.12 mm
The front surface radius is 8.22 mm
Max thickness = 0.303 mm at diameter = 8.24 mm
Compensated arithmetic mean thickness = 0.27 mm
Compensated harmonic mean thickness = 0.266 mm

The single vertical red line marks the transition between the two curves. The offset design results in the central curve running tangentially into the peripheral curve and so the position of the transition is not apparent in the thickness profile. Also the maximum thickness does not occur at the BOZD which would be the case for a conventional bicurve.

• Left click the mouse on the *Quit* command button

Example 7– the conicoid lens

- Left double click the mouse on the *Conicoid* option on the first screen and the conicoid option will then be displayed
- Left click the mouse on each of the windows and type the following in each window:

BVP	type **–8**
Refractive index	type **1.49**
Centre thickness	type **.2**
Apical radius	type **7.2**
p-value	type **.53**
Total diameter	type **9.5**

- Left click the mouse on the *Calculate* command button

The screen will display a radial thickness profile from the lens centre to the lens periphery. In this example the following values are displayed on screen:

Edge thickness = 0.29 mm
The Axial Edge Lift = 0.12 mm
The front surface radius is 8.22 mm
Max thickness = 0.29 mm at diameter = 9.5 mm
Compensated arithmetic mean thickness = 0.258 mm
Compensated harmonic mean thickness = 0.255 mm

- Left click the mouse on the *Quit* command button

If we now compare the conicoidal lens to an equivalent multicurve, we can see which design gives the thinnest lens. The comparison will be made between the above conicoid design with a tricurve with a BOZR equal to the conicoidal lens apical radius. The multicurve lens has the same TD, AEL, BVP, centre thickness and refractive index.

The tricurve back surface specification was derived using the *peripheral radius* program and the specification is as follows:

BVP –8.00D Refractive index 1.490 Centre thickness 0.2 mm
C3 7.20:7.00/7.68:8.00/8.11:9.50

giving an AEL of 0.12 mm at diameter 9.50 mm.

When this lens is entered into the *Radial thickness* program we discover:

Edge thickness = 0.291 mm
The front surface radius is 8.22 mm
Max thickness = 0.304 mm at diameter = 8 mm
Compensated arithmetic mean thickness = 0.27 mm
Compensated harmonic mean thickness = 0.265 mm

So the conicoid design is shown to produce a thinner lens.

Also note that the offset and conicoid lenses considered above have a similar back surface specification and identical BVP which all results in the same centre and edge thickness. The conicoid mean thicknesses are less than those of the equivalent offset lens which are slightly greater than the last tricurve example. The offset and tricurve lenses with their spherical back optic zone display an increase in thickness as we move from the lens centre due to the –8.00 D power. A thickness decrease does not develop until we are well into the peripheral region (diameter 8.24 and 8.00 mm respectively) and this thickness decrease is due to the flat peripheral curve(s). The aspheric lens, on the other hand, has a back surface that is flattening towards the periphery. From the moment we leave the lens apex this results in a slower rate of thickness increase and a maximum thickness at the lens edge. It is, therefore, not surprising that the aspheric lens possesses a smaller maximum thickness, arithmetic mean thickness and harmonic mean thickness.

- Left click the mouse on the *Exit* command button to leave this program

11.7 Axial thickness

The calculation of axial thickness is more straightforward and is more satisfactory when dealing with lenticulated lenses because any given diameter applies to both the front and back surfaces. In the case of radial thickness, a diameter of 8.00 mm on the back surface will increase as we move radially from the back to the front surface. Also if we measure in a direction normal to the front surface as recommended then, when the lens is lenticulated, the direction of measurement will change suddenly as we cross the front surface junction that divides the front optic zone from the peripheral carrier zone.

The program *Axthick* calculates axial thickness for bicurve through to pentacurve multicurve lenses, offset lenses and conicoidal back surface lenses. In all cases the program contains the option to deal with lenticulation and, where this is requested, will calculate the front optic zone radius (FOZR) required for specified junction and edge thicknesses.

As before, the first screen display lists the lens options that can be handled by the program. The second screen will deal with the option selected and will give a graphical display of the lens thickness variation from the lens centre to the lens periphery.

The use of a program like this one allows the practitioner to determine what centre thickness, FOZD and FPR to order for any high-powered contact lens instead of leaving the specification to be decided by the contact lens laboratory. This is another example of the computer putting the contact lens practitioner back in the driving seat so that s/he dictates the lens design to the laboratory instead of passively accepting standard laboratory designs.

The mean and harmonic mean axial thicknesses are calculated. These are not relevant for the purpose of considering gas transmission through the lens. They are however useful for comparative purposes and indicate the significant improvements achieved by lenticulation.

In order to check the program try the following.

Example 1 – the monocurve lens

- Left double click the mouse on the *Monocurve* option on the first screen and the monocurve option will then be displayed
- Make sure that *Lenticulation not required* is the active radio button
- Left click the mouse on each of the windows and type the following in each window:

BVP	type **–3.5**
Refractive index	type **1.444**
Centre thickness	type **.15**
BOZR	type **8.4**
Total diameter	type **13**

- Left click the mouse on the *Calculate* command button

The screen will display an axial thickness profile from the lens centre to the lens periphery. In this example the following values are displayed on screen:

Edge thickness = 0.473 mm
Max thickness = 0.473 mm at diameter = 13 mm
Compensated arithmetic mean thickness = 0.279 mm
Compensated harmonic mean thickness = 0.251 mm

All of this is illustrated in **Figure 11.19**.

- Left click the mouse on the *Clear* command button

In order to examine the possibilities for lenticulated lenses:

- Make sure that the *Lenticulation required* radio button is selected as the active option
- Left click the mouse on each of the windows and type the following in each window:

BVP	type **–3.5**
Refractive index	type **1.444**
BOZR	type **8.4**
Total diameter	type **13**
Junction thickness	type **.15**
Junction diameter (FOZD)	type **12**

- Left click the mouse on the *Calculate* command button

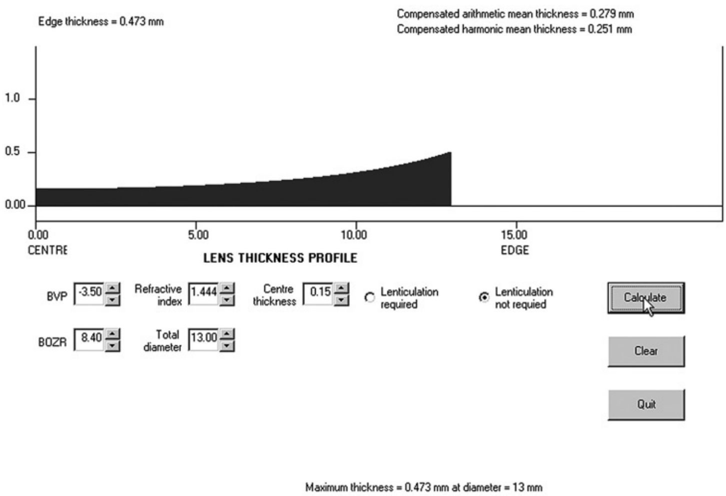

Figure 11.19 The second screen display for calculating the axial thickness profile of a monocurve soft contact lens

The graph remains blank and the following messages appear on the screen:

MINIMUM junction thickness required = 0.27 mm
TOO THIN – Try again with a thicker lens or a smaller junction diameter

This means that a junction thickness less than 0.27 mm will be associated with a centre thickness less than 0.03 mm. The screen display is illustrated in **Figure 11.20**.

- Use the vertical scroll bars on the side of the *Junction thickness* window to increase the thickness to 0.27
- Left click the mouse on the *Calculate* command button

The screen display now plots the axial thickness variation up to the transition and a new window appears requesting the edge thickness that you require.

- Left click the mouse on the *Edge thickness required* window and type **.2**
- Left click the mouse on the *Calculate peripheral radius* command button
  The graphical display will then be completed up to the lens edge and the following information will be displayed:

  Edge thickness = 0.2 mm
  Centre thickness required = 0.038 mm
  The front peripheral radius = 8 mm
  Max thickness = 0.27 mm at diameter = 12 mm
  Compensated arithmetic mean thickness = 0.15 mm
  Compensated harmonic mean thickness = 0.111 mm

The small vertical green line marks the position of the junction. All of this is illustrated in **Figure 11.21**.

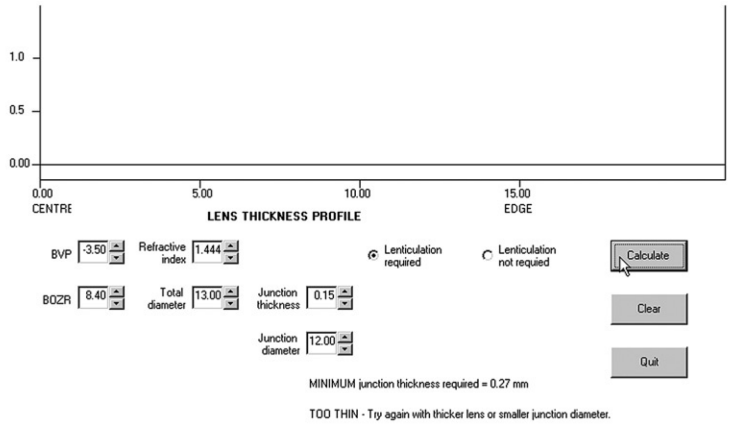

Figure 11.20 The second screen display for calculating the axial thickness profile of a monocurve soft contact lens with lenticulation. The junction thickness requested produces an unacceptably small centre thickness. The thickness profile is not displayed and a minimum junction thickness is given

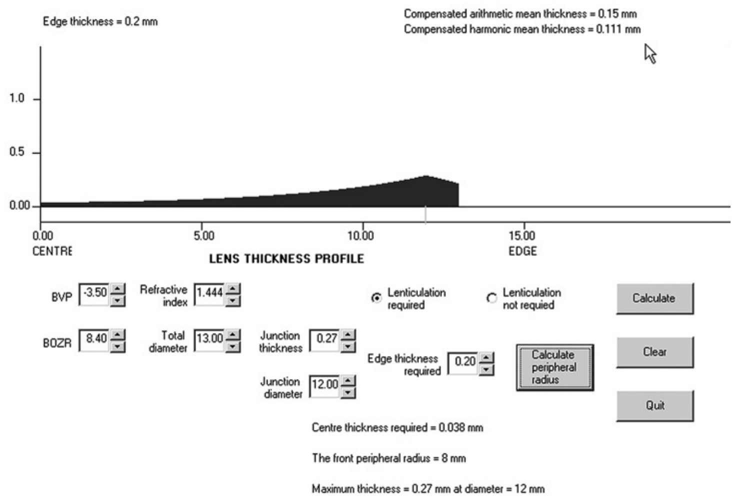

Figure 11.21 The second screen display for calculating the axial thickness profile of a monocurve soft contact lens with lenticulation. The junction thickness of 0.27 mm gives a lens centre thickness of 0.038 mm. The edge thickness requested can be achieved by using a front peripheral radius of 8 mm and this gives a positive carrier. The short green line marks the position of the junction (Plate 18)

Thus, when lenticulation is required, the practitioner decides what junction diameter (FOZD) and junction thickness s/he requires. The program calculates the lens centre thickness. If this is acceptable, the practitioner decides what edge thickness is required and the program calculates the front peripheral radius (FPR) required to achieve the requested edge thickness. The

practitioner can then order a lenticulated lens with a stated centre thickness, FOZD and FPR.

- Left click the mouse on the *Quit* command button

Example 2 – the bicurve lens

- Left double click the mouse on the *C2* option on the first screen and the bicurve option will then be displayed
- Make sure that the *Lenticulation not required* radio button is active
- Left click the mouse on each of the windows and type the following in each window:

BVP	type **4**
Refractive index	type **1.43**
Centre thickness	type **.17**
BOZR	type **8.5**
BOZD	type **11**
BPR	type **9.7**
Total diameter	type **14.5**

- Left click the mouse on the *Calculate* command button

The screen will display the message:

TOO THIN – Try again with thicker lens
Maximum thickness = 0.17 mm at diameter = 0 mm

The thickness profile graph indicates that the lens thickness reduces to zero just outside diameter 10 mm. This is all illustrated in **Figure 11.22**.

This lens will benefit from lenticulation.

- Left click the *Lenticulation required* radio button
- Left click the *Junction thickness* window and type **.15**
- Left click the *Junction diameter* window and type **11.2**
- Left click the *Calculate* command button

The screen display plots the axial thickness variation up to the junction and a new window appears requesting the edge thickness that you require.

- Left click the mouse on the *Edge thickness required* window and type **.4**
- Left click the mouse on the *Calculate peripheral radius* command button

The graphical display will then be completed up to the lens edge and the following information will be displayed:

Edge thickness = 0.399 mm
Centre thickness required = 0.354 mm
The front peripheral radius = 10.82 mm
Max thickness = 0.399 mm at diameter = 14.5 mm
Compensated arithmetic mean thickness = 0.27 mm
Compensated harmonic mean thickness = 0.254 mm

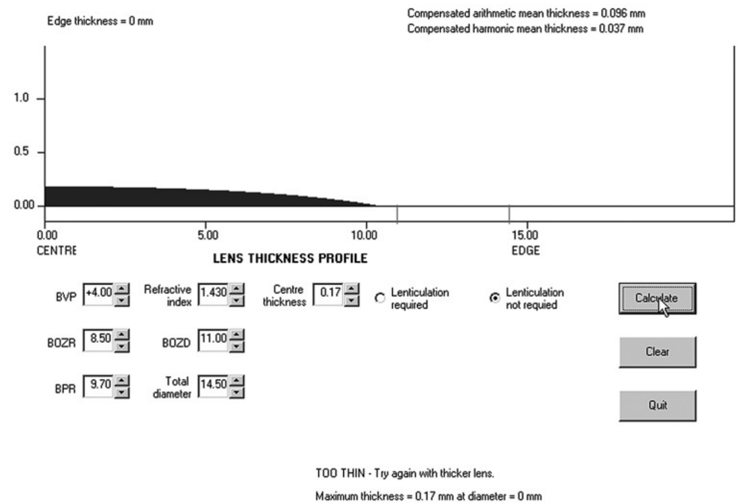

Figure 11.22 The second screen display for calculating the axial thickness profile of a bicurve soft contact lens. This +4.00 D lens is too thin when a centre thickness of 0.17 mm is selected (Plate 19)

The small vertical green line marks the position of the junction. The short vertical red lines mark the position of the transition and the lens edge. All of this is illustrated in Figure 11.23.

- Left click the mouse on the *Quit* command button

Example 3 – the tricurve lens

- Left double click the mouse on the *C3* option on the first screen and the tricurve option will then be displayed
- Make sure that the *Lenticulation required* radio button is active
- Left click the mouse on each of the windows and type the following in each window:

BVP	type **6**	*Refractive index*	type **1.4**
BOZR	type **7.7**	*BOZD*	type **7**
BPR1	type **8.5**	*BPD1*	type **8**
BPR2	type **9.45**	*Total diameter*	type **9**
Junction thickness	type **.15**		
Junction diameter	type **7.2**		

- Left click the mouse on the *Calculate* command button

The screen display plots the axial thickness variation up to the transition and a new window appears requesting the edge thickness that you require.

- Left click the mouse on the *Edge thickness required* window and type **.2**
- Left click the mouse on the *Calculate peripheral radius* command button

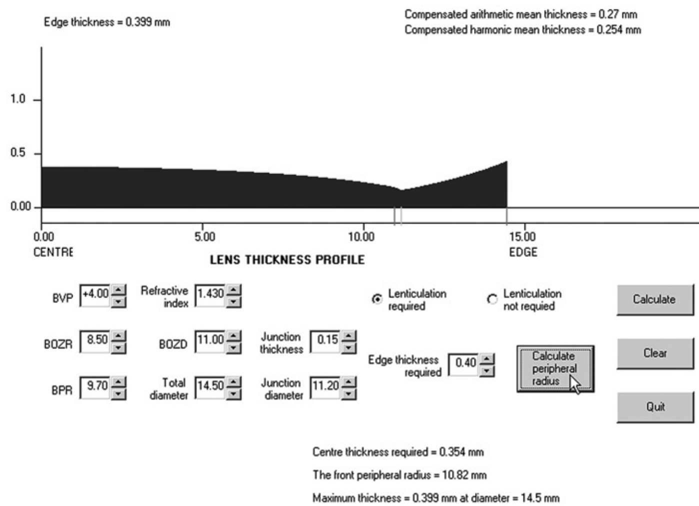

Figure 11.23 The second screen display for calculating the axial thickness profile of a bicurve soft contact lens with lenticulation. The selection of a junction thickness of 0.15 mm at FOZD = 11.20 mm gives a centre thickness of 0.345 mm. The selection of an edge thickness of 0.4 mm produces a negative carrier and this is achieved by using an FPR of 10.82 mm. The short vertical red lines mark the transition and lens edge. The short vertical green line marks the position of the junction (Plate 20)

The graphical display will then be completed up to the lens edge and the following information will be displayed:

Edge thickness = 0.2 mm
Centre thickness required = 0.263 mm
The front peripheral radius = 9.94 mm
Max thickness = 0.263 mm at diameter = 0 mm
Compensated arithmetic mean thickness = 0.202 mm
Compensated harmonic mean thickness = 0.197 mm

The small vertical green line marks the position of the junction. The short vertical red lines mark the position of the transitions and the lens edge.

- Left click the mouse on the *Quit* command button

Example 4 – the tetracurve lens

- Left double click the mouse on the *C4* option on the first screen and the tetracurve option will then be displayed
- Make sure that the *Lenticulation not required* radio button is active
- Left click the mouse on each of the windows and type the following in each window:

BVP	type **–6**	*Refractive index*	type **1.45**
BOZR	type **7.7**	*BOZD*	type **7**
BPR1	type **8.3**	*BPD1*	type **8**
BPR2	type **9.15**	*BPD2*	type **8.5**
BPR3	type **10.3**	*Total diameter*	type **9**
Centre thickness	type **.1**		

- Left click the mouse on the *Calculate* command button

The screen will display an axial thickness profile from the lens centre to the lens periphery. In this example the following values are displayed on screen:

Edge thickness = 0.169 mm
Max thickness = 0.21 mm at diameter = 8 mm
Compensated arithmetic mean thickness = 0.167 mm
Compensated harmonic mean thickness = 0.159 mm

The short vertical red lines mark the position of the transitions and the lens edge.

- Left click the mouse on the *Quit* command button

Example 5 – the pentacurve lens

- Left double click the mouse on the *C5* option on the first screen and the pentacurve option will then be displayed
- Make sure that the *Lenticulation not required* radio button is active
- Left click the mouse on each of the windows and type the following in each window:

BVP	type **8**	*Refractive index*	type **1.4**
BOZR	type **8.5**	*BOZD*	type **7**
BPR1	type **8.15**	*BPD1*	type **7.5**
BPR2	type **8.65**	*BPD2*	type **8**
BPR3	type **9.2**	*BPD3*	type **8.5**
BPR4	type **9.9**	*Total diameter*	type **9**
Centre thickness	type **.2**		

- Left click the mouse on the *Calculate* command button

The screen will display the message:

TOO THIN – Try again with thicker lens
Maximum thickness = 0.2 mm at diameter = 0 mm

The flatter peripheral curves on the back surface of the lens result in the lens thickness decreasing to zero before the fifth curve. This lens will benefit from lenticulation.

- Left click the *Lenticulation required* radio button
- Left click the *Junction thickness* window and type **.15**

- Left click the *Junction diameter* window and type **7.2**
- Left click the *Calculate* command button

The screen display plots the axial thickness variation up to the transition and a new window appears requesting the edge thickness that you require.

- Left click the mouse on the *Edge thickness required* window and type **.2**
- Left click the mouse on the *Calculate peripheral radius* command button

The graphical display will then be completed up to the lens edge and the following information will be displayed:

Edge thickness = 0.2 mm
Centre thickness required = 0.29 mm
The front peripheral radius = 9.99 mm
Max axial thickness = 0.29 mm at diameter = 0 mm
Compensated arithmetic mean axial thickness = 0.209 mm
Compensated harmonic mean axial thickness = 0.203 mm

The small vertical green line marks the position of the junction. The short vertical red lines mark the position of the transitions and the lens edge.

- Left click the mouse on the *Quit* command button

Example 6 – the offset bicurve lens

- Left double click the mouse on the *Offset* option on the first screen and the offset bicurve option will then be displayed
- Make sure that the *Lenticulation not required* radio button is active
- Left click the mouse on each of the windows and type the following in each window:

BVP	type **–8**
Refractive index	type **1.5**
Centre thickness	type **.3**
BOZR	type **7.6**
BOZD	type **6**
BPR	type **18.45**
Total diameter	type **9**

- Left click the mouse on the *Calculate* command button

The screen will display the message:

Edge thickness = 0.398 mm
Maximum thickness = 0.416 mm at diameter = 7.73 mm
Compensated arithmetic mean thickness = 0.379 mm
Compensated harmonic mean thickness = 0.375 mm

This negative lens will benefit from lenticulation with a positive carrier.

- Left click the *Lenticulation required* radio button
- Left click the *Junction thickness* window and type **.3**
- Left click the *Junction diameter* window and type **7**
- Left click the *Calculate* command button

The screen display plots the axial thickness variation up to the junction and a new window appears requesting the edge thickness that you require.

- Left click the mouse on the *Edge thickness required* window and type **.1**
- Left click the mouse on the *Calculate peripheral radius* command button

The graphical display will then be completed up to the lens edge and the following information will be displayed:

Edge thickness = 0.1 mm
Centre thickness required = 0.193 mm
The front peripheral radius = 6.98 mm
Max thickness = 0.3 mm at diameter = 7 mm
Compensated arithmetic mean thickness = 0.234 mm
Compensated harmonic mean thickness = 0.221 mm

The small vertical green line marks the position of the junction. The short vertical red lines mark the position of the transition and the lens edge.

- Left click the mouse on the *Quit* command button

Example 7 – the conicoidal lens

- Left double click the mouse on the *Conicoid* option on the first screen and the conicoid option will then be displayed
- Make sure that the *Lenticulation required* radio button is active
- Left click the mouse on each of the windows and type the following in each window:

BVP	type **5**
Refractive index	type **1.49**
Apical radius	type **8**
p-value	type **.33**
Total diameter	type **9.5**
Junction thickness	type **.15**
Junction diameter	type **7**

- Left click the *Calculate* command button

The screen display plots the axial thickness variation up to the junction and a new window appears requesting the edge thickness that you require.

- Left click the mouse on the *Edge thickness required* window and type **.3**
- Left click the mouse on the *Calculate peripheral radius* command button

The graphical display will then be completed up to the lens edge and the following information will be displayed:

Edge thickness = 0.3 mm
Centre thickness required = 0.242 mm
The front peripheral radius = 10.66 mm
Max axial thickness = 0.3 mm at diameter = 9.5 mm
Compensated arithmetic mean thickness = 0.213 mm
Compensated harmonic mean thickness = 0.207 mm

The small vertical green line marks the position of the junction.

- Left click the mouse on the *Quit* command button
- Left click the mouse on the *Exit* command button to leave this program

The examples selected above have been used to illustrate how the program can be useful. The thicknesses used should not be considered as recommended thicknesses.

For rigid corneal lenses, optimum thicknesses will depend on the material being used with the thickness increasing as the permeability of the material increases. An optimum junction thickness for PMMA is considered to be around 0.15 mm. An optimum edge thickness is around 0.18 mm. An optimum centre thickness will depend on the BVP of the lens but will be around 0.2 mm for negative and 0.3 to 0.4 mm for positive lenses.

11.8 Tear layer thickness and edge clearance

This program *TLT AEC* calculates the tear layer thickness at the corneal apex, the axial edge clearance and the radial edge clearance at a specified diameter assuming that the contact lens is sitting on the cornea in a central position. It will handle C2, C3, C4, C5, contralateral offset bicurve or conicoidal contact lenses. The program also offers the chance to change the lens back surface specification to achieve the required TLT and AEC. Once again, the first screen display indicates the lens design options that are selected by double clicking the design of interest.

The program uses a FOR-NEXT loop and calculates sags for progressively increasing diameters (the semi-diameter increases in 0.001 mm steps) for both the cornea and the contact lens. The program compares the sag of the contact lens with that of the cornea. If the corneal sag is larger than the sag of the contact lens, then the lens is a flat fit, the contact diameter is zero and the contact point between the lens and cornea is at the corneal apex. If the contact lens sag is larger, the lens is a steep fit. The program notes the difference between the two sags each time the FOR–NEXT loop is entered, i.e. for each diameter increment. It determines where this difference is maximal and this diameter is the contact diameter for the lens on the cornea. In the case of typical C2 to C5 lenses fitted on the steep side of alignment, this is likely to

be the BOZD. For a toric cornea the program calculates the contact diameter, TLT, AEC and REC along both corneal meridians. The program then displays the tear layer profile for both meridians. The display is similar to those shown in Chapter 6.

The user is required to enter the corneal apical radii and the p-values for the flat meridian and steep corneal meridians. These corneal characteristics can be derived from videokeratoscopes or modified keratometry measurements as discussed in Chapter 5. It must be noted that the investigations by Douthwaite *et al.* (1999) and Douthwaite (2002) have shown that not only do we expect the apical radius to be different in the two corneal principal meridians but we expect to see a difference in the two p-values. The near vertical principal meridian is likely to be slightly less aspheric than the near horizontal.

In order to test the program try the following.

Example 1 – the bicurve lens

- Left double click the mouse on the *C2* option on the first screen and the bicurve option will then be displayed
- Left click the mouse on each of the windows and type the following in each window:

Corneal Apical Radius Flat Meridian type **7.4**
Corneal p-value Flat Meridian type **.8**
Corneal Apical Radius Steep Meridian type **7.3**
Corneal p-value Steep Meridian type **.9**

BOZR type **7.35** *BOZD* type **7**
BPR type **9** *Diameter for AEC* type **8.5**

- Left click the mouse on the *Calculate flat meridian* command button

The screen will display the fluid lens profile for the flat meridian from the lens centre to the lens periphery. In this example the following values are displayed on screen:

Flat Meridian
TLT = 0.018 mm *AEC = 0.087 mm* *REC = 0.072 mm*
CD = 7 mm (Contact Diameter)

Let us suppose that we require a TLT of 0.02 mm and an AEC of 0.08 mm.

- Left click the mouse on the *Change specification* command button

This will clear the graph and you can alter any element of the lens back surface specification in order to attempt to achieve the required fit.

- Use the vertical scroll bars to acquire the following in each window:

BOZR **7.30** *BPR* **8.85**

- Left click the *Calculate flat meridian* command button

A new tear thickness profile will appear with the following values displayed on screen:

Flat Meridian
TLT = 0.025 mm *AEC = 0.079 mm* *REC = 0.066 mm*
CD = 7 mm (Contact Diameter)

We see that the BOZR of 7.35 mm produced a TLT nearer to the TLT = 0.02 mm value, with the BPR of 8.85 mm producing an AEC of 0.079 mm which is as near to the required value as we are likely to get. Therefore our best fitting lens back surface will be:

C2 7.35:7.00/8.85:8.50

- Use the vertical scroll bars to alter the BOZR to 7.35
- Left click the *Change specification* command button to clear the graph
- Left click the *Calculate flat meridian* command button

The screen will display the new fluid lens profile and the following values are displayed on screen:

Flat Meridian
TLT = 0.018 mm *AEC = 0.079 mm* *REC = 0.066 mm*
CD = 7 mm (Contact Diameter)

- Left click the *Continue* command button
- Left click the *Calculate* steep meridian command button

The screen will now display the fluid lens profile for both meridians from the lens centre to the lens periphery and the following values are displayed on screen for the steep meridian:

Steep Meridian
TLT = 0.018 mm *AEC = 0.115 mm* *REC = 0.094 mm*
The contact lens AEL = 0.101 mm

Note that the short red lines mark the position of the transition and the lens edge. No consideration is given to the extra edge clearance produced by any edge profile or edge polishing. The display is illustrated in **Figure 11.24**.

- Left click the mouse on the *Quit* command button

Example 2 – the tricurve lens

- Left double click the mouse on the *C3* option on the first screen and the tricurve option will then be displayed
- Left click the mouse on each of the windows and type the following in each window:

Corneal Apical Radius Flat Meridian type **7.6**
Corneal p-value Flat Meridian type **.8**
Corneal Apical Radius Steep Meridian type **7.45**
Corneal p-value Steep Meridian type **.85**

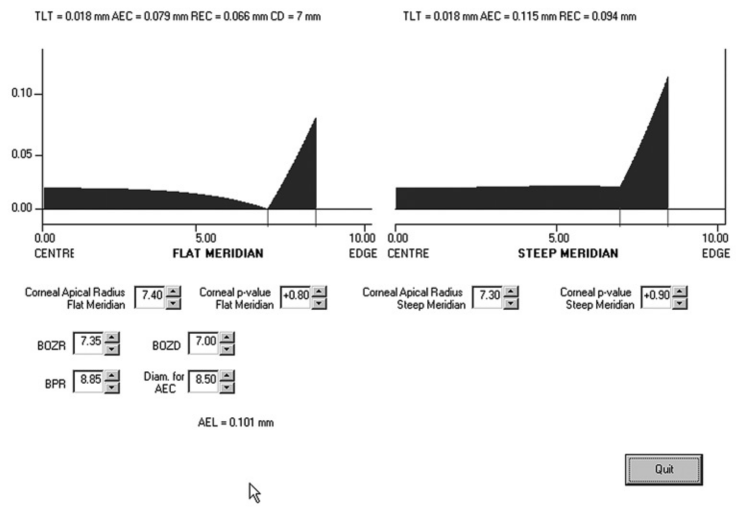

Figure 11.24 The second screen display for calculating the tear lens axial thickness profile produced by a bicurve RGP contact lens resting in a central position on a cornea of apical radii and p-values as selected in the display. The short vertical lines mark the position of the transition and the lens edge. The TLT, AEC and REC are given for the two corneal principal meridians. The contact diameter is displayed as is the contact lens AEL

BOZR	type **7.7**	*BOZD*	type **7**
BPR1	type **8.2**	*BPD1*	type **8**
BPR2	type **10**	*Diameter for AEC*	type **9**

- Left click the mouse on the *Calculate flat meridian* command button

The screen will display the fluid lens profile for the flat meridian from the lens centre to the lens periphery. In this example the following values are displayed on screen:

Flat Meridian
TLT = *0 mm* *AEC* = *0.108 mm* *REC* = *0.088 mm*
CD = *0 mm* (Contact Diameter)

- Left click the *Continue* command button
- Left click the *Calculate steep meridian* control button

The screen will display the fluid lens profile for both meridians and the following values are displayed on screen for the steep meridian:

Steep Meridian
TLT = *0 mm* *AEC* = *0.151 mm* *REC* = *0.122 mm*
The contact lens AEL = *0.118 mm*

Note that the short red lines mark the position of the transitions and the lens edge. The report giving a TLT of zero and contact diameter of zero indicates a flat fit on the flat meridian. The fluid lens profile indicates that the fit on

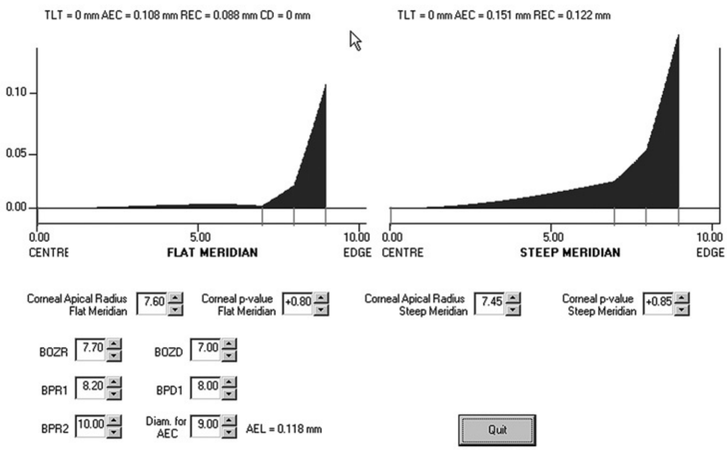

Figure 11.25 The second screen display for calculating the tear lens axial thickness profile produced by a tricurve contact lens resting in a central position on a cornea of apical radii and p-values as selected in the display

the flat meridian looks to be an alignment fit. What the program is telling us is that the fit is very, very slightly flat on the flat meridian. The display is illustrated in Figure 11.25.

It can be noted that the graphical display shows that despite an initial increase in the fluid lens thickness as we move from the vertex, the thickness decreases as we approach the first transition.

- Left click the mouse on the *Quit* command button

If we had selected a BOZR of 7.684, the lens would have touched the cornea at the apex and the transition (TLT = 0, CD = 7). This represents the best matching fit between a spherical lens and the flat meridian of this aspheric cornea.

Example 3 – the tetracurve lens

- Left double click the mouse on the *C4* option on the first screen and the tetracurve option will then be displayed.
- Left click the mouse on each of the windows and type the following in each window:

Corneal Apical Radius Flat Meridian	type **8**
Corneal p-value Flat Meridian	type **.7**
Corneal Apical Radius Steep Meridian	type **7.9**
Corneal p-value Steep Meridian	type **.8**

BOZR	type **8**	*BOZD*	type **7**
BPR1	type **8.25**	*BPD1*	type **8**
BPR2	type **10**	*BPD2*	type **8.6**
BPR3	type **12**	*Diameter for AEC*	type **9**

• Left click the mouse on the *Calculate flat meridian* command button

The screen will display the fluid lens profile for the flat meridian from the lens centre to the lens periphery. In this example the following values are displayed on screen:

Flat Meridian
TLT = 0.014 mm *AEC = 0.081 mm* *REC = 0.068 mm*
CD = 8 mm (Contact Diameter)

• Left click the *Continue* command button
• Left click the *Calculate steep meridian* control button

The screen will display the fluid lens profile for both meridians and the following values are displayed on screen for the steep meridian:

Steep Meridian
TLT = 0.014 mm *AEC = 0.114 mm* *REC = 0.095 mm*
The contact lens AEL = 0.108 mm

Note that the short red lines mark the position of the transitions and the lens edge. Here we see a lens that displays an alignment fit for the first peripheral curve on the flat meridian. The contact diameter is 8 mm, indicating that this first peripheral curve is very, very slightly steep. The display is illustrated in **Figure 11.26**. Is it likely that the fluorescein picture would allow such subtle relationships to be appreciated?

• Left click the mouse on the *Quit* command button

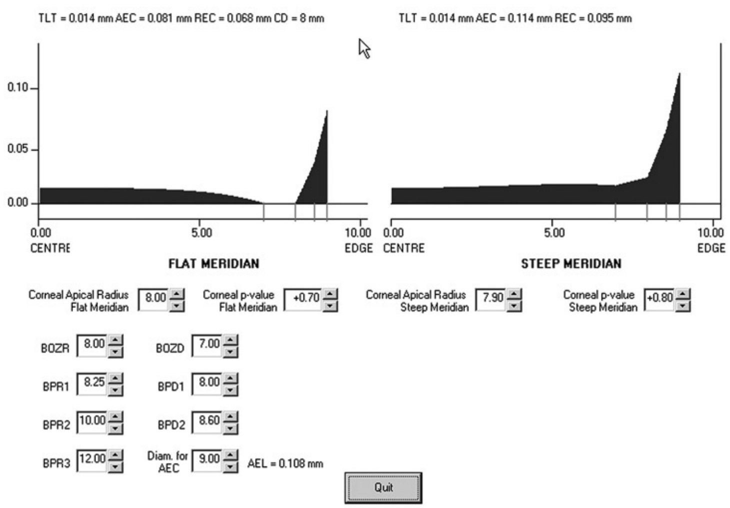

Figure 11.26 The second screen display for calculating the tear lens axial thickness profile produced by a tetracurve contact lens resting in a central position on a cornea of apical radii and p-values as selected in the display

Example 4 – the pentacurve lens

- Left double click the mouse on the *C5* option on the first screen and the pentacurve option will then be displayed
- Left click the mouse on each of the windows and type the following in each window:

Corneal Apical Radius Flat Meridian	type **7.9**
Corneal p-value Flat Meridian	type **.8**
Corneal Apical Radius Steep Meridian	type **7.85**
Corneal p-value Steep Meridian	type **.85**

BOZR	type **7.8**	*BOZD*	type **7**
BPR1	type **8.25**	*BPD1*	type **8**
BPR2	type **8.9**	*BPD2*	type **8.5**
BPR3	type **9.65**	*BPD3*	type **9**
BPR4	type **10.6**	*Diameter for AEC*	type **9.5**

- Left click the mouse on the *Calculate flat meridian* command button

The screen will display the fluid lens profile for the flat meridian from the lens centre to the lens periphery. In this example the following values are displayed on screen:

Flat Meridian
TLT = 0.021 mm *AEC = 0.107 mm* *REC = 0.087 mm*
CD = 7 mm (Contact Diameter)

- Left click the *Continue* command button
- Left click the *Calculate steep meridian* control button

The screen will display the fluid lens profile for both meridians and the following values are displayed on screen for the steep meridian:

Steep Meridian
TLT = 0.021 mm *AEC = 0.128 mm* *REC = 0.103 mm*
The contact lens AEL = 0.149 mm

Note that the short red lines mark the position of the transitions and the lens edge. The display is illustrated in **Figure 11.27**.
- Left click the mouse on the *Quit* command button

The edge clearance displayed in **Figure 11.27** is achieved in progressive steps across the peripheral curve bands. The stepped nature of the change in fluid lens thickness can be smoothed out by using an aspheric or pseudo-aspheric back surface as illustrated in the two remaining examples.

Example 5 – the offset bicurve lens

- Left double click the mouse on the *Offset* option on the first screen and the offset bicurve option will then be displayed

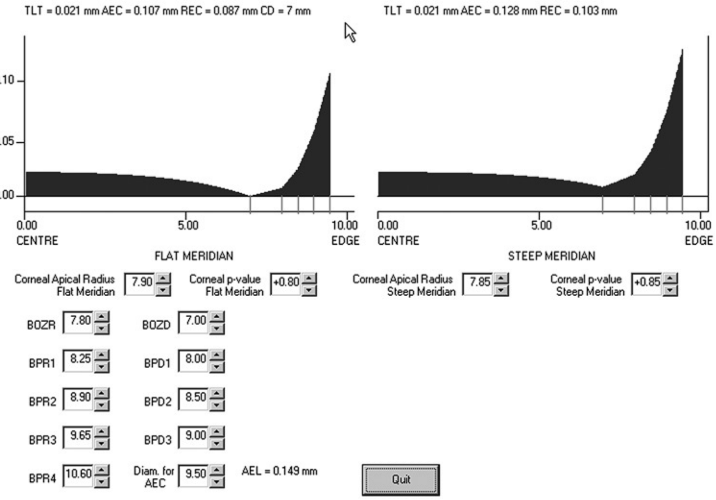

Figure 11.27 The second screen display for calculating the tear lens axial thickness profile produced by a pentacurve contact lens resting in a central position on a cornea of apical radii and p-values as selected in the display

- Left click the mouse on each of the windows and type the following in each window:

Corneal Apical Radius Flat Meridian type **7.4**
Corneal p-value Flat Meridian type **.8**
Corneal Apical Radius Steep Meridian type **7.35**
Corneal p-value Steep Meridian type **.9**

BOZR type **7.35** *BOZD* type **6.5**
BPR type **25** *Diameter for AEC* type **9.25**

- Left click the mouse on the *Calculate flat meridian* command button

The screen will display the fluid lens profile for the flat meridian from the lens centre to the lens periphery. In this example the following values are displayed on screen:

Flat Meridian
TLT = 0.015 mm *AEC = 0.105 mm* *REC = 0.084 mm*
CD = 6.76 mm (Contact Diameter)

- Left click the *Continue* command button
- Left click the *Calculate steep meridian* control button

The screen will display the fluid lens profile for both meridians and the following values are displayed on screen for the steep meridian:

Steep Meridian
TLT = 0.015 mm *AEC = 0.139 mm* *REC = 0.109 mm*
The contact lens AEL = 0.151 mm

Note that the short red lines mark the position of the transition and the lens edge. The short green line indicates the point of contact between the lens and cornea. Note that the contact diameter does not coincide with the BOZD due to the central and peripheral curves running tangentially into each other. Also there is a step free increase in fluid lens thickness from the contact point to the lens edge. The display is illustrated in **Figure 11.28**.

- Left click the mouse on the *Quit* command button

Example 6 – the conicoidal lens

- Left double click the mouse on the *Conicoid* option on the first screen and the conicoid option will then be displayed
- Left click the mouse on each of the windows and type the following in each window:

Corneal Apical Radius Flat Meridian	type **8**
Corneal p-value Flat Meridian	type **.8**
Corneal Apical Radius Steep Meridian	type **7.95**
Corneal p-value Steep Meridian	type **.85**
Contact lens apical radius	type **7.7**
p-value	type **0**
Diameter for AEC	type **9**

- Left click the mouse on the *Calculate flat meridian* command button

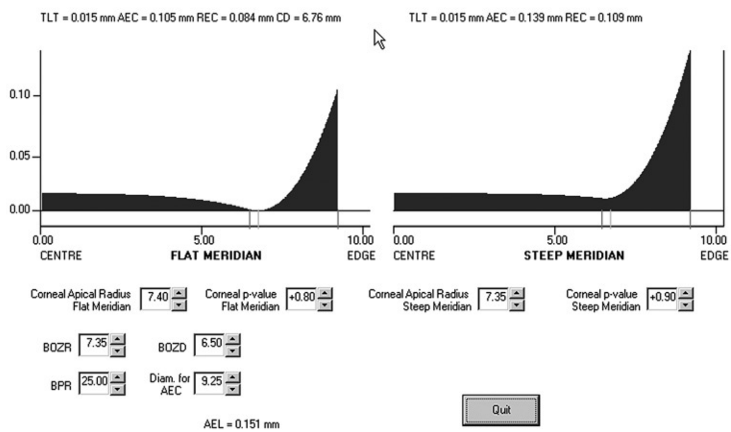

Figure 11.28 The second screen display for calculating the tear lens axial thickness profile produced by an offset bicurve contact lens resting in a central position on a cornea of apical radii and p-values as selected in the display. The short green vertical line represents the position of contact between the lens and the cornea on the flat principal meridian (Plate 21)

The screen will display the fluid lens profile for the flat meridian from the lens centre to the lens periphery for this paraboloidal back surface contact lens. In this example the following values are displayed on screen:

Flat Meridian
TLT = 0.007 mm *AEC = 0.05 mm* *REC = 0.042 mm*
CD = 4.86 mm (Contact Diameter)

- Left click the *Continue* command button
- Left click the *Calculate steep meridian* control button

The screen will display the fluid lens profile for both meridians and the following values are displayed on screen for the steep meridian:

Steep Meridian
TLT = 0.007 mm *AEC = 0.067 mm* *REC = 0.056 mm*
The contact lens AEL = 0.137 mm

Note that the short red line marks the position of the lens edge. The short green line indicates the point of contact between the lens and cornea. Note also that despite the fact that the lens back surface is parabolic (p-value = 0) with a small contact diameter of 4.86 mm, the edge clearance is less than optimum. This illustrates the problem with conicoidal back surfaces that the lens asphericity must be much greater than that of the cornea in order to achieve a reasonable edge clearance. The display is illustrated in **Figure 11.29**.

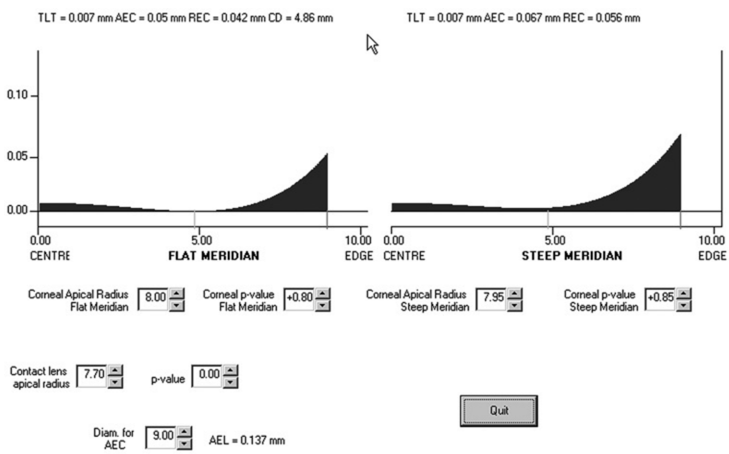

Figure 11.29 The second screen display for calculating the tear lens axial thickness profile produced by a conicoidal contact lens resting in a central position on a cornea of apical radii and p-values as selected in the display. The short green vertical line represents the position of contact between the lens and the cornea on the flat principal meridian (Plate 22)

To improve the edge clearance in this lens we could either increase the apical radius or go for a negative (hyperbolic) p-value.

- Left click the mouse on the *Quit* command button
- Left click the mouse on the *Exit* command button

11.9 Lens design

The *Lens Design* program is potentially the most useful program for fitting RGP lenses. The program simply requires the corneal apical radii and p-values for both principal meridians. It then requires the transition diameters and total diameter of the lens. These are likely to be the diameters of the practitioners favourite lens. The program then calculates all the back surface radii to give an optimal TLT and AEC. These are set at the default values of TLT =0.02 mm and AEC = 0.08 mm. The default values for the share of the edge lift/clearance in the multicurve lenses are the same as those in the *Peripheral radius* program and, as before, they can be altered if the user wishes to do this. The program and its use have been fully described in Chapter 6 and will not be repeated here. The examples discussed in Chapter 6 can be checked out in order to familiarize the user with the program.

11.10 Orthokeratology

The topic of orthokeratology was covered in Chapter 9. The *Orthokeratology* program provides a simple means of deriving an orthokeratology lens. This program is a variation of the design program discussed in the previous section. The majority of the windows are provided with default values. The user simply needs to enter the corneal apical radii and p-values of the two principal meridians and the refractive error that requires elimination. The program calculates a contact lens back surface specification and displays the tear layer profile for the flat and steep corneal meridians. Any of the default values can be changed to suit the individual practitioner or patient. The program offers two options which are a pentacurve or a polynomial design.

It is important to note that any practitioner considering orthokeratology fitting must be fully aware of the limitations and possible complications before offering this service to patients. This book does not cover these topics. The practitioner will need to read books and papers on the subject of orthokeratology and work with colleagues who have experience in orthokeratology lens fitting.

The default values in the programs should not be regarded as suitable for all patients. They are simply *ball park* figures to illustrate the shape of the tear layer profile.

In order to test the program try the following.

Example 1 – the pentacurve lens

- Left double click the mouse on the *C5* option on the first screen and the pentacurve option will then be displayed
- Left click the mouse on each of the windows and type the following in each window:

Corneal Apical Radius Flat Meridian	type **7.9**
Corneal p-value Flat Meridian	type **.7**
Corneal Apical Radius Steep Meridian	type **7.8**
Corneal p-value Steep Meridian	type **.8**
Refractive error change required	type **–2.5**

- Left click the mouse on the *Calculate flat meridian* command button

The screen will display the fluid lens profile for the flat meridian from the lens centre to the lens periphery. In this example the following values are displayed on screen:

Flat Meridian
TLT = 0.005 mm *AEC = 0.08 mm* *REC = 0.062 mm*
CD = 8 mm (Contact Diameter)

The contact lens back surface specification is also displayed as follows:

C5 8.60:6.00/7.08:7.00/7.93:8.00/8.24:9.00/9.24:10.60

If a lens was to be ordered the radii could be rounded to the nearest 0.05 mm or left as they are.

- Left click the *Calculate* steep meridian command button

The screen will now display the fluid lens profile for both meridians from the lens centre to the lens periphery and the following values are displayed on screen for the steep meridian:

Steep Meridian
TLT = 0.005 mm *AEC = 0.142 mm* *REC = 0.108 mm*

Note that the short red lines mark the position of the transitions and the lens edge. No consideration is given to the extra edge clearance produced by any edge profile or edge polishing. The display is illustrated in **Figure 11.30**.

On the flat corneal meridian, this lens has a central clearance of 0.006 mm. The contact lens central curve is flat in relation to the cornea giving a negative fluid lens. The first peripheral curve is the reverse geometry curve. The second peripheral curve starts with a corneal clearance of 0.01 mm and ends with a clearance of zero. The third peripheral curve is a matching alignment fit with its associated corneal region. The fourth and final peripheral curve is flat enough to generate an axial edge clearance of 0.081 mm.

Note that requesting a reverse curve clearance of zero results in an alignment fit for both the third and fourth back surface curves. The tear layer

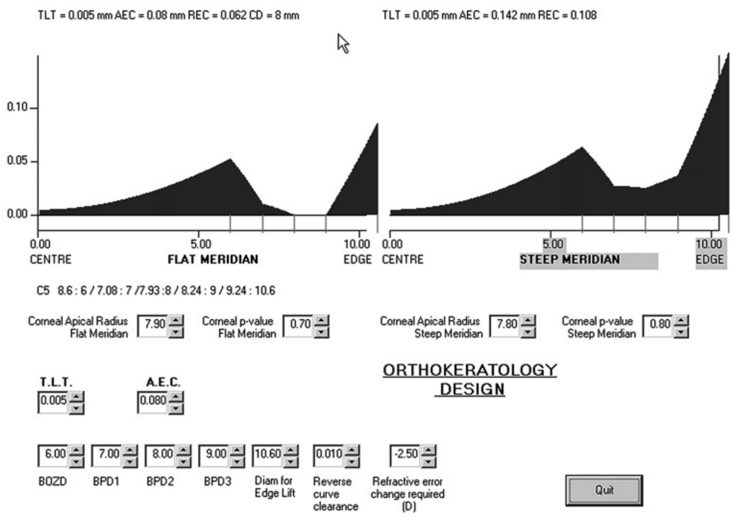

Figure 11.30 The tear layer profile for a C5 orthokeratology design

profile will look like that of a tetracurve design although there will still be differences in the BPRs for the third and fourth curves.

The effects of changing any of the radii and/or diameters can be investigated using the program *TLT AEC.*

- Left click the mouse on the *Quit* command button

Example 2 – the polynomial lens

Let us compare the polynomial design with the C5 lens fitted to the same cornea.

- Left double click the mouse on the *Polynomial* option on the first screen and the polynomial option will then be displayed.
- Left click the mouse on each of the windows and type the following in each window:

Corneal Apical Radius Flat Meridian type **7.9**
Corneal p-value Flat Meridian type **.7**
Corneal Apical Radius Steep Meridian type **7.8**
Corneal p-value Steep Meridian type **.8**
Refractive error change required type **–2.5**

- Left click the mouse on the *Calculate flat meridian* command button

The screen will display the fluid lens profile for the flat meridian from the lens centre to the lens periphery. In this example the following values are displayed on screen:

Flat Meridian
TLT = 0.006 mm *AEC = 0.081 mm* *REC = 0.063 mm*
CD = 9.18 mm (Contact Diameter)

The polynomial equation coefficients are displayed along with the contact lens back surface sags:

Sag1 = 0.376 mm at diameter 5 mm
Sag2 = 1.369 mm at diameter 9 mm
Sag3 = 1.871 mm at diameter 10.6 mm

- Left click the *Calculate* steep meridian command button

The screen will now display the fluid lens profile for both meridians from the lens centre to the lens periphery and the following values are displayed on screen for the steep meridian:

Steep Meridian
TLT = 0.006 mm *AEC = 0.142 mm* *REC = 0.108 mm*

Note that the short green lines mark the position of contact on the flat meridian. No consideration is given to the extra edge clearance produced by any edge profile or edge polishing. The display is illustrated in **Figure 11.31**.

On the flat corneal meridian, this lens has a central clearance of 0.006 mm. The contact lens central curve is spherical and is flat in relation to the cornea giving a negative fluid lens. The polynomial surface then experiences a steepening of the curve to give zero clearance at diameter 9.18 mm. The surface then undergoes dramatic flattening in order to generate an axial edge clearance of 0.081 mm.

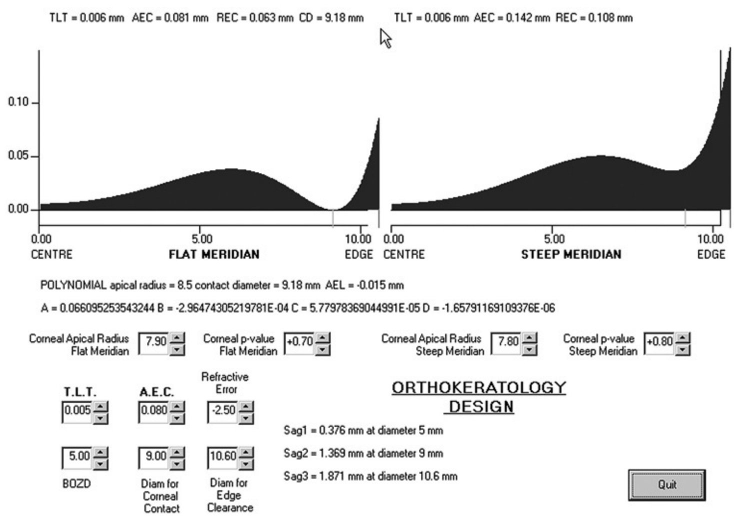

Figure 11.31 The tear layer profile for a polynomial orthokeratology design (Plate 23)

Would a polynomial surface like this one be more or less effective at eliminating refractive error when compared with a multicurve design?

• Left click the mouse on the *Quit* command button

These two orthokeratology programs include appropriate default values. The ability to alter the default values means that a contact lens practitioner has the option to incorporate design modifications.

11.11 Apical radius and p-value from keratometric readings

Mention was made of the use of the Zeiss telecentric keratometer to derive the apical radius and p-value of the corneal principal meridians. One mire extremity has the appearance illustrated in **Figure 5.18**. The centre of the cross is 18° from the instrument optical axis. The small square blocks each subtend angles of 13, 14.75, 16.5, 19.5, 21.25 and 23°.

1. The patient is asked to fix on the central fixation target and a conventional keratometry measurement is made on the two principal corneal meridians (the mean of four readings is recorded).
2. The mire extremities are aligned to the near horizontal principal meridian (mire images in step) and the subject is asked to fix on the innermost of the small squares (13°) and the keratometric reading is made with this oblique fixation (the recorded value is the mean of two readings).
3. The process is repeated with the patient fixating on the 14.75 and 16.5° squares. Finally the patient is asked to fix on the centre of the cross to give a measurement for an 18° oblique fixation. Again each recorded measurement is the mean of two readings.
4. Let us suppose that the mire extremity was along meridian 10 and was on the nasal side. The keratometer is then rotated through 180° so that the extremities are once more along meridian 10 but the hollow cross with the small squares is now on the temporal side. The oblique measurements are repeated.
5. For each oblique target there will be a recorded corneal radius for the nasal and temporal cornea. These two values are averaged to give a single radius reading for each individual fixation target. Thus each oblique radius recorded is the average of four readings.
6. We now have a single radius value for:

   Conventional keratometry
   Oblique keratometry at 13°
   Oblique keratometry at 14.75°
   Oblique keratometry at 16.5°
   Oblique keratometry at 18°

   These five radius values are entered into the program which then calculates the apical radius and the p-value.

To test the program *Asphericity by keratometer* try the following.

- Left click the mouse on each of the five windows and type the following in each window:

Conventional k-reading (mm)	type **7.7**
first tilted radius (mm) at angle 13 degrees	type **7.88**
second tilted radius (mm) at angle 14.75 degrees	type **7.92**
third tilted radius (mm) at angle 16.5 degrees	type **8**
fourth tilted radius (mm) at angle 18 degrees	type **8.05**

- Left click the mouse on the *Calculate* command button

The answer

The apical radius is 7.67 mm
The p-value is 0.69
the coefficient of determination is 0.997

appears on the screen display which is illustrated in **Figure 11.32**. The program has calculated the apical radius and p-value by examining the distance squared versus radius squared relationship. This will be a linear relationship if the corneal section is a conic section. In the case of a perfect conic section the coefficient of determination will be unity. In our example, a coefficient of determination of 0.997 indicates that we have a surface that is very, very close to one that possesses a conic section. The graphical relationship is also drawn as a scatterplot as part of the program display. A perfect

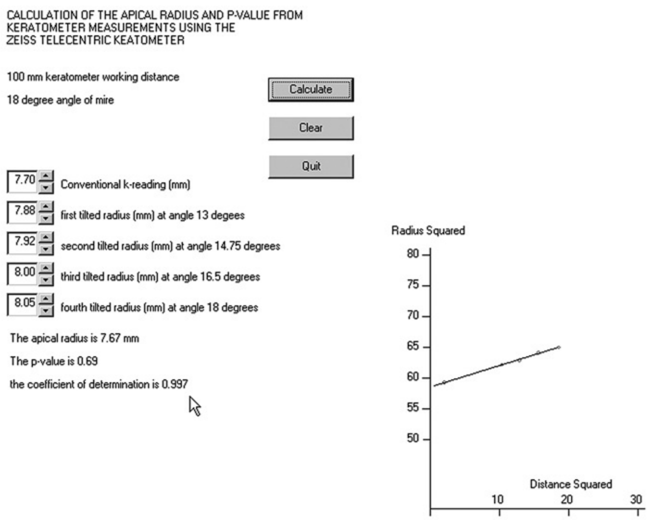

Figure 11.32 The screen display for calculation of the apical radius and asphericity of a corneal section. The coefficient of determination and the scatterplot indicate how close this corneal section approximates to a conic section

coefficient of determination of unity will produce a scatterplot where all five points are on the regression line. Any result with a poor coefficient of determination indicates that the corneal meridian being examined does not approximate to the conic form. In these circumstances, the use of an apical radius and p-value for the purposes of predicting the fitting relationship between the cornea and the contact lens is inappropriate.

• Left click the mouse on the *Quit* command button

11.12 The BVP to order for toric lenses

The calculation of the theoretical BVP to order when fitting either toric soft or toric RGP lenses has been fully described in Chapter 7. The *torics* program uses the principles described in Chapter 7.

11.12.1 Soft toric lenses

This option in the program is simply a calculation of the effective power of the spectacle correction at the eye and assumes a plano fluid lens for both meridians. The program calculates the contact lens BVP based on the spectacle refraction.

• Ensure that the *Toric soft lens* radio button is active
• Left click the mouse on each of the windows and type the following in each window:

Enter the k-reading of the flat meridian	type **8**
Enter the k-reading of the steep meridian	type **7.4**
SPHERE	type **–5**
CYLINDER	type **–3**
Enter spectacle Rx vertex distance	type **12**

• Left click the *Calculate* command button

The program then displays:

BVP along flat meridian = –4.72 D
BVP along steep meridian = –7.3 D

In other words the sphere/cyl power is:

–4.72 DS/–2.58 DC axis along the flat meridian

This is the effective power of the spectacle correction at the eye.

11.12.2 RGP toric lenses

This option in the program calculates the effective power of the spectacle correction at the eye and also calculates the power of the fluid lens for both

the flat and steep meridians. It does not need to discriminate between corneal and lenticular astigmatism.

- Ensure that the *Toric RGP lens* button is active
- Left click the mouse on each of the windows and type the following in each window:

Enter the k-reading of the flat meridian	type **8**
Enter the k-reading of the steep meridian	type **7.4**
SPHERE	type **–5**
CYLINDER	type **–3**
Enter spectacle Rx vertex distance	type **12**
BOZR for corneal flat meridian	type **8**
BOZR for corneal steep meridian	type **7.5**

- Left click the *Calculate* command button

The program then displays:

BVP along flat meridian = –4.72 D
BVP along steep meridian = –6.69 D

In other words the sphere/cyl power is:

–4.72 DS/–1.97 DC axis along the flat meridian

In both the soft and RGP options, the calculated BVP would be rounded to the nearest 0.25 D.

- Left click the mouse on the *Quit* command button

11.13 Toric rotation

One particular problem with toric lenses is unwanted rotation and its influence on refractive correction.

In the case of an RGP compensated toric lens, any unwanted rotation is not a problem because the compensated toric lens corrects its own induced astigmatism. The induced astigmatism is produced by the toric lens back surface and is corrected by the toric lens front surface. Thus any lens rotation will not alter this happy state of affairs.

Unfortunately any unwanted rotation for front surface RGP or any toric soft lenses will produce a situation where crossed cylinders are present and the resulting refractive state will require calculation based on Stokes construction. The program *Rotation* deals with this problem and offers three options. In the case of the soft contact lens, it is assumed that the fluid lens power is plano in both principal meridians due to the soft lens draping over the cornea and adopting the corneal curvature with no change in lens BVP due to flexure. A front surface toric RGP lens can also be considered using

this program but the lens powers will need to be the contact lens plus fluid lens power.

In order to illustrate the value of this program to normal clinical practice we will take a disposable soft toric lens example where the trial lens selected has a BVP which is not that required for either the sphere or the cylinder, the axis orientation is not what is required and the lens suffers unwanted rotation on the eye. This is about as bad as things can get!

The example is in fact the example considered in Section 7.13. We will use this example to work through the three options.

11.13.1 *Option 1 – Calculating the theoretical over-refraction*

The eye being examined has an ocular refraction of

–0.75 DS / –2.00 DC axis 5

The eye is fitted with a disposable toric soft lens which has a BVP of

–0.25 DS / –1.50 DC axis 10

This lens rotates 5° in an anticlockwise direction on the eye giving an axis orientation of 15 (remember that the standard notation scale runs in an anticlockwise direction). What will be the over-refraction in this example?

The screen display for Option 1 is illustrated in **Figure 11.33**.

In order to test the program try the following.

- Ensure that the *Theoretical cylinder* radio button is active
- Left click the mouse on each of the seven windows and type the following in each window:

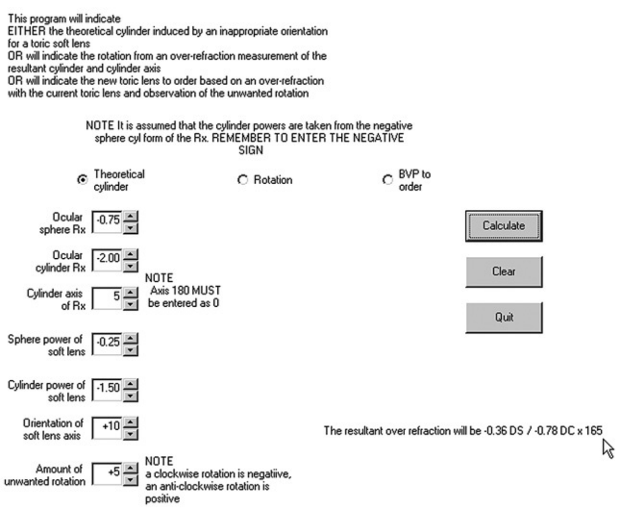

Figure 11.33 Option 1 of the *Rotation* program giving the theoretical over-refraction result when a soft lens takes up an inappropriate orientation

Ocular sphere Rx	type **–0.75**
Ocular cylinder Rx	type **–2**
Cylinder axis of Rx	type **5**
Sphere power of soft lens	type **–0.25**
Cylinder power of soft lens	type **–1.5**
Orientation of soft lens axis	type **10**
Amount of unwanted rotation	type **5**

Note that an anticlockwise rotation is positive because that is the rotation direction of the standard notation scale.

- Left click the mouse on the *Calculate* command button

The answer:

The resultant over-refraction will be –0.36 DS / –0.78 DC axis 165

appears on the screen display which is illustrated in **Figure 11.33**.

- Left click the mouse on the *Clear* command button

11.13.2 Option 2 – Calculating the unwanted rotation

Now we know the over-refraction result, we can use this to calculate the unwanted rotation. The situation can arise where the orientation marks are not visible but it is always possible to acquire an over-refraction result.
In order to test the program try the following.

- Ensure that the *Rotation* radio button is active
- Left click the mouse on each of the five windows and type the following in each window:

Ocular cylinder Rx	type **–2**
Cylinder axis of Rx	type **5**
Orientation of soft lens axis	type **10**
Over-refraction resultant cylinder	type **–0.78**
Over-refraction resultant axis orientation	type **165**

- Left click the mouse on the *Calculate* command button

The answer:

The toric lens has an unwanted rotation of 5 degrees ANTICLOCKWISE

appears on the screen display which is illustrated in **Figure 11.34**.
- Left click the mouse on the *Clear* command button

11.12.3 Option – Calculating the BVP to order

This is the most useful option. We have inserted a disposable soft toric lens that is not really the power that we need and it has taken up an inappropriate

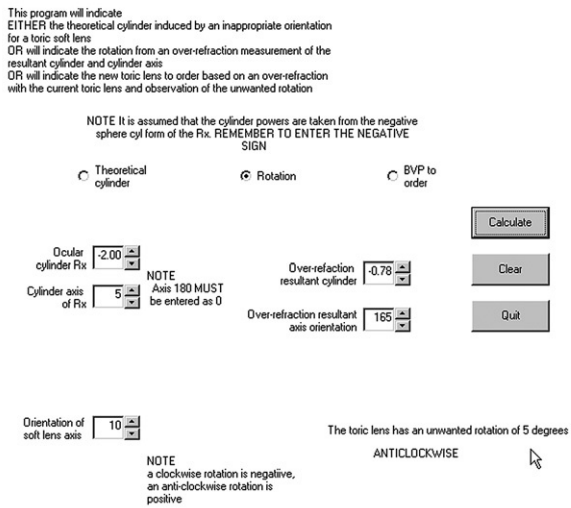

Figure 11.34 Option 2 of the *Rotation* program giving the unwanted rotation derived by comparing the ocular astigmatism with the resultant cylinder power and axis orientation

orientation. Let us suppose that we are happy with the fit but we still need to know what BVP to order.

In order to test the program try the following.

- Ensure that the *BVP to order* radio button is active
- Left click the mouse on each of the eight windows and type the following in each window:

Cylinder axis of Rx	type **5**
Sphere power of soft lens	type **–0.25**
Cylinder power of soft lens	type **–1.5**
Orientation of soft lens axis	type **10**
Amount of unwanted rotation	type **5**
Over-refraction resultant sphere	type **–0.36**
Over-refraction resultant cylinder	type **–0.78**
Over-refraction resultant axis orientation	type **165**

- Left click the mouse on the *Calculate* command button

The answer:

The BVP to order will be –0.75 DS / –2.01 DC axis 0

appears on the screen display which is illustrated in **Figure 11.35**.

Note that orientation *axis 0* is orientation of the cylinder axis 180.

The answer gives the soft lens BVP with a cylinder axis appropriate for a contact lens that has an unwanted rotation of 5° anticlockwise using the rule described in Section 7.9. When this lens rotates 5° anticlockwise on the eye, the correction becomes:

–0.75 DS / –2.01 DC axis 5

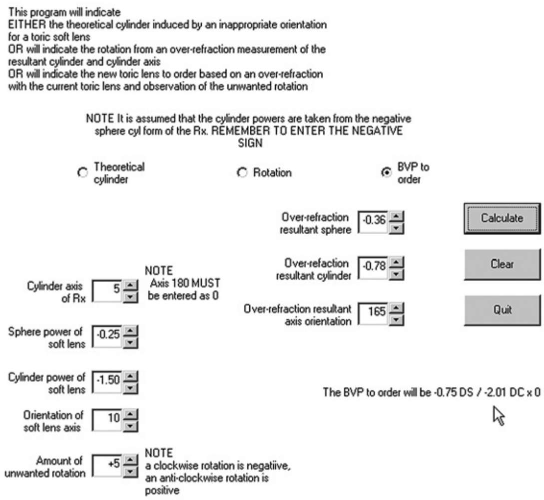

Figure 11.35 Option 3 of the *Rotation* program giving the theoretical BVP to order when the trial lens is of both an inappropriate power and inappropriate orientation on the eye

This is the ocular refraction of the eye being corrected.

• Left click the mouse on the *Clear* command button

11.13.4 A second example

If we take, as a second example, a situation where we have ordered what looks like an appropriate lens only to find that it takes up an inappropriate orientation then we need to decide how to check our results and what to order next.

The eye has an ocular refraction of

–3.25 DS / –2.50 DC axis 175

We order a soft toric lens with a BVP of

–3.25 DS / –2.50 DC axis 175

What will be:

1. the theoretical over-refraction result
2. the assessment of lens rotation based on the over-refraction result
3. the BVP to order for the replacement lens based on the over-refraction result

if the lens rotates 15° anticlockwise.

Repeat the calculations for a rotation of 15° clockwise.

Using the rotation program we come up with the following answers:

Option 1

• Make sure that the *Theoretical cylinder* radio button is active

- Left click the mouse into each window and type the following

Ocular sphere Rx	type **–3.25**
Ocular cylinder Rx	type **–2.5**
Cylinder axis Rx	type **175**
Sphere power of soft lens	type **–3.25**
Cylinder power of soft lens	type **–2.5**
Orientation of soft lens axis	type **175**
Amount of unwanted rotation	type **15**

Remember that an anticlockwise rotation is positive.

- Left click the mouse on the *Calculate* command button

The answer:

The resultant over-refraction will be +0.65 DS / –1.29 DC × 138

is displayed on the screen.

- Now change the

Amount of unwanted rotation window type **–15** (clockwise rotation)

- Left click the mouse on the *Calculate* command button

The answer:

The resultant over-refraction will be +0.65 DS / –1.29 DC × 32

is displayed on the screen.

- Left click the mouse on the *Clear* command button

Option 2

- Make sure that the *Rotation* radio button is active
- Left click the mouse into each window and type the following

Ocular cylinder Rx	type **–2.5**
Cylinder axis Rx	type **175**
Orientation of soft lens axis	type **175**
Over-refraction resultant cylinder	type **–1.29**
Over-refraction resultant axis orientation	type **138**

- Left click the mouse on the *Calculate* command button

The answer:

The toric lens has an unwanted rotation of 15 degrees ANTICLOCKWISE

is displayed on the screen.

- Now change the

Over-refraction resultant axis orientation window type **32**

• Left click the mouse on the *Calculate* command button

The answer:

The toric lens has an unwanted rotation of –15 degrees CLOCKWISE

is displayed on the screen

• Left click the mouse on the *Clear* command button

Option 3

• Make sure that the *BVP to order* radio button is active
• Left click the mouse into each window and type the following

Cylinder axis of Rx	type **175**
Sphere power of soft lens	type **–3.25**
Cylinder power of soft lens	type **–2.5**
Orientation of soft lens axis	type **175**
Amount of unwanted rotation	type **15**
Over-refraction resultant sphere	type **.65**
Over-refraction resultant cylinder	type **–1.29**
Over-refraction resultant axis orientation	type **138**

• Left click the mouse on the *Calculate* command button

The answer:

The BVP to order will be –3.23 DS / –2.52 DC × 160

is displayed on the screen.

This checks out with the original ocular refraction for a lens that has rotated 15° in an anticlockwise direction which moves the axis from 160 to 175.

• Now change the

Amount of unwanted rotation	type **–15**
Over-refraction resultant axis orientation	type **32**

• Left click the mouse on the *Calculate* command button

The answer:

The BVP to order will be –3.23 DS / –2.52 DC × 10

is displayed on the screen.

This checks out with the original ocular refraction for a lens that has rotated 15° in a clockwise direction which moves the axis from 10 to 175.

• Left click the mouse on the *Quit* command button

This completes the contact lens programs that have been written to take the tedium out of lengthy calculations. Hirji (1989) and Rabbets (1993) have

suggested the alternative approach of using a computer spreadsheet for designing contact lenses. This allows the practitioner to design a spreadsheet to meet individual needs. For any reader wishing to explore this avenue I recommend the paper by Ronald Rabbetts (1993) who uses a similar approach to lens design to that used in this chapter.

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